

ADVANCED PRACTICAL
ARITHMETIC

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THE ADVANCED
PRACTICAL ARITHMETIC

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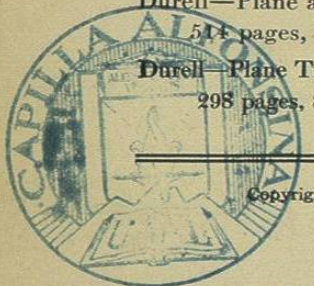
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PREFACE

THE object in preparing this arithmetic has been the same which the authors had in view when writing their school algebra, viz.: "to show more plainly, if possible, than has been done heretofore, the practical or common sense reason for each step or process." It is believed that this treatment is not only "adapted to the practical American spirit, but also gives the study of the subject a larger educational value." It is also believed that the scholarly possibilities and value of the treatment, instead of being diminished, are increased thereby. For instance, it is hoped that the matter presented in the chapters on the Applications of Percentage and Interest, and on Arithmetical History, will have a new value.

The main principle which has governed the authors in writing the book has also been the controlling factor in the treatment of many matters of detail which are still in debate among teachers of arithmetic and writers on the subject.

Almost all are agreed that the study of arithmetic should begin with the study of *concrete objects*; and that the use of *geometric diagrams* is a great help in presenting certain parts of the subject, as fractions. But the authors believe that it is a mistake to have the concrete objects, or even pictures of them, constantly before the pupil. Nor should diagrams be printed as part of the text. The pure arithmetical processes should be made easy and natural as soon as possible, on account of their brevity and simplicity. Con-

crete objects or diagrams, if kept before the eye constantly, tend to clog and hamper the mind; hence they should be recalled only so often and at such places as may be necessary in order to make the subject vivid and real.

In the same way *algebraic symbols and methods* have been introduced only when they give a clear and pronounced advantage, and thus arouse in the pupil the desire to know more of them.

It is believed that *rules* for processes are useful in many ways when they are arrived at after the proper preliminary work and when they are used with discretion.

In order to cultivate habits of *analysis* and exact statement, and yet to prevent these from becoming mere mechanical rote processes, different forms of analysis have been used adapted to different kinds of work. These are indicated by the use of different words as "Operation," "Explanation," "Solution," etc.

In like manner, *oral exercises* are sometimes put before written exercises, sometimes after them.

The subject-matter is not spirally arranged, but is adapted to spiral study. The subject is presented as an organic whole, yet one which can be learned by successive steps (see p. 6).

A large number of examples adapted to the theory of the book has been made and carefully graded. Especial attention is also called to the chapter on the Metric System.

The authors will be glad to receive any corrections or suggestions from teachers using this book.

FLETCHER DURELL,
EDWARD R. ROBBINS.

LAWRENCEVILLE, N. J., }
PHILADELPHIA, PA., }
May 1, 1901.

TO THE TEACHER.

1. The teacher should make sure at different times that the pupil carries in mind the *concrete object for which a symbol stands*. Now show the pupil, now have him show the concrete object. Show him diagrams illustrating the properties of fractions. Have him make these diagrams. But do this only *occasionally*, and always for some good reason.

2. In *oral work* and *explanations* insist on *careful and accurate statements*. For instance, in oral work do not allow a pupil to give the answer merely without a formal statement of the analysis or steps by which the result was obtained.

3. In *written problems* in which the *analysis is difficult* (as in Exercise 23) elicit the analysis from the pupil by oral questions, and afterward have the pupil write out the analysis and solution.

4. Insist on the use of *cancellation* wherever possible. Train the pupil to combine all the operations required in the solution of a problem in a *comprehensive plan* or scheme; then to factor and cancel wherever possible; never to multiply till it is necessary to do so.

5. Train the pupil also to make a *rough estimate* or forecast of the answer before beginning the exact numerical work. This not only tends to eliminate large errors, but is also a valuable habit, since, in practical life, fully one-half the applications of arithmetic are made in this way.

6. Impress upon the minds of pupils in various ways the *local value of digits* and the *limitations in the accuracy* of all arithmetical work based on measurements (see Arts. 19, 77, 78).

7. Study to vary methods to suit the needs of different pupils, both in presenting topics and in meeting difficulties. It is to be remembered that pupils as they come from different homes probably have more varied capacities with respect to the subject of arithmetic than to any other.

8. Do not be satisfied till by long practice and working innumerable examples, if necessary, the pupil has become a rapid and accurate computer. The power of handling figures with facility and accuracy is of the first importance both in practical life and in its influence on the further educational development of the pupil.

9. REVIEW CONSTANTLY.

SCHEDULE RECOMMENDED IN USING DURELL AND ROBBINS' ARITHMETIC.

YEAR.	FIRST HALF-YEAR.	SECOND HALF-YEAR.
1	<i>Oral work without text-book.</i>	<i>Elementary Pract. Arith.</i> , pp. 1-32 (much of it read to pupil by teacher; supplemented by other oral work).
2	<i>Elementary Pract. Arith.</i> , pp. 1-32. Review and second course.	<i>Elementary Pract. Arith.</i> , pp. 33-77 (more difficult parts of some lessons omitted).
3	<i>Elementary Pract. Arith.</i> , pp. 33-77. Review and second course.	<i>Elementary Pract. Arith.</i> , pp. 78-121 (more difficult parts of some lessons omitted).
4	<i>Elementary Pract. Arith.</i> , pp. 55-121. Reviewed and completed in details.	<i>Elementary Pract. Arith.</i> , pp. 122-194 (leading principles and easier exercises).
5	<i>Elementary Pract. Arith.</i> , pp. 91-194. Reviewed and completed in details.	<i>Advanced Pract. Arith.</i> to p. 158 (leading principles and easier exercises).
6	<i>Advanced Pract. Arith.</i> to p. 158 (fuller course).	<i>Advanced Pract. Arith.</i> , pp. 158-255 (leading principles and easier exercises).
7	<i>Advanced Pract. Arith.</i> , pp. 158-255 (fuller course).	<i>Advanced Pract. Arith.</i> , pp. 256 to end (leading principles and easier exercises).
8	<i>Advanced Pract. Arith. Review.</i> Rapid review to p. 256. Fill in details, pp. 256 to end.	<i>Elementary Algebra. Geometrical Drawing.</i>

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ARITHMETIC.

CHAPTER I.

NUMBER. NUMERATION. NOTATION.

1. Units.—For many purposes the most convenient way of dealing with quantity (as, for instance, with the length of a given line) is to take a certain definite part of the given quantity as a unit, and determine the number of times the unit must be used in order to make up the given quantity (or line).

Thus, in determining the length of a given linear object, as a rope, we do not depend merely on general impressions of its magnitude (formed by the eye or by moving the hand over it), but by taking a unit, as one inch or one foot, and determining the number of times the unit must be used in order to make up the line.

A boy dealing with a quantity of marbles in his possession does not do so merely by means of the aggregate impression which they make in his pocket, but by taking a single marble as a unit, and counting the number of marbles which he has.

This method of regarding quantity as made up of units gives greater ease and precision in all the ordinary uses made of an aggregate of material.

A unit is a certain quantity taken as a standard of reference when dealing with quantity of the same kind.

2. Kinds of Units.—Units are of different kinds.

Natural units are those which occur in the world about us, as one apple, one man, one year, one day.

Artificial units do not occur naturally, but are devised