EXERCISE 3.

Express in Arabic notation—

1. XV.
2. XX.
3. XXIV.
4. XXXII.
5. XIX.
6. XXIX.
7. XLIV.
8. LVI.
9. LXVIII.
10. LXXIX.
11. XCI.
12. XCIV.
13. CXVI.
14. CXLIX.
15. CLXXXIV.
16. CCXCIX.
17. CDLVI.
18. DCIX.
19. MCXLVII.
20. MCCXLIX.
21. MDXC.
22. MDCXLIII.
23. MDCCCXCVIII.
24. MMDCXLIX.
25. IVCDXLIV.
26. XDCCXXVI.
27. XLVCCLXVI.
28. DXCVIII.
29. MCCLCXLV.
30. MMDCCXVDCXXI.

Express in Roman notation—

31. 18.
32. 27.
33. 39.
34. 46.
35. 58.
36. 72.
37. 93.
38. 98.
39. 111.
40. 120.
41. 147.
42. 375.
43. 421.
44. 490.
45. 567.
46. 719.
47. 984.
48. 1302.
49. 1492.
50. 1776.
51. 1865.
52. 2674.
53. 200468.
54. 1321894.

CHAPTER II.

ADDITION.

26. Illustration.—If James has 5 apples and John has 4 apples, how many apples have they together?

If we take the 5 apples belonging to James and count on to them the 4 apples which John has, we get 6, 7, 8, 9 apples; that is, as final result, 9 apples. Or, if we are familiar with the results of counting together small groups, we may simply recall the result of a former counting together and say 5 apples and 4 apples are 9 apples.

In the latter case we substitute the less labor of recollection for the greater labor of counting the groups together. By the use of the memory we utilize the work which we have done at some former time, to obtain the number of units in two groups when taken together.

This process is called *addition.*

27. Definitions.—**Addition** is the process of obtaining in the simplest way a single number which shall contain as many units as there are units in two or more given numbers taken together.

The **sum** is the number obtained as the result of an addition.

The **addends** are the numbers added.

28. Symbols.—The symbol or sign used to denote addition is the erect cross, +, which reads "*plus.*" It means that the numbers between which it is placed are to be added.

The symbol, =, reads "equals," and is placed between two numbers to indicate that they are equal. Hence, it may be employed to denote the *equality* between a sum and the numbers added.

Thus, 5 + 4 = 9, reads "5 plus 4 equals 9."

29. Addition Table.—So convenient is the system of numeration and notation used for representing numbers, that all numbers, however large, may be resolved into digits, and the sum of any numbers obtained by taking the sums of pairs of digits. Hence, if the sum of each pair of digits be obtained and committed to memory, the addition of all larger numbers may be performed by their use. We have 1 unit + 1 like unit = 2 units (of the same kind), or, briefly, $1+1=2$; also, $1+2=3$, $1+3=4$, etc. Or, putting the pairs of digits in the position in which the pupil will need to use them, and leaving the sum in each case to be supplied by him, we have—

ADDITION TABLE.

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9
2	2	2	2	2	2	2	2	2
1	2	3	4	5	6	7	8	9
3	3	3	3	3	3	3	3	3
1	2	3	4	5	6	7	8	9
4	4	4	4	4	4	4	4	4
1	2	3	4	5	6	7	8	9
5	5	5	5	5	5	5	5	5
1	2	3	4	5	6	7	8	9
6	6	6	6	6	6	6	6	6
1	2	3	4	5	6	7	8	9
7	7	7	7	7	7	7	7	7
1	2	3	4	5	6	7	8	9
8	8	8	8	8	8	8	8	8
1	2	3	4	5	6	7	8	9
9	9	9	9	9	9	9	9	9
1	2	3	4	5	6	7	8	9

30. Addition Independent of Order (Commutative).—If a group of units (as a group of 8 boys) be counted, it is evident that the same numerical result (or number) will be obtained, in whatever order the units be counted. Since addition is but a short way of counting different groups together, it follows that two or more given groups may be added together in any order. Hence, $8+7$ gives the same result as $7+8$, and the Addition Table in Art. 29 gives the sum of each pair of digits, in whatever order the digits occur. To be able to add in either way frequently saves labor.

