number of letters it will take to write the full names of all the members of the class.

13. From the geographies or elsewhere, find the population of each of the New England States. Then find the total.

14. Find the same for the Middle States and for the South Atlantic States.

15. Find the population of the capital of your own State, and of the capitals of all the States which touch it, and then find the total.

16. Find the number of square miles in the six largest States, and then the aggregate.

17. Add seventy-six, three hundred nine, twelve thousand six hundred ten, and forty thousand sixteen.

18. The English army at Waterloo consisted of 26661 infantry, 8735 cavalry, 6877 artillery, and 33413 allies. What was the total?

19. A man owns bonds worth $43765, real estate worth $37050, merchandise valued at $17980, and other property worth $50379. What is the total value of his property?

20. New York contains 49170 sq. mi.; New Jersey, 7815; Pennsylvania, 45215; Delaware, 2050; Maryland, 12210; Virginia, 42450; West Virginia, 24780; and Texas contains 82090 sq. mi. more than all of these put together. How many square miles has Texas?

21. Add 753284 + 95603 + 887653 + 47328 + 867547 + 37895 + 90384 + 7056 + 19948 + 38756 + 938765.

22. Add 77563 + 987635 + 447 + 88956 + 327654 + 887654 + 963558 + 79658 + 9976 + 885432 + 796 + 147785.

CHAPTER III.

SUBTRACTION.

38. Illustration.—John has 7 marbles and gives James 4 of them. How many marbles has John left?

If we take a group of 7 marbles, and remove 4 marbles one at a time, counting off 6, 5, 4, 3, we obtain 3 marbles as the number of marbles left.

But if we are familiar with results of former countings-off, and can recall these, we can say that if 4 marbles be taken from 7 marbles, 3 marbles will be left.

This process is called *subtraction.*

Or we can recall from the addition table the number which, added to 4, makes 7, and say: since 4 + 3 makes 7, when 4 is taken from 7 the number 3 must be left. In either of these two latter processes we substitute the less labor of memory for the greater labor of counting off one number from another.

39. Definitions.—Subtraction is the process of finding with least labor what number is left when a number of units is taken away from a larger number of units of the same kind.

The larger number is called the **minuend.**

The smaller number, to be taken from the minuend, is called the **subtrahend.**

The number left is called the **difference** or **remainder.**

Thus, in the illustrative example of Art. 38, we have

7 marbles, *Minuend.*
4 marbles, *Subtrahend.*
3 marbles, *Difference.*

40. The sign of subtraction is the horizontal dash, —, which reads "minus." Placed between two numbers the minus sign means that the second number is to be subtracted from the first.

Thus, 9 − 5 reads "nine minus five," and means that 5 is to be subtracted from 9.

41. Subtraction Table.—Just as addition is performed to the best advantage by committing to memory certain primary sums (viz., the sum of each pair of digits), and performing the addition of all larger numbers by their use, so subtraction is performed to the best advantage by committing to memory certain primary differences, and performing the subtraction of all larger numbers by their use.

Thus, from 7 units we count off 4 like units and get 3 as a remainder, and, to save the labor of again counting off, commit the result to memory, 7 − 4 = 3.

Similarly we obtain and commit to memory every difference in which the subtrahend and remainder are both single digits. Arranging these differences in a table, placing the minuend over the subtrahend as they usually occur in actual subtraction, and leaving it to the pupil to supply the remainders, we have the table on the opposite page.

So convenient is the system of numeration adopted that numbers, however large, may readily be resolved into digits and pairs of digits, and all subtractions performed by means of this table.

42. I. Subtraction when each digit of the subtrahend is less than the corresponding digit of the minuend.

The process is illustrated by the following example:

Ex. Subtract 345 from 597.

Operation.	Explanation.
597, *Minuend.*	We place the subtrahend under the minuend so that units of the same order shall stand in the same column. Beginning at the right, 5 units from 7 units leaves 2 units, and we write 2 in the units place; 4 tens from 9 tens leaves 5 tens, and we write 5 in the tens place; 3 hundreds from 5 hundreds leaves 2 hundreds, and we write 2 in the hundreds place. Hence, we obtain the remainder 252.
345, *Subtrahend.*	
252, *Difference.*	

SUBTRACTION TABLE.

1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1
2	3	4	5	6	7	8	9	10	11
2	2	2	2	2	2	2	2	2	2
3	4	5	6	7	8	9	10	11	12
3	3	3	3	3	3	3	3	3	3
4	5	6	7	8	9	10	11	12	13
4	4	4	4	4	4	4	4	4	4
5	6	7	8	9	10	11	12	13	14
5	5	5	5	5	5	5	5	5	5
6	7	8	9	10	11	12	13	14	15
6	6	6	6	6	6	6	6	6	6
7	8	9	10	11	12	13	14	15	16
7	7	7	7	7	7	7	7	7	7
8	9	10	11	12	13	14	15	16	17
8	8	8	8	8	8	8	8	8	8
9	10	11	12	13	14	15	16	17	18
9	9	9	9	9	9	9	9	9	9

43. II. Subtraction when any figure of the subtrahend is greater than the corresponding figure of the minuend.

In this case, before subtracting, increase the figure in the minuend, which is too small, by borrowing a unit from the digit of next higher order of the minuend.

