# ARITHMETIC.

## CHAPTER I.

### NUMBER. NUMERATION. NOTATION.

**1. Units.**—For many purposes the most convenient way of dealing with quantity (as, for instance, with the length of a given line) is to take a certain definite part of the given quantity as a unit, and determine the number of times the unit must be used in order to make up the given quantity (or line).

Thus, in determining the length of a given linear object, as a rope, we do not depend merely on general impressions of its magnitude (formed by the eye or by moving the hand over it), but by taking a unit, as one inch or one foot, and determining the number of times the unit must be used in order to make up the line.

A boy dealing with a quantity of marbles in his possession does not do so merely by means of the aggregate impression which they make in his pocket, but by taking a single marble as a unit, and counting the number of marbles which he has.

This method of regarding quantity as made up of units gives greater ease and precision in all the ordinary uses made of an aggregate of material.

A **unit** is a certain quantity taken as a standard of reference when dealing with quantity of the same kind.

**2. Kinds of Units.**—Units are of different kinds.

**Natural** units are those which occur in the world about us, as one apple, one man, one year, one day.

**Artificial** units do not occur naturally, but are devised by man so as to extend the advantages arising from the use of units as widely as possible, as one foot, one-third of an apple, etc.

A **primary** unit is a single unit of a given kind, as one dollar.

A **derived** unit is an aggregate of single units, as five dollars (a "V"); or a part of a unit, regarded as a new unit, as one-third of a dollar.

A unit of one kind may become, in certain relations, a unit of another kind. Thus, an artificial unit may become, in some senses, a natural unit, as one dollar. Also, a derived unit may come to be regarded as a primary unit, as one week, one quarter (of a dollar).

### EXERCISE 1.

1. What unit of length is used in measuring the length of a room? The length of a pencil? Of a quantity of cloth?
2. What unit of length is used in measuring the distance between two cities? The diameter of the earth?
3. With what unit of capacity is milk measured? Grain? Strawberries? Potatoes?
4. Which unit of area is used in stating the size of a farm? Of a county?
5. Which of the following units are natural and which are artificial: year, second, week, foot, quart, yard, peck, mile, month, degree?

**3. Number** is a unit, or collection of like units.

When quantity is regarded as made up of like units, it becomes a number. Thus, when an aggregate of apples is regarded as made up of distinct apples, it becomes a number of apples.

Thus, also, when a line is regarded as made up of inches, it becomes a number of inches.

**4. Arithmetic** is the science which treats of number. It investigates the most advantageous ways of expressing quantities as numbers, and of using numbers when formed.

**5. Number Words.**—When we have determined a quantity as made up of units, and ascertained the number of the units in a given quantity, it is often useful to transfer



the number idea thus formed, to other persons, and thus give them a definite conception of the quantity dealt with, without labor on their part. Hence, words are useful by which to designate different aggregates of units, or numbers.

Number words are useful also to the person using them, in calling up the precise ideas connected with each aggregate of units.

The words used for the different aggregates of units (beginning with a single unit) are—

*one, two, three, four, five, six, seven, eight, nine, ten.*

For larger aggregates of units a system of grouping units and naming the groups formed is used, which is explained later.

**6. Counting** is the process of affixing to any group of units the number word belonging to that group, beginning with unity, and affixing its number word to each group, till the last unit of the entire group dealt with is reached.

**7. Number Symbols.**—Further economies and additional power in dealing with numbers are obtained by using a distinct *symbol* for each number apart from its number word. Thus, for the number words

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | one, | two, | three, | four, | five, | six, | seven, | eight, | nine, |
| we use | 1, | 2, | 3, | 4, | 5, | 6, | 7, | 8, | 9. |

These number symbols, or **figures**, have advantages as compared with number words, in that they are easier to write, and to recognize when written. They have many other derived advantages, when used in combinations, both in denoting and in operating with, numbers larger than nine.

