

Find the value of

13. $\frac{6}{7} - \frac{1}{3}$.	19. $\frac{7}{12} - \frac{19}{88}$.	25. $\frac{35}{76} - \frac{31}{73}$.
14. $\frac{4}{9} - \frac{5}{24}$.	20. $\frac{5}{18} - \frac{4}{81}$.	26. $\frac{125}{34} - \frac{29}{119}$.
15. $\frac{17}{11} - \frac{11}{17}$.	21. $\frac{61}{84} - \frac{55}{72}$.	27. $\frac{87}{88} - \frac{143}{148}$.
16. $\frac{2}{3} - \frac{2}{9}$.	22. $\frac{43}{48} - \frac{1}{44}$.	28. $\frac{39}{92} - \frac{17}{76}$.
17. $\frac{15}{14} - \frac{49}{49}$.	23. $\frac{67}{81} - \frac{59}{99}$.	29. $\frac{97}{105} - \frac{57}{98}$.
18. $\frac{13}{33} - \frac{8}{25}$.	24. $\frac{47}{120} - \frac{23}{75}$.	30. $\frac{119}{144} - \frac{191}{132}$.

123. Special Case.—The subtraction of certain fractions may be simplified in a manner similar to that given for the addition of fractions in Art. 120.

$$\text{Ex. } \frac{1}{3} - \frac{1}{5} = \frac{5-3}{15} = \frac{2}{15}, \text{ Difference.}$$

Let the student make a formal statement of this case.

124. Subtraction of Mixed Numbers.—If the fraction in the subtrahend is less than the fraction in the minuend, subtract the fractions and whole numbers separately, and combine the results.

Ex. 1. Subtract $\$3\frac{1}{4}$ from $\$5\frac{1}{2}$.

$$\begin{aligned} \$5\frac{1}{2} &= \$5\frac{1}{2} \\ \$3\frac{1}{4} &= \$3\frac{2}{4} \\ \hline \$2\frac{1}{4} & \text{ Difference.} \end{aligned}$$

But if the fraction in the subtrahend is larger than the fraction of the minuend, it is necessary to increase the fraction in the minuend by borrowing 1 from the integral part of the minuend.

Ex. 2. Obtain the difference, $17\frac{1}{4} - 12\frac{3}{8}$.

$$\begin{aligned} 17\frac{1}{4} &= 17\frac{2}{8} = 16\frac{10}{8} \\ 12\frac{3}{8} &= 12\frac{3}{8} \\ \hline 4\frac{7}{8} & \text{ Difference.} \end{aligned}$$

Similarly, Ex. 3. $3 - 1\frac{3}{8} = 2\frac{8}{8} - 1\frac{3}{8} = 1\frac{5}{8}$, Difference.

EXERCISE 39.

Subtract:

1. $1\frac{1}{4}$ from $3\frac{3}{8}$.	6. $\frac{1}{3} - \frac{1}{4}$.	11. $1\frac{1}{4} - \frac{1}{8}$.
2. $2\frac{3}{10}$ from $2\frac{3}{8}$.	7. $\frac{1}{5} - \frac{1}{8}$.	12. $1\frac{1}{2} - \frac{1}{4}$.
3. $4\frac{1}{8}$ from $5\frac{1}{12}$.	8. $\frac{1}{7} - \frac{1}{11}$.	13. $2\frac{1}{5} - \frac{1}{11}$.
4. $3\frac{3}{4}$ from $5\frac{5}{8}$.	9. $\frac{1}{8} - \frac{1}{18}$.	14. $3\frac{1}{8} - 2\frac{1}{8}$.
5. $6\frac{1}{4}$ from $9\frac{3}{8}$.	10. $\frac{1}{14} - \frac{1}{15}$.	15. $4\frac{1}{10} - 3\frac{1}{15}$.
16. $3\frac{1}{8} - 2\frac{1}{2}$.	24. $2 - \frac{1}{2}$.	
17. $6\frac{1}{8} - 2\frac{1}{3}$.	25. $3 - 1\frac{1}{3}$.	
18. $8\frac{1}{4} - 5\frac{1}{3}$.	26. $14 - 5\frac{5}{8}$.	
19. $7\frac{3}{11} - 6\frac{2}{3}$.	27. $8 - 3\frac{2}{11}$.	
20. $2\frac{5}{12} - 1\frac{1}{8}$.	28. $13\frac{7}{8} - 9\frac{3}{8}$.	
21. $4\frac{2}{3} - 3$.	29. $25\frac{1}{4} - 8\frac{1}{2}$.	
22. $5\frac{3}{4} - 4$.	30. $19\frac{1}{8} - 16\frac{7}{8}$.	
23. $6\frac{1}{2} - 2\frac{1}{2}$.	31. $12\frac{1}{4} - 7\frac{3}{4}$.	