Ex. 1. Subtract 129 from 653.

Operation.	Explanation.
653	Since we cannot subtract 9 units from 3 units, we take or borrow 1 ten from 5 tens and add it to the three units; then 9 units subtracted from 13 units leaves 4 units, which we write in the units place of the remainder. We now subtract the 2 tens from the 4 tens which
129	
524, *Remainder.*	

remain after taking away 1 ten, and obtain 2 tens, which we write in the tens place; 1 hundred taken from 6 hundreds leaves 5 hundreds. Hence, the entire remainder is 524.

It may be necessary to borrow several times in succession.

Ex. 2. Subtract 2358 from 5346.

OPERATION.	EXPLANATION.
5346	For 8 from 16 leaves 8.
2358	5 " 13 " 8.
2988, *Remainder.*	3 " 12 " 9.
	2 " 4 " 2.

Instead of borrowing a higher unit from the next figure of the minuend, the subtraction may be performed by adding 1 to the corresponding unit of the subtrahend. The result will be the same, since if two numbers be equally increased (as by the addition of 1 ten to the 4 tens and 2 tens in Ex. 1 above, making them 5 tens and 3 tens), the difference will remain unchanged. The process performed in this latter way is slightly easier, since addition is easier than subtraction.

Thus, in Ex. 1 the method of the subtraction would be

9 from 13 leaves 4.	In Ex. 2, 8 from 16 leaves 8.
3 " 5 " 2.	6 " 14 " 8.
1 " 6 " 5.	4 " 13 " 9.
Difference is 524.	3 " 5 " 2.
	Difference is 2988.

44. Verification.—To test the accuracy of the work, add the difference and the subtrahend. Their sum should equal the minuend.

Thus, in Ex. 2 above, add 2358 and 2988; their sum is 5346. Hence, the difference obtained, 2988, is correct unless mistakes have been made in the two processes, of such a nature that they compensate. This is not likely to occur. There is another similar method of verification which the student should discover for himself.

45. General Rule for Subtraction.—*Write the subtrahend under the minuend, placing units of the same order in the same column; begin at the right and subtract each figure of the subtrahend from the corresponding figure of the minuend, and place the result beneath;*

If any figure of the subtrahend is less than the corresponding figure of the minuend, increase the latter by 10, *and subtract; to compensate, diminish by* 1 *the figure of the next higher order in the minuend (or increase by* 1 *the figure of next higher order in the subtrahend), and continue the process.*

EXERCISE 7.

	1.	*2.*	*3.*	*4.*	*5.*
From	67 men.	357 boys.	531 balls.	256 mi.	614
take	25 men.	135 boys.	311 balls.	245 mi.	110

	6.	*7.*	*8.*	*9.*	*10.*
From	265 men.	948 boys.	728 balls.	876 mi.	840
take	161 men.	446 boys.	315 balls.	463 mi.	637

	11.	*12.*	*13.*	*14.*	*15.*	*16.*	*17.*
From	53	41	63	106	358	502	752
take	19	26	45	37	176	387	173

	18.	*19.*	*20.*	*21.*	*22.*	*23.*
From	362	543	760	924	571	5071
take	194	367	368	367	478	1837

	24.	*25.*	*26.*	*27.*	*28.*
From	4705	13756	21504	34576	28765
take	3846	8607	8476	16268	9175

	29.	*30.*	*31.*
From	7265432	301705401	2706510547308
take	3514765	170643053	1607432813954

32. From 71532056176032 take 47063127159374.

33. From 6755307165322 take 2946834073163.

34. Subtract 17650321470063280 from 29560732165032761.

35. Subtract 63075324107695472 4 from 850325076504032080.

36. Take 1234567890987654321 from 5432101234567890123.

46. Computers' Method of Subtraction.—There is another method of subtraction much used by professional computers, which the pupil should at least understand. It is illustrated by the ordinary process of making change. Thus, if a storekeeper receives a dollar bill in payment of a bill of 78 cents, he makes change by paying out first 2 cents, which, with the 78 cents, makes 80 cents; and then paying out 2 dimes, which, with the 80 cents, makes $1. By this method the required subtraction is converted into and performed as addition. In like manner any subtraction may be performed as an addition.

Thus, to subtract 723 from 968, we have—

968	Since 3 and 5 are 8.
723	2 " 4 " 6.
245, *Remainder.*	7 " 2 " 9.

We set down 245 as the remainder. Similarly

653	9 and 4 are 13.
129	2 " 2 " 4.
524, *Remainder.*	1 " 5 " 6.