The number symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 are called the nine **digits**. The absence of number is denoted by a symbol, 0, called **zero, naught,** or **cipher.**

Zero is sometimes regarded as a number.

**8. Large Numbers.**—In order to denote large numbers by words and symbols, it is necessary to devise a plan of so grouping units that a few words or symbols systematically used will represent any number, however large. It is plainly impracticable to denote each different number by an entirely new and distinct word or symbol.

## NUMERATION.

**9. Numeration** is the process of grouping an aggregate of units according to a convenient, systematic plan, and of naming the groups so formed; or briefly, numeration is the expression of numbers in words.

**10. Decimal System of Numeration.**—Let us suppose a heap of like objects, as silver dollars, and let us suppose that we desire to determine the number of these objects, and to express the number of them in words in a convenient, systematic way. We first count ten of the dollars and set them aside as a single group (equivalent to a ten-dollar bill), then count ten more dollars and set them aside, and continue making like groups until the number of dollars left is less than ten. Suppose eight tens are formed and six dollars are left. By thus forming groups of ten each, and regarding each such group as a new unit of a higher order, we can express the given group of units (or number of dollars) in words without employing any new number word beside those already given (Art. 5), except a word to denote the new unit group of higher order—viz., *ten*. For the number of dollars in the original heap is expressed in words as eight tens and six units of (or eighty-six) dollars.

Similarly, if there are ten or more groups of the new unit groups of higher order (*i. e.*, of groups of ten dollars each) in

NOTE.—The number ten is used because most of our savage ancestors counted by aid of their ten fingers. Hence the number ten became the primary group in numeration, and has been so used ever since. Any other number (except unity) might be used as the primary group in numeration. Two, six, eight, and twelve are among those which have been suggested, of which, twelve, perhaps, would be the best.



the original heap, we regard ten ten-units taken together as a new unit group of still higher order, and call it one *hundred*. Similarly ten hundreds are regarded as a new unit group of higher order and called a *thousand*.

**11. Numbers Larger than One Thousand.**—Similarly we may form other new unit groups, each ten times as great as the preceding, and called one *ten thousand*, one *hundred thousand*, one *million*, one *ten million*, etc. But in denoting these groups (greater than one thousand) entirely new number words are used only for those groups which are one thousand times as great as the group denoted by the last preceding new number word, as *million* (one thousand times as great as one thousand), *billion* (one thousand times as great as one million), *trillion*, etc. The intermediate unit groups are denoted by using "ten" and "hundred" as modifiers to other number words.

**12. Number Words Actually Used.**—Beside the number words already given, it is found convenient to use a few others, though these are not actually necessary. Thus, some number words are formed by using two primary numbers and fusing them into a single word.

Thus, for "ten" and "one" we have "eleven" (formed by fusing the Gothic words for one and ten, *ain lif*); for "ten" and "two" we have "twelve" (the Gothic words for two and ten, *twa lif*, fused); for "ten" and "three" we have "thirteen" (for "three" and "ten" fused). Similarly we obtain "fourteen," "fifteen," "sixteen," "seventeen," "eighteen," "nineteen." Also, for "two tens" we have "twenty," by fusion of the words "two" and "ten." Similarly are obtained "thirty," "forty," etc.

Hence the number words in actual use are *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*, *ten*, *eleven*, *twelve*, *thirteen*, *fourteen*, *fifteen*, *sixteen*, *seventeen*, *eighteen*, *nineteen*, *twenty*, *thirty*, *forty*, *fifty*, *sixty*, *seventy*, *eighty*, *ninety*, *hundred*, *thousand*, *million*, *billion*, *trillion*, *quadrillion*, *quintillion*, *sextillion*, etc.

By the systematic use of these few words any number may be expressed in words.

**13. Orders of Units.**—Thus, in the decimal numeration, we use a *unit*, and a series of *derived units*, each ten times as great as the preceding—viz., *one*, *ten*, *hundred*, *thousand*, *ten thousand*, etc.

