13. What is the greatest width of carpet that will exactly fit three rooms of widths 15 feet, 24 feet, and 33 feet respectively?

14. A merchant having 54 yards of one kind of cloth, 84 yards of another, and 132 yards of a third, wishes to cut them into patterns of equal length. What is the greatest possible length of each pattern?

15. With a 4-quart, a 5-quart, and a 6-quart vessel, what is the size of the smallest can which may be filled exactly by each?

16. Find the length of the shortest line that can be measured exactly by rods of lengths 6 feet, 8 feet, 10 feet, and 12 feet.

17. What is the length of the longest rod which will exactly measure 209 feet, 242 feet, and 341 feet?

18. A farm produces 442 bushels of oats, 728 bushels of corn, and 585 bushels of wheat. The grain is removed in equal cases and all are full. What is the greatest capacity of each case, provided there is no mixing of the grains?

19. How can the L. C. M. of two numbers, which are prime to each other, be found? Of two prime numbers?

20. How many common multiples may 2 or more numbers have?

21. Find the difference between the G. C. D. of 480 and 520, and the L. C. M. of 5, 6, 15, 20.

CHAPTER IX.

COMMON FRACTIONS.

104. Derived Units.—A certain unit, as one pound, having been chosen for the purpose of weighing objects in general, it is often convenient to obtain from this primary unit (one pound) other derived units to be used for weighing special classes of objects. Thus, one ton (or 2000 pounds) is used in weighing objects of small value in proportion to their bulk, such as hay, coal, etc., and one ounce is used in weighing objects of great value in proportion to their bulk, as spices, gold, drugs, etc. Similarly, from any primary unit, derived units may be obtained adapted to special uses.

When the derived unit is an exact part of the primary unit, it is termed a *fractional unit* or *fraction*.

105. Fractional Units.—Thus, for measuring long distances, the *mile* is the convenient unit; but for many purposes, as, for instance, in running races in athletic games, it is convenient to divide the unit into 4 equal parts, and call one of them one-fourth of a mile; similarly we form other fractional units from the miles, as one-eighth, one-half, one-sixteenth of a mile, etc. These are all fractional units or fractions, and are expressed by writing the number of parts into which the given unit is divided under the figure 1. Thus, $\frac{1}{8}$ of a mile means one-eighth of a mile.

If a given fractional unit be taken any number of times, the result is still a fraction, and is denoted by writing the number of times the unit is taken above the line instead of 1. Thus, "$\frac{5}{8}$ mile" is an abbreviation for "5 units" of the value $\frac{1}{8}$ mile.

Sometimes a fractional unit receives a special name, as when one-twelfth of a foot is called an "inch"; or it may be made into a physical object, as when a quarter of a dollar is coined and is known as a "quarter." But the great majority of fractions have no name beyond their numerical one, and many of them are used merely as aids in computations, or in mental estimates and comparisons, and have no physical existence.

Hence, the advantages in the use of fractional units lie in the ease with which such units can be devised for any purpose, temporary or permanent; the unlimited number of such units that can be formed; and the fact that when conceptions of their value as compared with the primary unit have been once formed, these conceptions can be used in connection with a set of similar fractions constructed from any other unit. Thus, having formed ideas of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{16}$, etc., of 1 inch, and of properties of these fractional units, this knowledge can be used at once in connection with similar fractions of any other primary unit, as 1 apple. This could not be done readily if each derived unit were denoted by a special name rather than in the above numerical, fractional way.

106. Fractions as Indicated Divisions.—Fractions may also be regarded as indicated divisions. It was found in Art. 84 that when a process consists of a number of multiplications and divisions, it is usually best not to perform any of the operations till all of them can be considered together and all possible cancellations made.

When the quotient of one number divided by another is indicated by writing the dividend above a line, and the divisor below, the indicated quotient is termed a *fraction* or ratio.

Hence, a **fraction** may be defined as

(1) *One or more of the exact parts of a unit,* or
(2) *the indicated quotient of one number divided by another.*

These two ways of regarding fractions are aspects of the same idea, the one or the other aspect to be used as advantage may dictate.

In investigating the properties of fractions, we will adopt one or the other point of view, as is most advantageous. When a property of fractions is obtained from one point of view, it will be left as an exercise to the student to show that the same property is true for fractions from the other point of view.

Let the student draw a line 3 inches long and divide it into 8 equal parts. Each part will be $\frac{3 \text{ inches}}{8}$ long. Let him also divide each of the three inches into eighths and take one-eighth from each inch. He will have 3 times $\frac{1}{8}$ inch, or $\frac{3}{8}$ inch. It will then be easy for him to see that $\frac{3 \text{ inches}}{8} = \frac{3}{8}$ inch.

107. Denominator and Numerator.—In a fraction, the *denominator* is the number below the line, the *numerator* is the number above the line.