47. Value of Subtraction.—The student should frequently call to mind the saving of labor effected by subtraction as compared with other processes of determining a remainder.

Thus, if a merchant knows that his original stock of potatoes was 1000 bushels, and his records show that he has sold 627 bushels, by a simple subtraction, and without the labor of counting off the number of bushels sold from the original number, or the labor of actual measurement of the number left, he can tell the number of bushels remaining, and whether he can supply a customer who wants 400 bushels.

Thus, also, if he has bought a hogshead of molasses containing 63 gallons, and has sold 27 gallons, by subtraction he can determine the number of gallons left, without the labor of actually measuring the remainder in gallons and counting them.

EXERCISE 8.

1. The following accounts were each paid with a dollar bill: how much change was due in each case?

40 cts.	60 cts.	14 cts.	85 cts.	61 cts.	57 cts.
55 cts.	70 cts.	27 cts.	39 cts.	78 cts.	83 cts.

2. If a bicycle cost $87 and sold for $98, how much was gained?

3. A house cost $3205 and sold for $3052. Find the loss.

4. A merchant having 2712 yards of cloth sold 1907. How many yards remained?

5. A farmer who raised 1600 bushels of corn retained 205 bushels. How many bushels did he sell?

6. I paid $110 for a horse and $78 for a wagon. I sold both for $169. Did I gain or lose, and how many dollars?

7. An official receives $2315 salary and $1692 in fees. He spends $2865. How many dollars does he save?

8. A grain dealer bought in one week 76321 bushels of grain, and in the next 33478 bushels. He then sold 67305 bushels. How many remained?

9. America was discovered in 1492. How many years was that before you were born? How many years was that before the year 2000?

10. A farmer bought a horse for $231, and harness for $87. He sold the horse for $256, and the harness for $54. How many dollars did he lose altogether?

11. Of 3728 men in an army, 276 were wounded, 193 were killed, and 705 deserted. How many remained at duty?

12. An estate of $23675 was divided among a widow who received $8525, a son who got $756 less than the widow, and a daughter who received the remainder. What was the daughter's part?

13. Three men invest $25600. The first invests $7356; the second $1728 more than the first. How much does the third invest?

14. Thomas Jefferson was born in 1743 and died in 1826.

How old was he? How old would he have been if he had lived till 1900?

15. From the sum of 6175 and 2857 take their difference.

16. I receive \$27, \$42, \$69, \$121, and pay out \$73, \$29, \$11, \$7, and \$130. How much remains?

17. There have been subscribed toward a million dollars by Mr. A. \$26310, by Mr. B. \$42225, by Mr. C. \$61700, by Mr. D. \$54655, by Mr. E. \$112950, and by Mr. F. \$87605. How much remains to be raised?

Find the number of dollars remaining in the bank in each of these three cases:

18.		*19.*		*20.*	
Deposits.	Withdrawals.	Deposits.	Withdrawals.	Deposits.	Withdrawals
\$137	\$38	\$75	\$195	\$9721	\$46
341	142	132	38	328	375
273	67	41	92	5263	8
564	9	67	140	56	417
	156	328	7	8	1376
	225	576	18	46	4251
	46		4	575	3765
	7		56	1250	48
	11		123		4
					976
					23

Compute the values of—

21. 18 + 15 − 26 + 17 − 30 + 16.

HINT.—Take the sum of those preceded by a + sign and of those preceded by a − sign; subtract the latter from the former. Thus, 18 + 15 + 17 + 16 = 66; 26 + 30 = 56; 66 − 56 = 10, *Ans.*

22. 35 + 19 − 26.

23. 75 − 23 + 14.

24. 96 − 48 − 27.

25. 67 + 84 − 125.

26. 128 − 104 + 71.

27. 376 − 291 + 167.

28. 895 − 397 − 299.

29. 39 − 65 + 42 + 31 − 28.

30. 501 − 373 − 192 + 215.

31. 983 + 185 − 467 − 324.

32. 5768 − 4297 + 3008.

33. 59 − 43 + 97 − 101 + 38.

34. 87 − 75 − 9 + 108 − 79 + 40.

35. 131 − 118 + 46 − 28 + 137 − 95.

36. 1767 + 487035 − 397516 + 42765.

37. 895632 − 765107 + 143200 − 97653 − 8765.

38. From nine hundred seven take seven hundred nine.

39. Subtract six thousand five hundred sixty-three from fourteen thousand one hundred eight.

40. To seven hundred sixteen add three hundred ninety and six thousand seventy-five. From this sum take three thousand two hundred ninety-nine.

41. Subtract the sum of five thousand forty-seven and seven hundred twenty, from the sum of four thousand six hundred and three thousand one hundred eight.

42. From the sum of twenty-six thousand eight hundred forty-two and ninety-three thousand four hundred eighty-two, take the difference between four hundred six thousand forty-five and two hundred ninety-six thousand three hundred nine.

Compute the values of:

43. 75 − (12 + 37).

44. 96 − (28 + 51).