These units are of different orders.

*One* is called the unit of the *first order;*
*ten* is called the unit of the *second order;*
a *hundred* is called the unit of the *third order*, etc.

In naming any number we begin with the highest order, and state the number of units of each order which the given number contains. We speak, for example, of the number "three thousand, six hundred, seventy-two."

## NOTATION.

**14. Notation** is the process of expressing a number in symbols according to a convenient, systematic plan.

Having grouped an aggregate (or number) of units according to a scale (the decimal scale, for instance), and given names to the number groups so as to express the number in words to other persons, we need also to express these groups in simple symbols so as to facilitate the extended use of the number.

**15. Positional System of Notation.**—The first nine numbers are denoted by the nine digits (Art. 7). A simple method of expressing larger numbers in symbols is illustrated if we express the number "three thousand, six hundred, seventy-two" as follows: 3672.

Here the number of units of each order is denoted by the appropriate digit (the thousands by 3, hundreds by 6, etc.), and the size of the unit for which each digit stands is indicated by writing in a row the digits employed, the highest order to the left, each successive lower unit group being one place to the right. The simplicity and power of this system



of notation should be carefully noted by the pupil and frequently recalled.

The simplicity is due to the fact that in denoting a number by figures, as 3672, each of the digits 6, 7, 2 has not only its own value, but is also employed to determine the order of the unit group denoted by 3—viz., thousands; similarly, 7 and 2 define the order of the unit denoted by 6—viz., hundreds, etc. Hence, when for 3 thousand, 6 hundred, 7 tens, and 2 units, we write 3672, the word "thousand" is replaced by 672, "hundred" by 72, "tens" by 2, units by the absence of another digit after 2. Hence, for instance, the symbol 2 as here used has four uses; it has its own value, and it helps determine the value of 3, 6, and 7. It is because of this manifold use of each symbol that we are able to substitute the four symbols of 3672 for the thirty-three symbols which compose the expression "three thousand, six hundred, seventy-two."

It is also to be noted that the symbolism 3672 is uniform in arrangement and spacing, while the expression of the number in number words is irregular in form and spacing.

These great advantages in expressing numbers in symbols give ease and power in the extended use of numbers and make a thorough science of numbers possible.

The student is aided to a full appreciation of the advantages of the positional decimal system of notation by comparing it with others that have been used to some extent, as the Roman notation (Art. 23 *et seq.*).

**16. Zero Symbol in the Positional Notation.**—When units of one or more orders do not occur in a given number, the absence is indicated by the use of the zero symbol in each place where such a unit is missing.

Thus, 5042 represents a number containing 5 thousands, 4 tens, and 2 units, but no hundreds.

**17. Number vs. Number Symbols.**—The student should carefully discriminate between a *number* and the *symbols* or words which represent a number.

Thus, a number (which is an aggregate or collection of units, as a heap of apples), may exist long before any words or symbols are used to represent it. It may also be represented by different sets of symbols, as by "twelve," or 12, or xii. These are not different numbers, but only different symbols for the same number. However, for the sake of brevity, the expression "number denoted by the figures 3276" is shortened into "the number 3276," but the student is not to be misled into regarding the number and 3276 as identical.

**18.** The **place** of a figure (in a given number) is the position which the figure occupies with reference to the other figures in the number. Thus, in the number 3672, the figure in the right-hand place, 2, is said to occupy the *first* place; 7, the *second* place; 6, the *third* place, etc.

Hence, moving a figure one place to the left increases its value tenfold; but moving a figure one place to the right divides its value by ten.

**19. Absolute and Local Value.**—The value of each figure in a number is determined by two things:

*First*, the value of the figure without regard to its position, called its **absolute** (or digit) value;

*Second*, the value given the figure by the place it occupies in the number, called its **local** value.