The denominator denotes the number of equal parts into which a unit is divided; the numerator denotes the number of equal parts which are taken. Thus, $\frac{5}{8}$ inch denotes that an inch is divided into 8 equal parts, and that 5 of these parts are taken.

Hence, the denominator determines the size of the fractional units; the numerator determines the number of them.

The denominator and numerator taken together are called the **terms** of a fraction.

108. Proper and Improper Fractions.—A **proper fraction** is one whose numerator is less than its denominator, as $\frac{1}{3}$ or $\frac{12}{13}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator, as $\frac{8}{8}$ or $\frac{27}{25}$.

109. Integers and Mixed Numbers.—An **integer** is a number of entire units, as 5 dollars, 18.

An integer may be expressed in the form of a fraction by writing 1 under the integer as a denominator.

A **mixed number** is a number which is partly integral, partly fractional, as $\$4\frac{3}{4}$.

Thus, a mixed number consists of two different kinds of units, one integral or entire, the other fractional.

110. Simple, Compound, and Complex Fractions.—A **simple fraction** is a fraction, both of whose terms are integers, as $\frac{7}{11}$.

A **compound fraction** is a fraction of a fraction, as $\frac{2}{3}$ of \$$\frac{3}{4}$.

A **complex fraction** is one having a fraction in its numerator or in its denominator or in both. Ex. $\frac{2\frac{1}{3}}{\frac{5}{6}}$, $\frac{7}{\frac{3}{4}}$

When fractions are classified as proper or improper, they are classified as to their *value* (as greater or less than unity). When they are classified as simple, compound, or complex, they are classified as to their *form* (that is, as to the combination of operations in them).

111. Notation and Numeration of Fractions.—The preceding statements explain sufficiently the method of reading a given fraction expressed in figures, and also the method of expressing in figures, a fraction given in words. Let the student write out a formal rule for each of these processes.

112. Fundamental Properties of Fractions.—In order to use fractions with facility for various purposes, it is often desirable to transform them in different ways. Thus, for instance, it may be desirable to change the size of the fractional unit, without changing the value of the fraction.

Hence, we have the following first properties of fractions.

A. *If the numerator and denominator of a fraction be both multiplied, or both divided, by the same number, the value of the fraction is not changed.*

Thus, $\frac{6}{8}$ inch = $\frac{3}{4}$ inch = $\frac{12}{16}$ inch.

This is a mere restatement of the principle used in canceling out a factor common to both divisor and dividend.

It will aid the pupil in the present application of this principle to draw a line 6 inches in length, and to mark it off in fourths, eighths, and sixteenths of an inch, and then observe that six eighths, three fourths, and twelve sixteenths are exactly equivalent in length.

B. *Multiplying the denominator of a fraction by a given number divides the value of the fraction by that number.*

Thus, if we have \$$\frac{5}{4}$ and multiply the denominator by 3, we have \$$\frac{5}{12}$, the value of which is one-third the value of the original fraction.

For, multiplying the denominator of a fraction by a number increases the number of parts into which the original unit is divided, and hence diminishes the size of each fractional unit correspondingly.

Let the pupil show by drawing a line and subdividing it, that $\frac{5}{2}$ inch is four times as long as $\frac{5}{8}$ inch.

C. *Dividing the denominator of a fraction by a given number multiplies the value of the fraction by the same number.*

Thus, if we have the fraction \$$\frac{5}{4}$ and divide the denominator by 2, the fraction becomes \$$\frac{5}{2}$, the value of which is twice as large as the value of the original fraction.

For, dividing the denominator of a fraction by a number diminishes the number of parts into which the unit is divided, and hence increases the size of each fractional unit correspondingly.

Let the pupil show by drawing a line, and subdividing it, that $\frac{7}{16}$ of an inch is one-fourth of $\frac{7}{4}$ of an inch.

The following questions suggest two other first properties of a fraction which the student may state and prove.

If the numerator of a fraction be multiplied by a given number, what change is made in the value of the fraction?

If the numerator of a fraction be divided by a given number, what change is made in the value of the fraction?

These first properties of a fraction may all be combined as a single general principle, thus:

Multiplying or dividing the **numerator** *of a fraction by a number makes the* **same** *change in the value of the fraction that it makes in the value of the numerator; but multiplying or dividing the* **denominator** *of a fraction makes an* **opposite** *change in the value of the fraction from that which it makes in the value of the denominator.*

EXERCISE 29.

Name the kind of fraction in each case and read the following fractions:

1. $\frac{3}{4}$, $\frac{7}{10}$, $\frac{5}{8}$, $\frac{14}{9}$, $\frac{27}{19}$, $5\frac{1}{2}$, $\frac{2}{3}$ of $\frac{1}{8}$, $\frac{4}{5}$ of $2\frac{1}{6}$.