45. (96 − 28) + 51.

46. 29 + (75 − 19).

47. 300 − (175 + 98).

48. 975 − (328 + 400 − 275).

49. 788 − 275 − (300 − 96).

50. 1887 − 438 + 756 − 432.

51. 1887 − (438 + 756 − 432).

52. 976 − (85 + 176) − (276 − 88).

53. 8865 − (775 + 896 − 483) − (99 + 387).

54. (99765 + 73876 − 47956) − (88763 − 47958 + 38176).

NOTE.—Let the pupil's parent furnish more examples like the first seventeen of this exercise. Compare altitudes of mountains; population of cities; of States. Pupils can often form examples for each other, and then correct the papers or slates of one another.

CHAPTER IV.

MULTIPLICATION.

48. Illustration.—A woman buys 7 yards of cloth at $4 a yard. How many dollars does she pay for the cloth?

$4
4
4
4
4
4
4
$28

The cost of the cloth may be obtained by addition; the sum of a column of seven 4's is 28.

If, however, the student is familiar with the results of former additions of columns composed of the same digit, he may remember that seven 4's added make 28. It is easier to recall the result of the former addition than to add the column again. We substitute the less labor of recollection for the greater labor of addition and for the still greater labor of counting together the different sets of 4 units each.

Similarly the sum of any set of equal numbers may be found by recalling the results of former additions. This process is called *Multiplication.*

49. Multiplication is the process of finding the sum of a set of numbers, all equal to each other, by the abbreviated method of recalling the results of former additions.

The **multiplicand** is one of the equal numbers which are to be added.

$4, *Multiplicand.*
7, *Multiplier.*
$28, *Product.*

In the example given, $4 is the multiplicand.

The **multiplier** is the number which indicates how many equal numbers are to be added.

In the above example, 7 is the multiplier.

The **product** is the result obtained by the multiplication.

In the above example, $28 is the product.

The multiplicand and multiplier are called the *factors* of the product.

Multiplication is usually viewed in its abbreviated form, and may then be defined as follows:

Multiplication is the process of finding a number (the product) which shall equal another number (the multiplicand) repeated as many times as there are units in a third number (the multiplier).

50. The **sign of multiplication** is the inclined cross, ×. Placed between two quantities it means that the one is to be multiplied by the other. Thus 4 × 7 means "4 multiplied by 7," or "7 multiplied by 4" (that is, "4 times 7"). When the numbers to be multiplied are placed one over the other, the lower one is regarded as the multiplier.

51. Multiplication Table.—If the product of each pair of digits be obtained and committed to memory, the product of all other numbers, however large, may be obtained by the use of these few primary products.

For, so convenient is the system of numeration and notation which we have adopted, that all numbers, however large, may be resolved into digits, and their products obtained by taking the products of different pairs of digits.

While it is sufficient to know the products of pairs of numbers up to 9, it is convenient to extend the table a little further, and to learn the product of each pair of numbers up to 12.

By the addition of columns of like digits the following results are obtained:

MULTIPLICATION TABLE.

Twice	Three times	Four times	Five times	Six times	Seven times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

Eight times	Nine times	Ten times	Eleven times	Twelve times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

Since multiplication occupies the leading place in almost every process in arithmetic, the multiplication table should be so thoroughly mastered that the pupil can give instantly, without a moment's reflection, the product of any pair of digits. In committing the table to memory he will be aided by various simple expedients, thus:

In the *table for 5*, each product ends in 5 or 0.

In the *table for 9*, the sum of the digits of each product is 9 (except in 99), the tens digit increasing 1 and the units digit decreasing 1 in each successive product.

In the *table for 11*, the two digits in each product are alike up to 99.

The *labor of committing the table to memory is also diminished one-half* by remembering that, for instance, the product 9×7 is the same as 7×9.

EXERCISE 9.

ORAL.

1. There are 4 quarts in a gallon. How many quarts in 7 gallons? In 9 gallons? In 12 gallons?

2. There are 7 days in a week. How many days in 5 weeks? In 7 weeks? In 11 weeks? In 12 weeks?

3. How many working days are there in 8 weeks? In 10 weeks? In 12 weeks?

4. If I study 9 hours each day, how many hours will I study in 4 days? In 6 days? In 9 days? In 11 days?

5. If wood is worth $5 a cord, what must be paid for 3 cords? For 5 cords? For 9 cords? For 12 cords?

6. A boy spends $8 a month. How much will he spend in 4 months? In 6 months? In 9 months?

7. If 1 bucket of water costs 0 cents, what will 5 buckets cost? 7 buckets? 12 buckets?

8. What is the product of 8×7? 9×4? 11×6? 7×6? 5×9? 8×5? 12×9? 5×12? 4×7? 11×12? 8×9?

9. Find the value of $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$ without the table. Then by multiplication. Which is easier?