Thus, in 3672, the figure 6, for instance, has an absolute value, in that it represents 6 units, and a local value, in that each of the units represented by it is the hundred unit.

The student should not, as often happens, unconsciously form the habit of regarding the digits which form a given number as of equal importance and significance in a numerical result. This habit often arises perhaps from the fact that the digits as written are of equal size, and local value apparently neglected. He should frequently substitute (mentally) for 2, in 3672, a figure only one-tenth as large as 2, leave 7 unchanged, substitute for 6 a figure ten times as long as it is, for 3 one a hundred times as long. Or he may picture, back of 7, seven bundles of ten strokes each, back of 6, six bundles, each composed of ten ten-bundles, etc.

## NUMERATION AND NOTATION.

**20. Periods.**—In order to write and read large numbers with facility, it is customary to separate the different orders of units used into sets of three each, called **periods**. Periods



are formed by beginning at the right and marking off three figures in each period by the use of a comma. In reading numbers it is customary to express the aggregate of each period in terms of the lowest unit in that period.

| Number of Period. | 6th. | 5th. | 4th. | 3d. | 2nd. | 1st. |
|---|---|---|---|---|---|---|
| Name of Period. | Quadrillions. | Trillions. | Billions. | Millions. | Thousands. | Units. |
| Order of Units. | Hundreds of, Tens of, Units of | Hundreds of, Tens of, Units of | Hundreds of, Tens of, Units of | Hundreds of, Tens of, Units of | Hundreds of, Tens of, Units of | Hundreds of, Tens of, Units. |
| Number = | 5 4, | 2 0 3, | 6 7 5, | 4 0 0, | 0 7 6, | 5 4 2. |

The number expressed in symbols is

54,203,675,400,076,542,

and is read,

Fifty-four quadrillion, two hundred three trillion, six hundred seventy-five billion, four hundred million, seventy-six thousand, five hundred forty-two.

The names of the periods above quadrillions are quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, etc.

In actual practice, however, periods of the higher orders are little used.

**21.** I. **To express in figures a number given in words.**

*Write the proper figure for the number of units of each order, putting a zero in each vacant place. Mark the figures off into periods of three figures each, beginning at the right.*

Ex. Express in figures the number three billion, five hundred six million, seven thousand, twenty-two. We obtain 3,506,007,022.

**22.** II. **To express in words (*i. e.,* to read) a number given in figures.**

*By use of commas and beginning at the right, separate the figures given into periods of three figures each. Beginning at the left, read each group, giving it the name of the period to which it belongs.*

*Omit the name of the units period in reading.*

Ex. Read 5062380749.

We have 5,062,380,749, which is read, five billion, sixty-two million, three hundred eighty thousand, seven hundred forty-nine.

## EXERCISE 2.

Read:

*1.* 28.
*2.* 27.
*3.* 63.
*4.* 92.
*5.* 125.
*6.* 378.
*7.* 554.
*8.* 107.
*9.* 705.
*10.* 450.
*11.* 910.
*12.* 711.
*13.* 818.
*14.* 666.
*15.* 1352.
*16.* 3128.
*17.* 4201.
*18.* 3700.
*19.* 4025.
*20.* 7030.
*21.* 8004.
*22.* 13456.
*23.* 74901.
*24.* 28074.
*25.* 30212.
*26.* 30077.
*27.* 60103.
*28.* 70007.

*29.* 63360 inches.
*30.* 97056 men.
*31.* 38020 miles.
*32.* 25003 days.
*33.* 86400 seconds.
*34.* 10101 tons.

*35.* 129345.
*36.* 704508.
*37.* 201009.
*38.* 300102.
*39.* 295004.
*40.* 300071.
*41.* 3564320.
*42.* 13705028.
*43.* 37564005.
*44.* 20024106.
*45.* 10902070.
*46.* 703201001.
*47.* 250341702.
*48.* 402000271.
*49.* 300070005.
*50.* 777505003.
*51.* 909090909.
*52.* 65004030.