EXERCISE 40.

ORAL.

1. $\frac{1}{3} - \frac{1}{7} = ?$	5. $\frac{1}{2} - \frac{1}{3} = ?$	9. $1\frac{1}{2} - \frac{2}{3} = ?$	13. $2 - 1\frac{3}{4} = ?$
2. $\frac{1}{4} - \frac{1}{5} = ?$	6. $\frac{2}{3} - \frac{1}{4} = ?$	10. $1\frac{3}{4} - \frac{1}{2} = ?$	14. $5 - 3\frac{1}{2} = ?$
3. $\frac{2}{3} - \frac{1}{5} = ?$	7. $\frac{3}{8} - \frac{1}{4} = ?$	11. $1\frac{1}{6} - \frac{1}{4} = ?$	15. $5\frac{3}{8} - 2\frac{1}{2} = ?$
4. $\frac{3}{5} - 1 = ?$	8. $\frac{3}{8} - \frac{1}{3} = ?$	12. $3 - 1\frac{1}{2} = ?$	16. $3\frac{3}{8} - 1\frac{3}{8} = ?$

17. How much larger is $\frac{2}{3}$ than $\frac{1}{3}$?
 18. How much larger is $1\frac{1}{2}$ than $\frac{1}{2}$? $2\frac{1}{2}$ than $1\frac{1}{2}$?
 19. How much less is $2\frac{3}{4}$ than 3? $2\frac{1}{4}$ than $3\frac{1}{4}$?

EXERCISE 41.

REVIEW.

Find the value of:

1. $\frac{2}{3} + \frac{1}{2} - \frac{1}{3}$.	6. $2\frac{1}{8} + 1\frac{1}{2} + 2\frac{1}{8}$.	11. $7\frac{3}{8} - 4\frac{1}{2} + 3\frac{3}{8}$.
2. $\frac{1}{3} + \frac{2}{3} - \frac{3}{10}$.	7. $\frac{2}{3} + 7\frac{1}{4} - 5\frac{3}{5}$.	12. $6\frac{5}{8} + 3\frac{1}{2} - 7\frac{5}{8}$.
3. $\frac{2}{3} - \frac{1}{4} + \frac{1}{3}$.	8. $2\frac{1}{2} + 3\frac{3}{8} - 3\frac{1}{2}$.	13. $3\frac{1}{5} - 1\frac{7}{10} + \frac{2}{5}$.
4. $1\frac{1}{2} - \frac{2}{3} + \frac{5}{6}$.	9. $7\frac{1}{2} - 2\frac{1}{2} - 4\frac{1}{2}$.	14. $4\frac{1}{5} - 3\frac{2}{5} + 5\frac{1}{5}$.
5. $3\frac{3}{4} - 1\frac{1}{2} - \frac{1}{4}$.	10. $8\frac{3}{8} - 3\frac{1}{4} + 2\frac{3}{8}$.	
15. $3\frac{1}{2} + 2 - 4\frac{1}{8} + 7\frac{1}{4} - 4$.	16. $7\frac{1}{8} - 4\frac{1}{2} + 2\frac{1}{2} - 4 + 8\frac{5}{8} - 7\frac{3}{8}$.	
17. $3\frac{1}{2} - 1 - 1\frac{1}{8} + 9\frac{3}{4} - 3\frac{1}{2} + 3 - 2\frac{1}{2}$.		

18. If from a piece of cloth containing 50 yards there have been sold at one time $17\frac{3}{4}$ yards, at another $12\frac{3}{8}$, and at another $7\frac{1}{2}$, how many yards remain?

19. If during the month I spend $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{10}$, and $\frac{1}{15}$ of my salary for the month previous, what part have I left?