2. $\frac{5}{5}$, $\frac{17}{18}$, $\frac{18}{17}$, $1\frac{11}{23}$, $\frac{112}{345}$, $\frac{527}{621}$, $\frac{13}{5}$ of $7\frac{1}{3}$, $\frac{851}{362}$.

3. $\frac{5\frac{1}{3}}{7}$, $\frac{19}{14\frac{2}{3}}$, $\frac{2\frac{1}{4}}{7\frac{1}{5}}$, $\frac{\frac{1}{3}}{\frac{7}{8}}$, $\frac{3\frac{1}{4}}{8\frac{1}{2}}$, $\frac{1\frac{2}{3}}{\frac{7}{11}}$, $\frac{4\frac{2}{3}}{2\frac{1}{2}}$.

Write the following fractions:

4. Three-fifths; six-tenths; ten-thirds.
5. Nine-seventeenths; eleven-fortieths.
6. Eight and one-third; six and a half.
7. Ten and five-sixths; seventeen-hundredths.
8. One and a fourth over six-sevenths.
9. Two-thirds of four and three-elevenths.
10. Thirty-one and one-ninth over fourteen-fiftieths.
11. What fractions of an inch are commonly used by a carpenter?
12. What fractions of a yard are commonly used by a storekeeper?
13. What fractions of a pound are used by a grocer?

EXERCISE 30.

ORAL.

1. How many thirds in a yard? In 4 yards?
2. How many fifths in 2 yards? In 10 yards?
3. How many eighths in 6 miles? In half a mile?
4. How many twelfths in 7 years? In half a year?
5. How many sixteenths in 5 inches? In a quarter-inch?
6. Express $\frac{3}{4}$ of a dollar as eighths of a dollar.
7. Express $\frac{5}{6}$ of a yard as eighteenths of a yard.
8. Express $\frac{2}{3}$ of a year as twelfths of a year.
9. Multiply \$$\frac{4}{5}$ by 3; \$$\frac{3}{8}$ by 5; $\frac{1}{3}$ yard by 7.
10. Divide \$$\frac{15}{16}$ by 3; \$$\frac{8}{9}$ by 4; $\frac{6}{13}$ by 3.
11. Multiply $\frac{5}{8}$ by 4 in two ways; also $\frac{11}{15}$ by 5.
12. Divide $\frac{3}{7}$ by 2; $\frac{5}{8}$ by 3; $\frac{4}{5}$ by 7.
13. Multiply $\frac{3}{4}$ by 5; $\frac{25}{36}$ by 18; $\frac{14}{15}$ by 45.

TRANSFORMATIONS OF FRACTIONS.

113. I. To Reduce a Mixed Number to an Improper Fraction.—A mixed number is a number expressed by means of two units, one integral, the other fractional. Thus, \$$7\frac{3}{4}$ expresses 7 units of \$1 each, and 3 units of \$$\frac{1}{4}$ each.

It is often convenient to express such a number in terms of the fractional unit alone. In the above example, this would be done by expressing \$7 as fourths of a dollar, or \$$\frac{28}{4}$, and adding the 3 fourths to the 28 fourths, giving \$$\frac{31}{4}$ as equivalent to \$$7\frac{3}{4}$.

In general, to reduce a mixed number to an improper fraction, *multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the sum over the denominator.*

Ex. Reduce $23\frac{5}{7}$ to an improper fraction.

SOLUTION.

$23\frac{5}{7} = \frac{161}{7} + \frac{5}{7} = \frac{166}{7}$, *Result.*

EXERCISE 31.

Reduce the following mixed numbers to equivalent improper fractions:

1. $4\frac{1}{4}$ in.	*9.* $24\frac{4}{9}$.	*17.* \$$58\frac{3}{4}$.	*25.* $101\frac{23}{26}$.
2. $7\frac{2}{3}$ ft.	*10.* $25\frac{3}{13}$.	*18.* \$$65\frac{7}{8}$.	*26.* $400\frac{5}{48}$.
3. $5\frac{4}{7}$ mi.	*11.* $45\frac{1}{8}$.	*19.* \$$73\frac{1}{10}$.	*27.* $307\frac{1}{17}$.
4. $7\frac{5}{9}$ rds.	*12.* $17\frac{14}{15}$.	*20.* $270\frac{3}{5}$.	*28.* $11\frac{175}{347}$.
5. $8\frac{3}{10}$ yds.	*13.* $121\frac{1}{2}$.	*21.* $350\frac{10}{11}$.	*29.* $18\frac{49}{75}$.
6. $10\frac{7}{11}$ gal.	*14.* $235\frac{1}{4}$.	*22.* $700\frac{13}{15}$.	*30.* $19\frac{99}{100}$.
7. $9\frac{11}{12}$ qts.	*15.* $310\frac{4}{5}$.	*23.* $309\frac{17}{21}$.	*31.* $28\frac{417}{503}$.
8. $16\frac{4}{5}$ yr.	*16.* $108\frac{5}{12}$.	*24.* $111\frac{19}{20}$.	*32.* $43\frac{101}{222}$.