Compute the values of each of the following:

10. $3 \times 2 \times 5$.
11. $4 \times 3 \times 8$.
12. $5 \times 2 \times 9$.
13. $6 \times 2 \times 7$.
14. $7 \times 1 \times 8$.
15. $2 \times 5 \times 8$.
16. $3 \times 4 \times 2$.
17. $3 \times 3 \times 8$.
18. $8 \times 1 \times 12$.
19. $6 \times 2 \times 11$.
*20.** $4 \times 3 + 7$.
21. $6 \times 7 + 1$.
22. $3 \times 8 - 5$.
23. $4 \times 9 - 8$.
24. $9 \times 5 + 7$.
25. $2 \times 9 - 6$.
26. $3 \times 8 - 7$.
27. $9 \times 7 - 5$.
28. $7 \times 7 - 7$.
29. $8 \times 8 + 10$.
30. $5 \times 6 - 5$.
31. $7 \times 8 + 6$.
32. $4 \times 12 - 8$.
33. $11 \times 12 + 1$.
34. $9 \times 9 - 9$.
35. $7 \times 9 + 5$.
36.† $9 - 2 \times 3$.
37. $27 - 4 \times 6$.
38. $4 + 7 \times 3$.
39. $8 + 5 \times 6$.
40. $9 + 12 \times 3$.
41. $30 - 7 \times 4$.
42. $60 - 5 \times 12$.
43. $70 - 6 \times 11$.
44. $4 + 8 \times 12$.
45. $8 + 9 \times 6$.
46. $3 + 10 \times 5$.
47.‡ $2 + 3 \times 5 - 7$.
48. $12 - 3 \times 2 - 2$.
49. $5 \times 5 - 5 + 8$.
50. $14 - 6 + 3 \times 4$.
51. $3 \times 0 + 7 \times 2$.
52. $8 \times 7 - 6 \times 4$.
53. $7 \times 6 + 9 \times 1$.
54. $9 \times 12 - 8 \times 6$.
55. $7 - 3 \times 2 + 6 \times 8$.
56. $3 \times 4 + 5 - 2 \times 7$.
57. $9 \times 0 + 5 \times 12 - 8$.
58. $2 \times 5 \times 7 - 3 \times 4 \times 4$.
59. $8 - 2 \times 3 - 7 \times 0 + 4 \times 8$.
60. $13 + 7 \times 2 - 3 \times 5 + 6 \times 3$.
61. $75 - 7 \times 5 - 6 \times 6 + 6$.
62. $30 + 7 \times 9 - 8 \times 7 + 4 \times 0$.
63. $100 - 7 \times 6 - 5 + 4 \times 12 - 20$.

52. Use of Abstract and Concrete Numbers in Multiplication.—The use of abstract numbers is of the first importance in multiplication.

For if we made no use of abstract number and dealt with concrete numbers only, as marbles, apples, dollars, feet, etc., it would be necessary, for instance, to verify the multiplication table for each particular kind of concrete number before using the table to multiply that particular kind of

* In each example, the multiplication must be performed first.

† This is $9 - 6 = 3$. ‡ This is $2 + 15 - 7 = 10$.

concrete number. But if we form the multiplication table for abstract number (represented, for instance, by strokes or dots), we may then apply the table to any particular kind of concrete number.

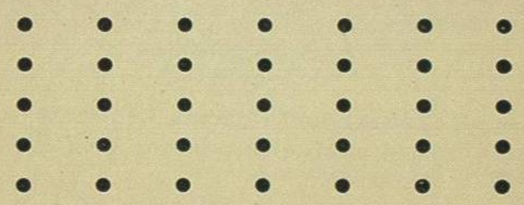
In the use of abstract and concrete number, certain limitations, however, are to be observed. Thus, the multiplicand may be either an abstract or concrete number, but the number of times the multiplicand is taken (that is, the multiplier) must be an abstract number. Thus, 7 apples can be multiplied by 8, but not by 8 marbles. Hence,

1 *The multiplicand may be either an abstract or concrete number.*

2. *The multiplier must be an abstract number.*

3. *The product is the same kind of number as the multiplicand;* i. e., *if the multiplicand is abstract, the product is abstract; if the multiplicand is concrete, the product is concrete and of the same kind.*

From these laws it follows that there are certain limitations also in interchanging the multiplicand and multiplier. If both are abstract numbers, it is evident that they are interchangeable. For if we have a series of 5 rows, each containing 7 dots, and each dot stand for an abstract unit, then the



total number of dots in the group is denoted either by 7×5 or 5×7; that is, *the factors of an abstract number are commutative.* Hence, we can perform the multiplication of two abstract numbers in either order, as may be most advantageous.

But if, of two factors, one is concrete and the other abstract, as $4 × 7, they are not commutative, and, taking the second number as the multiplier, we cannot write $4 × 7 = 7 × $4, since the latter form requires us to use a concrete number as a multiplier.