20. A carpenter finds some boards of following lengths, $17\frac{1}{3}$, $12\frac{1}{4}$, $15\frac{3}{8}$, $16\frac{1}{2}$, $11\frac{1}{3}$, and $18\frac{1}{2}$ feet respectively. What was the total length?

21. From an account amounting to $\$175\frac{1}{2}$, I have drawn at different times, $\$19\frac{1}{2}$, $\$24\frac{3}{8}$, $\$4\frac{1}{2}$, $\$11\frac{1}{4}$, $\$41\frac{3}{8}$, and $\$35\frac{1}{2}$. How many dollars remain?

III. MULTIPLICATION OF FRACTIONS.

125. To Multiply a Fraction by a Whole Number.—Fractional units are multiplied by a whole number in the same manner that other units are, viz.: by obtaining the product of the number of units and the multiplier. Thus,

$$7 \text{ dollars} \times 5 = 35 \text{ dollars.}$$

Similarly, 7 twelfths (of any unit) $\times 5 = 35$ twelfths (of this unit).

$$\text{Or, } \frac{7}{12} \times 5 = \frac{35}{12}, \text{ Product.}$$

Hence, to find the product of a whole number and a fraction, take the product of the numerator by the whole number; set the result over the denominator; simplify by cancellation.

$$\text{Ex. } \frac{7}{24} \times 15 = \frac{7 \times \overset{5}{15}}{\underset{8}{24}} = \frac{35}{8} = 4\frac{3}{8}, \text{ Product.}$$

126. To Multiply a Fraction by a Fraction.—If it be required to multiply $\$7\frac{1}{4}$ by $\frac{3}{5}$, we know that the product of $\$7\frac{1}{4}$ by 1 equals $\$7\frac{1}{4}$; hence, since, if we make the multiplier $\frac{1}{5}$ as large, the product will be $\frac{1}{5}$ as great, $\$7\frac{1}{4}$ multiplied by $\frac{1}{5}$ equals $\frac{1}{5}$ of $\$7\frac{1}{4}$, or $\$2\frac{7}{20}$ (see Art. 112, B).

Hence, $\$7\frac{1}{4}$ multiplied by $\frac{3}{5}$ equals 3 times $\$2\frac{7}{20}$ or $\$2\frac{1}{10}$ (see Art. 112, B), or $\$1\frac{1}{20}$.

$$\text{or, in brief, } \frac{7}{4} \times \frac{3}{5} = \frac{7 \times 3}{4 \times 5} = \frac{21}{20} = 1\frac{1}{20}, \text{ Product.}$$

Hence, in multiplying one fraction by another, we do two

things, (1) we diminish the size of the fractional unit (from one-fourth to one-twentieth of a dollar in the above example), (2) we increase the number of the units.

It may help the student to realize the above process to take 7 quarter dollars, multiply by $\frac{1}{5}$ by substituting nickels for quarters, then multiply by 3 by increasing the number of nickels threefold.

From the above, we obtain the following convenient mechanical method for multiplying two fractions:

Multiply the numerators together for the numerator of the product, and the denominators for the denominator of the product, abbreviating the work as far as possible by cancellation.

127. Continued Multiplication of Fractions.—To multiply three or more fractions together, multiply all the numerators together for a new numerator, and all the denominators for a new denominator, canceling when possible. For the (indicated) product of two fractions may be multiplied by another fraction, and so on for any number of fractions.

Ex. 1. Multiply $\frac{9}{10} \times \frac{16}{31} \times \frac{7}{48}$.

$$\frac{9}{10} \times \frac{\overset{16}{32}}{81} \times \frac{\overset{7}{35}}{48} = \frac{7}{27}$$

A **Compound Fraction** is a fraction of a fraction. Hence, a compound fraction is the product of two fractions, looked at from another point of view.

Ex. 2. Find value of $\frac{3}{5}$ of $\frac{4}{7}$.

$$\frac{3}{5} \text{ of } \frac{4}{7} = \frac{3}{5} \times \frac{4}{7} = \frac{12}{35}, \text{ Result.}$$

EXERCISE 42.