33. What are the two units of measurement in each example? What is the unit of the result?

114. II. To Reduce an Improper Fraction to a Mixed Number.—It is often desirable to reverse the process of the preceding article and convert a number expressed in terms of a fractional unit into a number expressed as far as possible in terms of the primary integral unit. Thus, to express \$$\frac{39}{4}$ in terms of the unit \$1 as far as possible, since in \$1 there are $\$\frac{4}{4}$, in $\$\frac{39}{4}$ there are as many unit dollars as 4 is contained times in 39, or $9 with $\$\frac{3}{4}$ remaining,

$$\therefore \$\tfrac{39}{4} = \$9\tfrac{3}{4}.$$

Hence, in general, to reduce an improper fraction to a mixed number, *divide the numerator by the denominator, and to the quotient annex the remainder placed over the denominator.*

Ex. Reduce $\frac{221}{12}$ to a mixed number.

Since 12 is contained in 221, 18 times with a remainder 5,

$$\tfrac{221}{12} = 18\tfrac{5}{12}, \text{ *Result.*}$$

EXERCISE 32.

Reduce the following improper fractions to equivalent mixed numbers:

1. $\frac{43}{7}$ qt.	*8.* $\$\frac{293}{10}$.	*15.* $\frac{728}{27}$.	*22.* $\frac{1000}{39}$.
2. $\frac{59}{8}$ mi.	*9.* $\$\frac{459}{20}$.	*16.* $\frac{598}{33}$.	*23.* $\frac{2001}{55}$.
3. $\frac{75}{4}$ in.	*10.* $\$\frac{877}{15}$.	*17.* $\frac{508}{35}$.	*24.* $\frac{3076}{63}$.
4. $\frac{124}{5}$ day.	*11.* $\$\frac{889}{25}$.	*18.* $\frac{875}{41}$.	*25.* $\frac{4809}{80}$.
5. $\frac{163}{7}$ wk.	*12.* $\$\frac{756}{11}$.	*19.* $\frac{487}{45}$.	*26.* $\frac{7508}{73}$.
6. $\frac{176}{9}$ gal.	*13.* $\frac{694}{13}$.	*20.* $\frac{920}{51}$.	*27.* $\frac{9569}{101}$.
7. $\frac{249}{12}$ ft.	*14.* $\frac{545}{17}$.	*21.* $\frac{970}{61}$.	*28.* $\frac{9630}{127}$.

115. III. **To Reduce a Fraction to its Lowest Terms.**—A fraction is reduced to an equivalent fraction in its lowest terms when its numerator and denominator have no common factor, that is, are prime to each other.

Reduction of a fraction to its lowest terms often saves labor in the further use of the fraction.

When a fraction is in its lowest terms, it is also easier to form a definite mental picture or conception of its value. Thus, $\frac{411}{548}$ cannot be realized definitely; but if the fraction be reduced to its lowest terms, $\frac{3}{4}$, an exact idea of its magnitude can be formed at once.

A fraction is reduced to its lowest terms by the use of Property A (Art. 112) of fractions. In general, *divide the numerator and denominator of the fraction by their G. C. D.*

Ex. 1. Reduce $\frac{8}{12}$ to its lowest terms.

Dividing 8 and 12 by their G. C. D., 4, we obtain

$$\tfrac{8}{12} = \tfrac{2}{3}, \text{ *Result.*}$$

Ex. 2. Reduce $\frac{411}{548}$ to its lowest terms.

In this case the G. C. D. of the numerator and denominator is not evident on inspection, and must be obtained by the long division method (Art. 100). Thus,

```
411)548(1
    411
    137)411(3       ∴ G. C. D. = 137.
        411
```

$$\therefore \frac{411}{548} = \frac{137 \times 3}{137 \times 4} = \frac{3}{4}.$$

116. Ratios, or Expressing one Number as a Part or Fraction of Another.—To express one number as the part of another number it is necessary to *take the number which is the part as the numerator and the other number as the denominator of a fraction* (that is, divide the number expressing a part by the number expressing the whole).

Ex. What part of a mile is 440 yards?

Since a mile contains 1760 yards,

440 yards equal $\frac{440}{1760}$ of a mile $= \frac{1}{4}$ of a mile.

EXERCISE 33.