In such cases, however, we can obtain the advantage of the commutative principle in the actual process of the multiplication, by setting aside the concrete unit involved, and making the multiplication abstract till it is performed, and then restoring the concrete unit to the product; or we can, for purposes of computation, transfer the concrete unit from one factor to the other, thus—

$$\$4 \times 7 = \$1 \times 4 \times 7 = \$1 \times 7 \times 4 = \$7 \times 4.$$

If each dot in the rectangular array of dots given on opposite page represent one dollar, this is the same as saying that 7 columns, each containing 4 units of 1 dollar each, are the same as 4 rows, each containing 7 units of 1 dollar each; or 7 yds. of cloth, at $4 each, cost as many dollars as 4 yds. at $7 each, and the one computation may be exchanged for the other, if any advantage is gained thereby.

We now proceed to show that by the use of the multiplication table any two numbers, however large, may be multiplied together.

53. I. When the Multiplier is a Single Digit.—The process is illustrated by the following example:

Ex. Multiply $264 by 7.

OPERATION.

$264, *Multiplicand.*
7, *Multiplier.*
$1848, *Product.*

EXPLANATION.

We write the multiplier, 7, under the units figure, 4, of the multiplicand, and multiply the 4 units by 7, obtaining 28 units, or 2 tens and 8 units, set down the 8 units in the units place, and reserve the 2 tens to be added to the next partial product. We then multiply the 6 tens by 7 and obtain 42 tens, to which we add the 2 tens reserved, and obtain 44 tens, or 4 hundreds and 4 tens. Setting down the 4 tens in the tens place, and reserving the 4 hundreds, we next multiply 2 hundreds by 7, and obtain 14 hundreds. To this we add the 4 hundreds reserved, and obtain 18 hundreds, which we set down in the proper place. Hence, the product of $264 by 7 is $1848.

The student should observe that the multiplication has been performed by taking the product of single pairs of digits separately and combining the partial products obtained. If the student will set down the number 264 seven times in a column, and obtain the sum by addition, he will realize the labor saved by the process of multiplication, even in a simple example like this.

EXERCISE 10.

1. There are 24 hours in one day. How many are there in 3 days? In 5 days? In 7 days? In 9 days?

2. One bushel of corn weighs 56 pounds. How many pounds will 4 bushels weigh? 9 bushels? 6 bushels?

3. There are 66 feet in one chain. How many feet in 4 chains? In 7 chains? In 8 chains?

4. A barrel of flour weighs 196 pounds. How many pounds do 4 barrels weigh? 5 barrels? 9 barrels?

5. There are 365 days in one year. How many days in 3 years? In 5 years? In 7 years?

6. If 725 men have each $4, how many have they all? If each has $6? $8?

Multiply:

7. 256 boys by 7.
8. 439 girls by 8.
9. 716 feet by 6.
10. 1729 men by 5.
11. $2803 by 6.
12. $6753 by 8.
13. $3926 by 9.
14. 42307 yds. by 7.

	15.	*16.*	*17.*	*18.*
Multiply	9013 feet	14706 inches	3278 min.	35544 rods
by	6	8	4	7

	19.	*20.*	*21.*	*22.*	*23.*
Multiply	2758	3759	46789	78697	53716
by	3	4	5	6	7

	24.	*25.*	*26.*	*27.*	*28.*
Multiply	7791	4567	3803	6175	40461
by	7	8	9	7	3

	29.	*30.*	*31.*	*32.*
Multiply	701508	517032	876531	90470680327
by	4	5	6	8

What must be paid for the following purchases:

33. 12 yards silk at $3 a yard, and 17 yards cloth at $2 a yard?

34. 16 spools thread at 5 cts. a spool, and 7 yards cord at 2 cts. a yard?

35. 8 cows at $28 each, and 6 oxen at $46 each?

36. 9 horses at $127 each, and 4 carriages at $108 each?

Find amounts of these two memoranda:

37. 3 mo. rent at $28.
6 tons coal at $6.
7 loads wood at $5.
12 barrels oil at $5.

38. 376 hats at $2.
428 coats at $8.
714 tickets at $6.
125 employees at $5.

Suppose here, too, the pupils make examples for each other. Let parents help at home. Teacher can give several examples whose answers can be told at a glance, such as: What is the difference between six times 5324, and 7 times the same number? Multiply 93076 first by 3, and then by 7, and add the results, etc., etc.

54. II. When the multiplier is a digit with one or more zeroes annexed.

Ex. Multiply $843 by 40.

OPERATION.
$843
40
$33720

EXPLANATION.

In this case we multiply by the digit, 4, and annex the zeroes to the product. For if 40 groups of 843 dollars each are to be added together, we can, if convenient, separate the 40 groups required into 10 groups of 4 each. $843 × 4, or $3372, will then give the number of units in each of the 10 groups, and the product of $3372 by 10 will give the entire number of units in the product of $843 × 40.