Multiply:

$$1. \frac{3}{5} \times 4.$$

$$2. \frac{4}{7} \times 14.$$

$$3. \frac{5}{6} \times 9.$$

$$4. 10 \times \frac{3}{16}.$$

$$5. 12 \times \frac{5}{21}.$$

$$6. 14 \times \frac{16}{35}.$$

$$7. \frac{15}{4} \times 33.$$

$$8. \frac{7}{8} \times \frac{3}{4}.$$

$$9. \frac{4}{5} \times \frac{1}{2}.$$

$$10. \frac{7}{11} \times \frac{3}{5}.$$

$$11. \frac{5}{6} \times \frac{12}{15}.$$

$$12. \frac{7}{8} \times \frac{2}{3}.$$

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| 13. $\frac{27}{49} \times \frac{36}{64}$. | 22. $\frac{2}{3}$ of 17. | 31. $\frac{2}{11}$ of $\frac{27}{10}$ of $\frac{12}{13}$. |
| 14. $\frac{48}{49} \times \frac{63}{64}$. | 23. $\frac{3}{4}$ of 14. | 32. $\frac{2}{3}$ of $\frac{3}{4} + \frac{1}{8}$. |
| 15. $\frac{3}{5} \times \frac{4}{9} \times \frac{15}{16}$. | 24. $\frac{7}{8}$ of 20. | 33. $\frac{5}{8}$ of $\frac{9}{10} - \frac{5}{8}$. |
| 16. $\frac{14}{15} \times \frac{20}{21} \times \frac{9}{16}$. | 25. $\frac{9}{10}$ of 56. | 34. $5 + \frac{2}{3}$ of 25. |
| 17. $\frac{20}{49} \times \frac{63}{70} \times \frac{7}{18}$. | 26. $\frac{7}{12}$ of 90. | 35. $8\frac{1}{2} - \frac{4}{5}$ of $\frac{35}{16}$. |
| 18. $\frac{72}{121} \times \frac{55}{56} \times \frac{77}{96}$. | 27. $\frac{9}{11}$ of $\frac{3}{5}$. | 36. $\frac{8}{9}$ of $\frac{27}{8} - \frac{1}{4}$. |
| 19. $\frac{15}{46} \times \frac{69}{70} \times \frac{28}{27}$. | 28. $\frac{4}{5}$ of $\frac{19}{11}$. | 37. $\frac{1}{2} + \frac{2}{11}$ of $63 + \frac{3}{5}$. |
| 20. $\frac{48}{185} \times \frac{147}{64} \times \frac{49}{27}$. | 29. $1\frac{1}{2}$ of $\frac{29}{33}$. | 38. $\frac{2}{3}$ of $75 - 14\frac{2}{3}$. |
| 21. $\frac{323}{495} \times \frac{77}{95} \times \frac{21}{32}$. | 30. $\frac{7}{9}$ of $\frac{4}{15}$ of $\frac{23}{3}$. | 39. $\frac{9}{16}$ of $\frac{5}{8}$ of $\frac{44}{13}$. |

128. To multiply a mixed number by another mixed number, it is best to reduce both mixed numbers to improper fractions.

Ex. 1. What will $3\frac{3}{8}$ yards of cloth cost at $\$1\frac{1}{2}$ a yard?

$$\$1\frac{1}{2} \times 3\frac{3}{8} = \$2\frac{1}{2} = \$4\frac{1}{2}, \text{ Cost.}$$

If, however, an integer (especially if it be a large one) is to be multiplied by a mixed number, the labor of multiplication is often diminished by proceeding as follows:

Ex. 2. What will 47 yards of cloth cost at $\$2\frac{3}{8}$ a yard?

OPERATION.

$$\begin{array}{r} 47 \\ 2\frac{3}{8} \\ \hline 8)141 \\ 17\frac{3}{8} \\ \hline 94 \\ \hline 111\frac{3}{8}, \text{ Product.} \end{array}$$

EXPLANATION.

We obtain first $\frac{3}{8}$ of 47, or $17\frac{3}{8}$; then multiply 47 by 2, obtain 94, and add the results. The entire cost is therefore $\$111\frac{3}{8}$.

EXERCISE 43.