Reduce each fraction to the equivalent fraction in its lowest terms:

1. $\frac{12}{21}$.	*8.* $\frac{72}{88}$.	*15.* $\frac{96}{176}$.	*22.* $\frac{968}{1320}$.
2. $\frac{20}{36}$.	*9.* $\frac{54}{99}$.	*16.* $\frac{104}{143}$.	*23.* $\frac{960}{1024}$.
3. $\frac{25}{45}$.	*10.* $\frac{60}{70}$.	*17.* $\frac{228}{950}$.	*24.* $\frac{1177}{2675}$.
4. $\frac{6}{48}$.	*11.* $\frac{84}{96}$.	*18.* $\frac{336}{528}$.	*25.* $\frac{1728}{2448}$.
5. $\frac{16}{56}$.	*12.* $\frac{48}{108}$.	*19.* $\frac{234}{315}$.	*26.* $\frac{1848}{2352}$.
6. $\frac{30}{54}$.	*13.* $\frac{70}{98}$.	*20.* $\frac{1152}{1792}$.	*27.* $\frac{2520}{2835}$.
7. $\frac{35}{63}$.	*14.* $\frac{90}{105}$.	*21.* $\frac{2310}{3570}$.	*28.* $\frac{2914}{3243}$.

29. A man invested \$360 and gained \$144. What part of the cost did he gain?

30. What part of 336 is 144? Is 168?

31. What part of 245 is 196? Is 210?

32. What part of a year is 146 days?

33. What part of a ton is 1275 pounds?

34. What part of a mile is 1320 yards?

35. \$230 is what part of \$322?

36. \$781 is what part of \$923?

37. Property valued at \$187200 was taxed for \$7020. Find the ratio of the assessment to the valuation.

38. Receiving \$8568 annually, I spent \$1904. What part of my income did I spend? What part did I save?

What part of:

39. 540 is 378?

40. 864 is 630?

41. 1775 is 1491?

42. 3154 is 2158?

117. IV. **To reduce two or more fractions to equivalent fractions having a common denominator.**

Similar fractions are fractions which have the same denominator. Hence, similar fractions express numbers in terms of the same fractional unit. Thus, $\$\frac{3}{10}$, $\$\frac{5}{10}$, $\$\frac{7}{10}$, are similar fractions.

If a series of fractions have different denominators, it is often useful to reduce them to fractions having the same denominator, that is, to express them in terms of the same unit. By combining their numerators, they may then, in many cases, be converted into a single fraction, and much labor saved by treating them in this form.

Also, in case it is required to compare the values of two or more dissimilar fractions, a direct comparison is often difficult or impossible. If, however, the fractions be reduced to a common denominator, their values can be compared at once by comparing the numerators obtained.

In reducing fractions to a common denominator, it is

important to reduce them to their least common denominator, in order to save as much labor as possible. The least common denominator of a set of fractions is the L. C. M. of their denominators.

In general, to reduce fractions to equivalent fractions having the L. C. D., *find the L. C. M. of the denominators of the given fractions; divide this L. C. M. by the denominator of each fraction; multiply each numerator by the corresponding quotient; the results will be the new numerators; write the L. C. D. under each new numerator.*

On which of the principles, A, B, C, of Art. 112, is this process based?

Ex. 1. Reduce $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, to their L. C. D.

The L. C. M. of 4, 8, and 12, is 24.

Dividing 24 by each of the numbers 4, 8, 12, we obtain quotients 6, 3, 2.

Multiply the numerators 3, 5, 7, by the corresponding quotients and setting each result over 24, we obtain

$\frac{18}{24}$, $\frac{15}{24}$, $\frac{14}{24}$, *Result.*

Ex. 2. Which is greater, $\$\frac{7}{12}$ or $\$\frac{11}{18}$?

Reducing the fractions to their L. C. D.,

$$\$\tfrac{7}{12} = \$\tfrac{21}{36}.$$
$$\$\tfrac{11}{18} = \$\tfrac{22}{36}.$$

Since $\frac{22}{36}$ is greater than $\frac{21}{36}$, $\$\frac{11}{18}$ is greater than $\$\frac{7}{12}$.

118. When the denominators of two or more fractions are prime to each other, the L. C. D. is the product of all the denominators, and the shortest way of reducing the fractions to their L. C. D. is to *multiply each numerator by all the denominators except its own, and set the result over the common denominator.*

Ex. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$ to their L. C. D.

The L. C. D. $= 3 \times 5 \times 7$.

Hence, we have $\frac{2 \times 5 \times 7}{105}$, $\frac{4 \times 3 \times 7}{105}$, $\frac{6 \times 3 \times 5}{105}$,

or $\frac{70}{105}$, $\frac{84}{105}$, $\frac{90}{105}$, *Result.*

EXERCISE 34.