Similarly, if the multiplier is composed of any two factors, the multiplication, if it is desirable, may be separated into two steps. We first multiply the multiplicand by one factor of the multiplier, and then multiply the product so obtained by the other factor. Thus, to multiply 2608 by 63, since $63 = 9 \times 7$, we may first multiply 2608 by 9 and obtain 23472, and then multiply 23472 by 7 and obtain 164304 as the product of the two original numbers. We may proceed similarly if the multiplier is separable into three or more factors.

55. III. **When the multiplier contains two or more digits.**

Ex. 1. Multiply $384 by 237.

OPERATION.		ABBREVIATED FORM.	
$384		$384	
237		237	
2688,	*First Partial product.*	2688	
11520,	*Second* " "	1152	
76800,	*Third* " "	768	
$91008,	*Entire Product.*	$91008,	*Product.*

EXPLANATION.—We regard the multiplier as composed of three parts, viz., 7 units, 3 tens or 30, and 2 hundreds or 200, multiply by each separately, and then add the partial products obtained. It is customary to omit the zeroes which indicate the order of the second, third, etc., partial products.

It is to be noted that if one or more of the figures of the multiplier is a zero, the corresponding partial product is a zero and need not be written.

Ex. 2. Multiply 56308 by 4007.

OPERATION.

56308
4007
394156
225232
225626156, *Product.*

56. Verification.—One method of verifying the work of multiplication—that is, of testing it with a view to detecting any errors that may have been made—is to let the multiplier and multiplicand change places, and perform the multiplication again. We are at liberty to do this by Art. 52.

Ex. Multiply 437 by 26, and test the accuracy of the result.

OPERATION.	VERIFICATION.
437	26
26	437
2622	182
874	78
11362	104
	11362

As the two products obtained are identical, and it is not likely that mistakes have been made in such a way in the two different processes that they exactly compensate, we assume that the result obtained is correct.

If the student is familiar with the process of division, it is left as an exercise for him to devise another method of verifying multiplication by its use.

57. A general rule for multiplication may now be formally stated. When the multiplier is a single figure,

Write the multiplier under the units figure of the multiplicand;

Multiply each figure of the multiplicand by the multiplier;

If the product is less than 10, place it under the figure multiplied; if greater, set down the right-hand figure, adding the other figure to the next partial product.

When the multiplier is greater than 9,

Write the multiplier under the multiplicand, with units of the same order in the same column;

Begin with the units figure of the multiplier and multiply the multiplicand by each figure of the multiplier in succession, placing the right-hand figure of each partial product under the term by which it was obtained;

Add the partial products.

EXERCISE 11.

	1.	*2.*	*3.*	*4.*	*5.*
Multiply	$75	$83	96 boys	128 pens	365 days
by	16	25	33	46	58

	6.	*7.*	*8.*	*9.*	*10.*
Multiply	$387	$476	375 feet	406 days	365 days
by	67	89	123	314	236

	11.	*12.*	*13.*	*14.*	*15.*	*16.*
Multiply	$760	$809	550	828	512	891
by	325	426	433	567	471	635

	17.	18.	19.	20.
Multiply	32071	76203	93761	56714
by	50	420	305	270
	21.	22.	23.	24.
Multiply	56702	67094	76543	67038
by	508	1320	2045	3029
	25.	26.	27.	28.
Multiply	67563	76805	80984	66894
by	1300	7503	3370	3502
	29.	30.	31.	32.
Multiply	46780	33607	23716	55007
by	1432	5071	7632	3896
	33.	34.	35.	36.
Multiply	34256	68327	32765	33856
by	4017	4009	12706	10708

37. How would you do the first example in this exercise by addition? How the 24th? How the 35th? Can you now understand how useful multiplication is?

38. There are 24 hours in a day, and 365 days in a year. How many hours in a year? In 6 years? In 75 years?

39. What will 743 horses cost at $130 each?

40. One mile contains 1760 yards. How many yards in 425 miles? In 720 miles?

41. There are 5280 feet in a mile, and 12 inches in a foot. How many inches in a mile? In 25 miles?

42. Methuselah lived 969 years. How many days did he live?

43. A book of 674 pages contains 38 lines on each page. How many lines in the book?

44. If there are 14 words in each line, how many words in the book of Example 43?

45. A bicyclist rides 16 miles an hour for 36 days of 7 hours each. How many miles did he ride altogether?

46. Which is the greater, and how much,

$756 \times 243 + 965$, or $3328 \times 79 - 69300$?

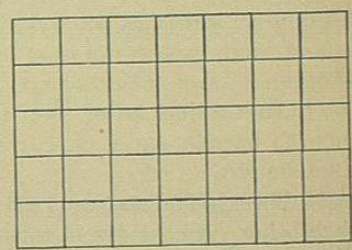
47. Multiply $261 \times 325 - 83476$ by $66750 - 307 \times 209$.

58. Value of Multiplication.—The value of the process of multiplication as compared with addition should be frequently recalled in connection with examples like the following:

Ex. If it costs $23562 to build a single mile of railroad, how much will it cost to build 273 miles?