31. Abstract and Concrete Number.—The work of dealing with numbers is further facilitated by the use of the idea of abstract numbers. For if we dealt with concrete units only, as marbles, apples, men, etc., as we find them in the world about us, we should, for instance, need to verify the addition table for each particular kind of concrete quantity before using the table in adding numbers composed of units of that sort.

Or, to put it in another way, in order to be sure that 7 + 5 makes 12 under all circumstances, it would be necessary to take 7 apples and 5 apples and count them together and obtain 12 apples; to take 7 oranges and 5 oranges and count them together and obtain 12 oranges; and to proceed in like manner with marbles, bushels, men, and every kind of concrete units. Instead of this, we take 7 units of any kind and represent them, say, by 7 strokes, and 5 units of the same kind represented by 5 other strokes, and count them together and obtain 12 units of the same kind, as the sum—a result true for like units of any particular kind.

Hence, if we learn the addition table for units in general, we may then use it in adding numbers made up of like units of any particular kind.

A **concrete number** is a number made up of like units of any one particular kind.

An **abstract number** is a number used without reference to any particular thing or unit;

as when we say,

7 units + 9 like units = 16 units of the same kind.

Hence, while the addition table is given for abstract numbers, it applies only to concrete numbers of the same kind.

Thus, it is not possible to add 5 days and 6 apples.

32. Arrangement of Numbers to be Added.—The great value of the positional system of denoting numbers is forcibly illustrated in the process of adding large numbers.

Thus, if a merchant has $623 in one bank, $9024 in another, and $151 in another, and it is required to determine how many dollars he has in all the banks together, we can arrange the three numbers one over the other, putting units of the same order in the same column, and perform the addition by adding each column of units separately. This could not be done if the Roman notation were used in denoting the above numbers.

33. I. **Addition when the sum of each column is less than 10** may be illustrated by the use of the example stated in the preceding article.

Setting down the numbers with like units in the same column, we have—

Operation.	Explanation.
623 dollars. 9024 " 151 " 9798 dollars, *Sum.*	Adding the units column first, we have 1 and 4 are 5, and 5 and 3 are 8; we set down the sum 8 under the units column. Adding the tens column, we obtain the sum 9, which we set down in the tens place. The sums of the other columns are obtained and set down similarly.

Abstract numbers are added in the same manner.

Thus,

5762 units.
2125 like units.
10112 " "
17999 like units, *sum.*

The enormous saving of labor obtained by the use of the addition table is realized if we conceive of trying to obtain this result by counting merely—that is, by taking 5762 units and counting on 2125 units, and then counting on 10112 units to the result obtained.

34. II. **Addition when the sum of any one column is greater than 9** is illustrated by the following:

Ex. A farmer owns three tracts of land, of which the first contains 598 acres, the second 1236 acres, and the third 8759 acres. How many acres does the farmer own?

Operation.	Explanation.
598 acres. 1236 " 8759 " 10593 acres, *Sum.*	Arranging the numbers as before, and adding the units column, we obtain the sum 23. The 3 is put under the column of units, and the 2 tens are carried and added with the other tens. The sum of tens column (together with the 2 tens that are carried) is then obtained, and found to be 19. The 9 is set down and 1 (or 10 tens, *i. e.*, 1 hundred) carried to the hundreds column. Proceeding in like manner with the other columns, the entire sum is found to be 10593 acres.

35. Verification.—To prevent error in the work, it is best to perform each process of addition in at least two ways, and observe whether the results are identical. The second process is called a *verification* of the first. There are several different methods of verifying an addition, but the best for ordinary use is to *add the given number by columns in an opposite direction from that first used;*

As, first from the bottom upward, and then from the top downward. If a column be added the second time in an opposite direction from the first, the computer is much more likely to discover any mistake that may have been made, than by simply adding a column twice in the same direction, since in the latter case a mistake made in the first addition is likely to be repeated.

An addition may also be verified by separating into groups the numbers to be added, adding each group separately, and then taking the sum of the partial sums obtained.

36. General Rule for Addition.—*Write the numbers to be added so that figures of the same order shall stand in the same column; begin at the right and add each column separately, placing the sum underneath if it is less than ten; if the sum of any column exceeds nine, set down the right-hand figure only, and add the other figure to the next column to the left.*

EXERCISE 4.