Reduce to equivalent fractions having a common denominator:

1. $\frac{3}{4}, \frac{5}{6}$.
2. $\frac{1}{6}, \frac{3}{8}$.
3. $\frac{5}{4}, \frac{7}{10}$.
4. $\frac{2}{3}, \frac{3}{4}$.
5. $\frac{4}{5}, \frac{8}{9}$.
6. $\frac{11}{12}, \frac{13}{15}$.
7. $\frac{5}{16}, \frac{11}{20}$.
8. $\frac{13}{38}, \frac{23}{24}$.
9. $\frac{7}{9}, \frac{7}{10}$.
10. $\frac{1}{2}, \frac{2}{3}, \frac{1}{4}$.
11. $\frac{1}{3}, \frac{3}{2}, \frac{5}{6}$.
12. $\frac{3}{4}, \frac{1}{6}, \frac{3}{8}$.
13. $\frac{5}{8}, \frac{2}{9}, \frac{11}{12}$.
14. $\frac{5}{12}, \frac{2}{15}, \frac{7}{30}$.
15. $\frac{7}{12}, \frac{5}{16}, \frac{1}{24}$.
16. $\frac{17}{18}, \frac{23}{24}, \frac{15}{32}$.
17. $\frac{3}{2}, \frac{2}{3}, \frac{1}{5}$.
18. $\frac{1}{4}, \frac{3}{7}, \frac{1}{6}$.
19. $\frac{2}{9}, \frac{5}{4}, \frac{11}{12}$.
20. $\frac{4}{7}, \frac{9}{11}, \frac{7}{2}$.
21. $\frac{3}{11}, \frac{8}{15}, \frac{4}{21}$.
22. $\frac{3}{13}, \frac{4}{15}, \frac{5}{16}$.
23. $\frac{4}{15}, \frac{3}{25}, \frac{7}{20}$.
24. $\frac{5}{11}, \frac{3}{8}, \frac{5}{6}$.
25. $2\frac{2}{3}, 1\frac{3}{4}, 4\frac{1}{6}$.
26. $2\frac{1}{4}, 5\frac{1}{10}, 7\frac{3}{8}$.
27. $\frac{7}{12}, \frac{5}{18}, \frac{11}{24}, \frac{13}{30}$.

Which is the larger,

28. $\frac{5}{6}$ or $\frac{7}{8}$?
29. $\frac{3}{5}$ or $\frac{2}{3}$?
30. $\frac{1}{4}$ or $\frac{2}{9}$?
31. $\frac{7}{13}$ or $\frac{8}{15}$?
32. $\frac{7}{12}$ or $\frac{4}{7}$?
33. $\frac{9}{16}$ or $\frac{15}{28}$?

34. Which is the largest and which is the least,
$\frac{13}{27}, \frac{17}{36}$, or $\frac{11}{24}$? Also $\frac{13}{28}, \frac{16}{35}$, or $\frac{19}{40}$?

35. At a certain convention a measure which required a favorable ballot of 5 to 3, to pass, received 96 votes for and 57 votes against it. Did it pass?

36. A certain bill required two-thirds majority to become a law, and received 390 out of a total of 584 votes. Did it pass?

37. A boy has read 144 pages of a book, containing 300 pages. What part of the book remains to be read?

38. The 1st day of September is the 244th day of an ordinary year. Is the part gone as much as $\frac{2}{3}$ of the year? Is the year as much as $\frac{7}{9}$ gone? Does $\frac{10}{31}$ of the year remain?

OPERATIONS WITH FRACTIONS.

I. Addition of Fractions.

119. General Case.—We have found (see Art. 31) that any numbers which refer to the same unit may be added. Thus,

17 apples + 28 apples = 45 apples.

Numbers which refer to the same fractional unit may be added in the same way. Thus, five eighths (of a unit) + two eighths (of same unit) = seven eighths (of this unit),

$$\text{or } \frac{5}{8} + \frac{2}{8} = \frac{7}{8}.$$

If fractions are dissimilar, in order to add them it is necessary first to make them similar, by reducing them to a common denominator. Hence, to add fractions, *reduce the given fractions to equivalent fractions having the least common denominator; add the numerators, and write the sum over the L. C. D.; in all cases simplify the result, and, if it is an improper fraction, reduce it to a mixed number.*

Ex. Add $\frac{2}{3}, \frac{5}{6}, \frac{3}{8}$.

The L. C. D. is 24.

$$\frac{2}{3} + \frac{5}{6} + \frac{3}{8} = \frac{16}{24} + \frac{20}{24} + \frac{9}{24}$$
$$= \frac{45}{24} = \frac{15}{8} = 1\frac{7}{8}, \textit{ Sum.}$$

EXERCISE 35.