If this example were solved by addition, it would require the setting down of 23562 two hundred and seventy-three times, and the addition of five columns of figures, each containing 273 figures. The process of multiplication (by resolving the numbers into pairs of digits and recalling the results of a few simple additions) gives the same result with scarcely one hundredth of the labor.

The saving of labor effected by multiplication is still further realized when some adequate idea of the extent of its application is formed. The largeness of its field of application is due to the fact that wherever possible, groups of units are made uniform in the number of units which each group contains. Thus, each foot in linear measure is composed of 12 inches, all the milk in a can is sold for the same number of cents per quart, each week contains 7 days, etc. Also, where the number of units is not the same in different groups considered, it is frequently made uniform by taking the average of all the groups, so that multiplication may be applied. Thus, measurements to determine areas, volumes, etc., are usually so taken that the multiplicative principle may be applied. Thus, in measuring the area of a floor which is, say, 7 yards long and 5 yards wide, we do not mark off the floor into actual square yards and count or add the number of them, but we measure the length and breadth of the floor, and thus, in effect, arrange the square yards contained in the floor into 5 rows, each containing 7 square yards, and then obtain the number of square yards in the area by the multiplication $7 \times 5 = 35$.



EXERCISE 12.

1. Multiply 750 by 430 and the product by 48.

2. From $87 \times 43 \times 56$ subtract 235×260.

3. Take $370 \times 80 \times 420$ from $567 \times 203 \times 50$.

4. What is the value of $46 \times 70 - 28 \times 45 + 160 \times 23$?

5. What will 763 acres of land cost at $85 an acre?

6. A man 600 miles from New York walked toward that city 28 days, 17 miles each day. How far away was he then?

7. A railroad train travels 45 miles an hour for 38 days of 24 hours each. How many miles must it still run to have gone 45000 miles?

8. Supposing there are 85 apples in a bushel, how many will there be in a crop of the same kind, of 560 bushels?

9. There are 32 quarts in a bushel. How would you determine the number of grains in a bushel of wheat without counting them all? In 75 bushels?

10. There are 43 rows of trees in an orchard and 61 trees in each row. How many trees in the whole orchard? How many in another orchard having twice as many rows and twice as many trees in each row?

11. Thirteen books contain respectively 261, 295, 304, 247, 283, 311, 219, 276, 253, 309, 267, 238, 294 pages. If they each had 293 pages, how many more pages would there be?

12. Mr. Dash sold Mr. Blank 24 rolls of cloth, each containing 45 yards, at $2 a yard. Mr. Blank sold Mr. Dash 6 lots of land, each containing 7 acres, at $52 an acre. Which gentleman owes the other and how much?

13. In a train-load of flour there are 78 cars, each containing 184 barrels and each barrel weighing 196 pounds. Find total weight.

14. Two bicyclists are 1800 miles apart and ride toward each other. One rides 11 miles an hour for 6 hours of each day, and the other rides 9 miles an hour for 8 hours every day. After riding thus for 13 days, how far apart are they?

15. A speculator bought 585 acres of land at $74 an acre. He sold at one time 87 acres at $64 an acre; at another time, 137 acres at $93 an acre; and at a third sale, 178 acres at $77 an acre. At what price must he sell all the remaining acres to gain $4280 on the transaction?

16. What would he have gained by selling the remaining acres at $75 an acre?

17. Which is larger, 753×427 or 691×538?

18. Find the difference between $97 \times 86 - 347$ and $97 \times 347 - 86$.

19. Bought 370 barrels of apples at $3 a barrel, and sold them so as to gain $2 on each barrel. What was the cost? What was my profit? What was the selling price?

20. A book-keeper whose salary is $4000 a year spends at the rate of $7 a day for 365 days every year. How much can he save in 28 years?

21. A man buys 326 sheep at $4 each and sells them so as to gain $125. What was the selling price?

22. A farm of 381 acres sold for $76 an acre, the owner thereby gaining $486. What was its cost to him?

23. When is there gain? When loss?

24. A capitalist bought 376 acres of land at $83 an acre. He sold from it, 96 acres at $92; 139 acres at $85; 63 acres at $81; and the remainder at $107. What was his total profit?

25. There are 2 pints in a quart, 4 quarts in a gallon, and 63 gallons in a hogshead. Without counting them all, how could one determine the number of drops of water in a hogshead? In 525 hogsheads?

26. Which is the greatest and which the least:

$$17 \times 13 + 15 \times 11 - 18 \times 19,$$
$$17 + 13 \times 15 - 11 \times 18 \times 19, \text{ or}$$
$$(17 - 13) \times 15 \times 11 - 18 \times 19?$$

27. Explain the several operations and the *order* of operations in: $3 + (4 + 5) \times 6 - (7 + 8 \times 9) + 10 \times 11$. And also in: $3 + 4 + (5 \times 6) - 7 + (8 \times 9 + 10) \times 11$. Find the value of each.