*53.* 1305217456.
*54.* 17271005301.
*55.* 298012003819.
*56.* 435710302456081.
*57.* 300310070004255.
*58.* 8000500123005760.



Write in words:

*59.* 750.
*60.* 342.
*61.* 1500.
*62.* 3027.
*63.* 2006.
*64.* 7102.
*65.* 30201.
*66.* 65311.
*67.* 82005.
*68.* 90102.
*69.* 88217.
*70.* 57008.
*71.* 214008.
*72.* 703307.
*73.* 1575014.
*74.* 20501310.
*75.* 42001025.
*76.* 120320020.
*77.* $5071.
*78.* 9003 ft.
*79.* $40267.
*80.* $21006.
*81.* 4001 days.
*82.* $250405005.

*83.* Write the largest number that can be expressed by three figures; by six figures. Read each of them.

*84.* Write the smallest number that can be expressed by five figures; by eight figures. Read each of them.

Express in figures:

*85.* Thirty-seven.
*86.* Eighty-three.
*87.* One hundred forty.
*88.* Two hundred ten.
*89.* Nine hundred eleven.
*90.* One thousand six.
*91.* Four hundred seventeen.
*92.* Six hundred ninety-three.

*93.* Eight hundred twenty-five.
*94.* Two thousand four hundred sixty-one.
*95.* Five thousand two hundred eight.
*96.* Seven thousand three hundred twenty.
*97.* Nine thousand five hundred.
*98.* Twelve thousand two hundred sixty.
*99.* Seventeen thousand six hundred one.
*100.* Twenty-three thousand ninety-seven.
*101.* Forty thousand three hundred nineteen.
*102.* Seventy-one thousand three.
*103.* Eighty thousand eleven.
*104.* One hundred two thousand four hundred twelve.
*105.* Three hundred twenty-seven thousand seventeen.
*106.* Four hundred thousand two hundred five.
*107.* Seven hundred seven thousand seventy-seven.
*108.* Seventy-seven thousand seven hundred seven.

*109.* Six million one hundred seven thousand four hundred sixty-nine.

*110.* Twelve million two hundred nineteen thousand eighty-one.

*111.* Three hundred eleven million seven hundred sixteen thousand four hundred forty-four.

*112.* Six hundred million two thousand fifteen.

*113.* Eleven million eleven thousand eleven.

*114.* Seven billion twenty million fourteen thousand sixty.

## ROMAN NOTATION.

**23. Number Symbols of Roman Notation.**—Beside the system of numeration and notation already explained (commonly called the Arabic system, owing to the fact that the peoples of Europe first learned it through the Arabs), there is another system still used to some extent, called the Roman system, because of its origin among the Romans.

The Roman system of notation uses seven capital letters of the alphabet as number symbols—viz.,

I, V, X, L, C, D, M.

To these, in order, the following values are assigned:

1, 5, 10, 50, 100, 500, 1000.

**24. Combination of Number Symbols in the Roman Notation.**—When the above symbols are used in combination, the value of each symbol in a combination is determined by the following laws:

*1. Each repetition of a letter repeats its value.*

Thus, XXX denotes 30, CC denotes 200, etc.

*2. When a letter is placed after another letter of greater value, its value is to be added to that of the greater letter.*



Thus, VI represents 5 + 1, or 6; XVI denotes 16; LXXXI denotes 81; DCC = 700.

*3. When a letter is placed before another letter of greater value, its value is taken from that of the greater letter.*

Thus, IV denotes 4; XL denotes 40; XC denotes 90.

A letter *between* two letters, each of which is of greater value than itself, is regarded as preceding the last letter.

Thus, XIV denotes 14; XIX denotes 19.