Add:

1. $\frac{3}{4}$ and $\frac{1}{6}$.
2. $\frac{4}{9}$ and $\frac{5}{6}$.
3. $\frac{7}{12}$ and $\frac{8}{15}$.
4. $\frac{3}{5}, \frac{4}{15}, \frac{1}{3}$.
5. $\frac{7}{8}, \frac{5}{6}, \frac{1}{12}$.
6. $\frac{1}{4}, \frac{1}{6}, \frac{17}{18}$.
7. $\frac{9}{10}, \frac{7}{15}, \frac{1}{3}$.
8. $\frac{5}{12}, \frac{11}{15}, \frac{7}{20}$.
9. $\frac{8}{15}, \frac{7}{18}, \frac{13}{30}$.
10. $\frac{9}{8}, \frac{5}{24}, \frac{17}{18}$.
11. $\frac{1}{16}, \frac{17}{20}, \frac{29}{80}$.
12. $\frac{2}{3}, \frac{4}{5}, \frac{7}{30}, \frac{9}{10}$.
13. $\frac{8}{15}, \frac{9}{20}, \frac{10}{21}, \frac{25}{28}$.
14. $\frac{4}{9}, \frac{5}{12}, \frac{11}{27}, \frac{23}{36}$.
15. $\frac{7}{8}, \frac{9}{10}, \frac{2}{3}, \frac{5}{6}, \frac{11}{15}$.
16. $\frac{5}{18} + \frac{7}{9} + \frac{11}{16} + \frac{5}{8} + \frac{11}{6} + \frac{7}{12}$.
17. $\frac{31}{45} + \frac{17}{30} + \frac{13}{36} + \frac{41}{90} + \frac{58}{75} + \frac{99}{100}$.
18. $\frac{75}{144} + \frac{29}{72} + \frac{31}{48} + \frac{23}{36} + \frac{107}{108}$.
19. $\frac{47}{48} + \frac{97}{112} + \frac{52}{63} + \frac{9}{56} + \frac{35}{72} + \frac{107}{126}$.
20. $\frac{5}{8} + \frac{7}{12} + \frac{10}{33} + \frac{19}{44} + \frac{29}{60} + \frac{124}{165} + \frac{99}{110}$.

120. Cases of Abbreviated Addition of Fractions.—1. The addition of two fractions each of whose numerators is unity, and whose denominators are prime to each other, may be abbreviated.

Thus, $\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$.

Or, in general, *the sum of the two denominators gives the numerator of the sum of the fractions; and the product of the two denominators gives the denominator of the sum of the fractions.*

2. The addition of a series of small fractions may often be facilitated by first adding the fractions in groups of two or three, and then taking the sum of the results.

Ex. Add $\frac{2}{3} + \frac{3}{4} + \frac{1}{3} + \frac{5}{12}$.

$$\frac{2}{3} + \frac{1}{3} = 1$$
$$\frac{3}{4} + \frac{5}{12} = \frac{7}{6}$$
$$2\tfrac{1}{6}, \text{ Sum.}$$

EXERCISE 36.

Add:

1. $\frac{1}{3} + \frac{1}{7}$.
2. $\frac{1}{5} + \frac{1}{8}$.
3. $\frac{1}{6} + \frac{1}{10}$.
4. $\frac{1}{3} + \frac{1}{13}$.
5. $\frac{1}{12} + \frac{1}{17}$.
6. $\frac{4}{5} + \frac{1}{3} + \frac{1}{5}$.
7. $\frac{3}{11} + \frac{6}{7} + \frac{5}{11}$.
8. $\frac{4}{15} + \frac{1}{8} + \frac{2}{5}$.
9. $\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$.
10. $\frac{1}{7} + \frac{1}{8} + \frac{1}{14}$.
11. $\frac{1}{9} + \frac{1}{6} + \frac{1}{18}$.
12. $\frac{1}{12} + \frac{1}{11} + \frac{1}{13}$.
13. $\frac{3}{10} + \frac{2}{5} + \frac{1}{2} + \frac{4}{5}$.
14. $\frac{4}{9} + \frac{3}{8} + \frac{5}{9} + \frac{1}{16}$.
15. $\frac{3}{5} + \frac{2}{7} + \frac{1}{15} + \frac{3}{14}$.
16. $\frac{14}{15} + \frac{1}{6} + \frac{2}{3} + \frac{4}{5}$.

121. Addition of Mixed Numbers.—To add mixed numbers, *first add the whole numbers, then add the fractions, then take the sum of the two results obtained.*

Ex. Add $11\frac{5}{8}$ and $10\frac{7}{12}$.

$$11\tfrac{5}{8} = 11\tfrac{15}{24}$$
$$10\tfrac{7}{12} = 10\tfrac{14}{24}$$
$$22\tfrac{5}{24}, \text{ Sum.}$$

EXERCISE 37.