Add:

1.	*2.*	*3.*	*4.*	*5.*
24 men.	71 miles.	132 hours.	375 boys.	$7356
52 men.	26 miles.	427 hours.	603 boys.	$1422

6.	*7.*	*8.*	*9.*	*10.*
34659	281367	546923	475013	73 men.
40140	707532	440013	503876	18 men.

11.	*12.*	*13.*	*14.*	*15.*
95 pens.	58 pages.	77 balls.	876 books.	$5734
47 pens.	65 pages.	58 balls.	309 books.	$6189

16.	*17.*	*18.*	*19.*	*20.*
35914	558093	307697	797512	98765
85377	483307	985457	123389	56789

21.	*22.*	*23.*	*24.*	*25.*
351 ft.	453 lines.	710 rods.	628 marks.	$14313
173 ft.	536 lines.	349 rods.	359 marks.	$76823
428 ft.	792 lines.	594 rods.	482 marks.	$55855

26.	*27.*	*28.*	*29.*	*30.*
34598	446789	512417	47565	98763
71623	615323	387694	62317	89689
24534	558394	757575	89497	78959

31.	*32.*	*33.*	*34.*	*35.*	*36.*
123	612	4567	2349	53427	77777
597	759	8912	3716	43251	86546
638	387	3456	5438	70698	71234
245	621	7890	3251	37056	56789
761	348	1357	7791	83799	66666

37.	*38.*	*39.*	*40.*
87653	95673	459543127	45325987654
12761	80170	745241436	91357048653
30508	19787	538579375	67324537167
39175	95068	477889901	45757863448
42078	49632	664543176	37666539399
66779	47819	836842566	85765684784

41.	*42.*	*43.*	*44.*	*45.*
27561	328005	35742	935684	138
3425	2796	35	71063	76
9	30	1276	29	924
342	420	328	576	92576
2700	7651	14514	3217	4231
6021	38	38	550	47
756	2071	1207	341256	561

46.	*47.*	*48.*	*49.*	*50.*
3278	9123	1179	6846	8784
8673	7168	7856	7845	7383
1075	3127	3412	3374	6486
3218	6503	3876	4063	3179
4716	9076	6719	5732	1954
7509	3795	5043	6118	2832
4123	4038	6132	9475	4085
8340	9987	7168	5684	5644
9999	7698	1876	7893	6756
6327	4389	1999	4952	7278
8909	6070	9871	3107	8393
5632	4395	7040	1736	9428
3067	7778	9328	8321	4595
1678	8666	9937	9614	6976
3915	4791	7064	7578	7833

37. Addition as a Science and as an Art.—The simplifications which arise from treating quantity as made up of *units;* from the grouping aggregates of units according to a simple systematic plan, so that they can be denoted by a *few number words* and a *few number symbols;* from the use of the system of *positional notation;* from the resulting possibility of performing the additions of all numbers, however large, by the use of the *addition table*—these simplifications together result in making addition all that can be desired as a practical *science.*

For by it, for instance, the general of an army in his tent can determine (and have before him in a form easy to comprehend and use) a representation of the number of men in each part and the whole of his army. A government can by it readily determine the number of its school-children or population, or state its wealth in numbers, etc.

But to be mastered as an *art,* the process of addition requires long practice. To add long columns of figures with absolute accuracy and great rapidity is a power which is obtained only after long and varied practice. The next exercise gives examples adapted to develop this power.

With practice the student will form habits (often instinctive and peculiar to the individual) of adding the figures in a column in certain special ways.

Thus, he may add them in groups of two, three, or four figures; or he may pick out in a column each group of figures that make 10 or 20, add these by themselves, add the other digits by themselves, and take their sum; or he may add two columns at a time. Practice and attention are the main factors which go to form a skillful calculator.

EXERCISE 5.

Let the teacher dictate numbers of one figure to the class, to be added mentally during the dictation and the sum reported immediately. Thus:

1. Add . . . 7, 1, 6, 3, 5, 4, 5, 8, 7, 4, 3, 8; sum is 61.
2. Add . . 8, 4, 3, 1, 7, 9, 6, 5, 3, 1, 7, 9, 6, 5, 6; sum is 80.
3. Add . . . 4, 5, 2, 9, 7, 1, 6, 5, 3, 7, 8, 9, 1, 8, 4, 6, 5; sum is 90.