*4. A bar (or dash) placed over a letter increases its value one thousand fold.* Hence we have

| *Thousands.* | *Hundreds.* | *Tens.* | *Units.* |
|---|---|---|---|
| M | C | X | I (=1) |
| MM | CC | XX | II (=2) |
| MMM | CCC | XXX | III (=3) |
| $\overline{\text{IV}}$ | CD | XL | IV (=4) |
| $\overline{\text{V}}$ | D | L | V (=5) |
| $\overline{\text{VI}}$ | DC | LX | VI (=6) |
| $\overline{\text{VII}}$ | DCC | LXX | VII (=7) |
| $\overline{\text{VIII}}$ | DCCC | LXXX | VIII (=8) |
| $\overline{\text{IX}}$ | CM | XC | IX (=9) |

**25. Uses of the Roman System of Notation.**—The Roman system of notation is used at times in connection with other systems to prevent confusion when several different groupings of an aggregate of material are made. Thus, Arabic numerals are used in numbering the articles of this book, and the Roman numerals in numbering the chapters.

Roman numerals are also used on monuments and formal documents to give variety and distinction.

The Roman system of numeration also has an educational value. It is useful during the study of arithmetic to compare processes in the Arabic notation with what they would be in the clumsy Roman notation, in order to appreciate the simplicity and power of the former.

### EXERCISE 3.

Express in Arabic notation—

*1.* XV.
*2.* XX.
*3.* XXIV.
*4.* XXXII.
*5.* XIX.
*6.* XXIX.
*7.* XLIV.
*8.* LVI.
*9.* LXVIII.
*10.* LXXIX.
*11.* XCI.
*12.* XCIV.
*13.* CXVI.
*14.* CXLIX.
*15.* CLXXXIV.
*16.* CCXCIX.
*17.* CDLVI.
*18.* DCIX.
*19.* MCXLVII.
*20.* MCCXLIX.
*21.* MDXC.
*22.* MDCXLIII.
*23.* MDCCCXCVIII.
*24.* MMDCXLIX.
*25.* IVCDXLIV.
*26.* XDCCXXVI.
*27.* XLVCCLXVI.
*28.* DXCVIII.
*29.* MCCLCXLV.
*30.* MMDCCXVDCXXI.

Express in Roman notation—

*31.* 18.
*32.* 27.
*33.* 39.
*34.* 46.
*35.* 58.
*36.* 72.
*37.* 93.
*38.* 98.
*39.* 111.
*40.* 120.
*41.* 147.
*42.* 375.
*43.* 421.
*44.* 490.
*45.* 567.
*46.* 719.
*47.* 984.
*48.* 1302.
*49.* 1492.
*50.* 1776.
*51.* 1865.
*52.* 2674.
*53.* 200468.
*54.* 1321894.

# CHAPTER II.

## ADDITION.

**26. Illustration.**—If James has 5 apples and John has 4 apples, how many apples have they together?

If we take the 5 apples belonging to James and count on to them the 4 apples which John has, we get 6, 7, 8, 9 apples; that is, as final result, 9 apples. Or, if we are familiar with the results of counting together small groups, we may simply recall the result of a former counting together and say 5 apples and 4 apples are 9 apples.

In the latter case we substitute the less labor of recollection for the greater labor of counting the groups together. By the use of the memory we utilize the work which we have done at some former time, to obtain the number of units in two groups when taken together.

This process is called *addition.*

**27. Definitions.**—**Addition** is the process of obtaining in the simplest way a single number which shall contain as many units as there are units in two or more given numbers taken together.

The **sum** is the number obtained as the result of an addition.

The **addends** are the numbers added.

**28. Symbols.**—The symbol or sign used to denote addition is the erect cross, +, which reads "*plus.*" It means that the numbers between which it is placed are to be added.

The symbol, =, reads "equals," and is placed between two numbers to indicate that they are equal. Hence, it may be employed to denote the *equality* between a sum and the numbers added.

Thus, $5 + 4 = 9$, reads "5 plus 4 equals 9."