Add:

1. $3\frac{2}{3} + 2\frac{1}{2}$.
2. $4\frac{5}{6} + 2\frac{3}{4}$.
3. $5\frac{1}{5} + 7\frac{3}{10}$.
4. $6\frac{3}{8} + 2\frac{5}{12}$.
5. $4\frac{5}{9} + 4\frac{2}{15}$.
6. $5\frac{11}{12} + 6\frac{17}{18}$.
7. $8\frac{5}{24} + 1\frac{7}{36}$.
8. $1\frac{2}{7} + 3\frac{5}{14} + 7\frac{1}{2}$.
9. $3\frac{4}{5} + 4\frac{5}{6} + 6\frac{7}{10}$.
10. $8\frac{7}{12} + 5\frac{4}{15} + 10\frac{9}{20}$.

11. $2\frac{3}{7} + 4\frac{1}{3} + 6\frac{2}{5} + \frac{5}{6} + 1\frac{31}{35}$.
12. $5\frac{4}{5} + 3\frac{1}{6} + 2\frac{3}{4} + 4\frac{7}{10} + 3\frac{14}{15} + \frac{11}{12}$.
13. $3\frac{1}{5} + 4\frac{1}{3} + 5\frac{3}{25} + 7\frac{4}{9} + 5\frac{7}{15} + \frac{3}{10} + \frac{1}{2}$.
14. $\frac{3}{8} + \frac{7}{6} + 1\frac{1}{4} + 2\frac{1}{9} + \frac{35}{36} + 4\frac{7}{12} + 8\frac{1}{3}$.
15. $9\frac{1}{2} + 7\frac{2}{3} + \frac{4}{5} + 1\frac{5}{6} + 7\frac{7}{8} + 4\frac{1}{9} + \frac{9}{10}$.
16. $3\frac{3}{8} + \frac{5}{6} + \frac{4}{9} + 2\frac{7}{12} + 3\frac{11}{18} + \frac{7}{16} + 14\frac{2}{3}$.

ORAL EXERCISE.

17. $\frac{1}{2} + \frac{2}{3}$.
18. $\frac{3}{4} + \frac{5}{6}$.
19. $\frac{1}{5} + \frac{1}{7}$.
20. $\frac{4}{7} + \frac{3}{8}$.
21. $\frac{3}{10} + \frac{4}{15}$.
22. $\frac{1}{3} + \frac{1}{10}$.
23. $\frac{1}{13} + \frac{1}{11}$.
24. $1\frac{1}{3} + 2\frac{1}{4}$.
25. $1\frac{2}{3} + 3\frac{3}{4}$.
26. $3\frac{1}{5} + 2\frac{3}{4}$.
27. $4\frac{3}{5} + 2\frac{2}{3}$.
28. $1\frac{1}{6} + 5\frac{3}{8}$.
29. $7\frac{3}{16} + 5\frac{7}{12}$.
30. $2\frac{11}{18} + 3\frac{4}{27}$.
31. $\frac{2}{3} + \frac{4}{5} + \frac{1}{10}$.
32. $\frac{3}{5} + \frac{7}{10} + \frac{2}{15}$.
33. $\frac{8}{9} + \frac{7}{12} + \frac{1}{4}$.
34. $\frac{1}{2} + \frac{5}{2} + \frac{3}{4} + \frac{5}{4}$.
35. $\frac{4}{3} + \frac{1}{6} + \frac{3}{4} + \frac{1}{2}$.
36. $\frac{7}{4} + 1 + \frac{1}{6} + \frac{1}{24}$.
37. $\frac{3}{14} + 3 + \frac{5}{7} + \frac{1}{2}$.

II. Subtraction of Fractions.

122. General Case.—As in addition of fractions, if the fractions are dissimilar, it is necessary to make them similar before subtracting. Hence, in general, to subtract one fraction from another, *reduce the fractions to their L. C. D.; subtract the numerator of the subtrahend from the numerator of the minuend and place the difference over the L. C. D.; simplify the result.*

Ex. Subtract $\frac{3}{8}$ from $\frac{7}{12}$.

$$\frac{7}{12} - \frac{3}{8} = \frac{14}{24} - \frac{9}{24} = \frac{5}{24}, \text{ Difference.}$$

EXERCISE 38.

Subtract

1. $\frac{2}{3}$ from $\frac{5}{6}$.
2. $\frac{5}{9}$ from $\frac{7}{12}$.
3. $\frac{8}{15}$ from $\frac{4}{5}$.
4. $\frac{11}{18}$ from $\frac{17}{24}$.
5. $\frac{8}{21}$ from $\frac{9}{14}$.
6. $\frac{7}{25}$ from $\frac{29}{30}$.
7. $\frac{8}{35}$ from $\frac{19}{42}$.
8. $\frac{9}{10}$ from $\frac{54}{55}$.
9. $\frac{3}{35}$ from $\frac{11}{50}$.
10. $\frac{6}{25}$ from $\frac{14}{15}$.
11. $\frac{8}{9}$ from $\frac{23}{24}$.
12. $\frac{75}{64}$ from $\frac{205}{96}$.