EXERCISE 6.

1. In six bins there are 45 bu., 82 bu., 96 bu., 124 bu., 43 bu., and 215 bu. How many bushels are there in all?

2. A man owns a farm of five fields which contain 23, 46, 51, 17, and 30 acres respectively. How many acres in the farm?

3. During the six days of one week a merchant received on sales the following amounts: $765, $350, $917, $479, $807, $987. What was the total for the week?

4. There are ten schools in a city, and they enroll 171, 230, 165, 187, 301, 287, 517, 176, 215, 351 pupils respectively. How many school-children in that city?

5. In a township there are six farms which contain 175, 400, 236, 355, 278, 196 acres. How many acres in the township?

6. Find the total expenses of running a bank, if the items for a week are as follows: Salaries and wages $875, postage $11, rent $46, stationery $23, printing $40, books $8, and legal fees $127.

7. A man at death left $4500 to the widow, $1635 to each of three sons, and $958 to a daughter. What was the value of the estate?

8. From A to B is 812 miles, from B to C is 406 miles, from C to D is 615 miles, and from D to A is 786 miles. What is the distance around the whole circuit?

9. Let the teacher give the number of days in each of the months, and the class find the number of days in a year.

10. Let each pupil tell the number of people in his family, and then the whole class find the number of people in all the homes.

11. In the same way let each report the number of examples he has solved, and then the class compute the aggregate.

12. Direct each pupil to count the letters in his full name. Then by telling the number of them, the class can find the

number of letters it will take to write the full names of all the members of the class.

13. From the geographies or elsewhere, find the population of each of the New England States. Then find the total.

14. Find the same for the Middle States and for the South Atlantic States.

15. Find the population of the capital of your own State, and of the capitals of all the States which touch it, and then find the total.

16. Find the number of square miles in the six largest States, and then the aggregate.

17. Add seventy-six, three hundred nine, twelve thousand six hundred ten, and forty thousand sixteen.

18. The English army at Waterloo consisted of 26661 infantry, 8735 cavalry, 6877 artillery, and 33413 allies. What was the total?

19. A man owns bonds worth $43765, real estate worth $37050, merchandise valued at $17980, and other property worth $50379. What is the total value of his property?

20. New York contains 49170 sq. mi.; New Jersey, 7815; Pennsylvania, 45215; Delaware, 2050; Maryland, 12210; Virginia, 42450; West Virginia, 24780; and Texas contains 82090 sq. mi. more than all of these put together. How many square miles has Texas?

21. Add 753284 + 95603 + 887653 + 47328 + 867547 + 37895 + 90384 + 7056 + 19948 + 38756 + 938765.

22. Add 77563 + 987635 + 447 + 88956 + 327654 + 887654 + 963558 + 79658 + 9976 + 885432 + 796 + 147785.

CHAPTER III.

SUBTRACTION.

38. Illustration.—John has 7 marbles and gives James 4 of them. How many marbles has John left?

If we take a group of 7 marbles, and remove 4 marbles one at a time, counting off 6, 5, 4, 3, we obtain 3 marbles as the number of marbles left.

But if we are familiar with results of former countings-off, and can recall these, we can say that if 4 marbles be taken from 7 marbles, 3 marbles will be left.

This process is called *subtraction.*

Or we can recall from the addition table the number which, added to 4, makes 7, and say: since 4 + 3 makes 7, when 4 is taken from 7 the number 3 must be left. In either of these two latter processes we substitute the less labor of memory for the greater labor of counting off one number from another.

39. Definitions.—Subtraction is the process of finding with least labor what number is left when a number of units is taken away from a larger number of units of the same kind.

The larger number is called the **minuend.**

The smaller number, to be taken from the minuend, is called the **subtrahend.**

The number left is called the **difference** or **remainder.**

Thus, in the illustrative example of Art. 38, we have

7 marbles, *Minuend.*
4 marbles, *Subtrahend.*
3 marbles, *Difference.*

40. The sign of subtraction is the horizontal dash, —, which reads "minus." Placed between two numbers the