Multiply:

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| 1. $21 \times 2\frac{7}{8}$. | 7. $10 \times 4\frac{3}{11}$. | 13. $6\frac{1}{4} \times 1\frac{1}{15}$. |
| 2. $24 \times 3\frac{5}{8}$. | 8. $28 \times 7\frac{5}{8}$. | 14. $8\frac{8}{9} \times 5\frac{8}{11}$. |
| 3. $27 \times 1\frac{3}{8}$. | 9. $7\frac{1}{3} \times 80$. | 15. $20\frac{4}{3} \times \frac{39}{44}$. |
| 4. $30 \times 4\frac{3}{10}$. | 10. $9\frac{4}{5} \times 100$. | 16. $47\frac{3}{16} \times 6\frac{2}{3}$. |
| 5. $6 \times 3\frac{3}{4}$. | 11. $13\frac{2}{11} \times 51$. | 17. $64\frac{8}{15} \times 7\frac{2}{11}$. |
| 6. $7 \times 2\frac{1}{5}$. | 12. $2\frac{1}{2} \times 4\frac{2}{5}$. | 18. $101\frac{2}{3} \times 9\frac{14}{15}$. |

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| 19. $\frac{3}{7}$ of $\frac{5}{8}$ of $23\frac{1}{10}$. | 22. $\frac{19}{27} \times 2\frac{1}{4}$ of $15\frac{1}{2}$. |
| 20. $\frac{7}{8}$ of $\frac{1}{2}$ of $33\frac{1}{4}$. | 23. $8\frac{2}{3} \times 5\frac{1}{4} \times 3\frac{1}{18}$. |
| 21. $\frac{1}{5}$ of $7\frac{7}{8} \times 7\frac{7}{11}$. | 24. $\frac{1}{24}$ of $9\frac{3}{4} \times 11\frac{5}{11}$. |

Find the areas of the following floors (or ceilings) in square feet:

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| 25. 16 feet long, $12\frac{2}{3}$ feet wide. | 27. 21 feet by $8\frac{1}{3}$ feet. |
| 26. 15 feet long, $13\frac{1}{2}$ feet wide. | 28. 19 feet by $15\frac{2}{3}$ feet. |
| 29. 30 feet by $20\frac{1}{2}$ feet. | 30. 28 feet by $18\frac{5}{8}$ feet. |

Find the areas of the walls of the following rooms:

31. Distance around (perimeter) is 80 feet, and height is $10\frac{1}{4}$ feet.
32. 36 feet in perimeter, and $7\frac{2}{3}$ feet high.
33. 41 feet in perimeter, and $8\frac{5}{12}$ feet high.
34. $46\frac{1}{2}$ feet in perimeter, and $7\frac{3}{4}$ feet high.

EXERCISE 44.

ORAL.

Find:

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| 1. $\frac{3}{8}$ of 90. | 5. $1\frac{2}{3} \times 1\frac{3}{4}$ | 9. $\frac{3}{4}$ of 19. |
| 2. $\frac{1}{2}$ of 55. | 6. $\frac{2}{3}$ of 50. | 10. $1\frac{1}{2} \times 2\frac{3}{4}$. |
| 3. $\frac{5}{8}$ of $1\frac{1}{2}$. | 7. $\frac{3}{5}$ of 27. | 11. $3\frac{3}{4} \times 2\frac{1}{4}$. |
| 4. $2\frac{1}{2} \times \frac{1}{10}$. | 8. $\frac{1}{12}$ of 80. | 12. $8\frac{7}{11} \times 1\frac{2}{3}$. |

13. What will $7\frac{3}{4}$ pounds of sugar cost at 6 cents a pound?
14. If $3\frac{5}{12}$ yards of cloth are necessary for a coat, how many yards will be required for 10 coats?
15. A man paid to each of nine laborers $2\frac{1}{4}$ dollars. How much did he pay to all?
16. If tea is worth $\$2\frac{1}{2}$ a pound, what will $\frac{5}{8}$ of a pound cost?
17. If ribbon is worth $\$2\frac{1}{3}$ a yard, what will $3\frac{3}{4}$ yards cost? What will $\frac{1}{4}$ a yard cost?

EXERCISE 45.

Required the cost of:

- 27 books @ $\$1\frac{1}{4}$ apiece.
- 42 pairs of shoes @ $\$2\frac{3}{4}$ each.
- 125 dozen pencils @ $\$1\frac{1}{3}$ a dozen.

4. $7\frac{7}{8}$ tons of hay @ $\$11\frac{1}{5}$ a ton.
5. 64 lamps @ $\$5\frac{3}{8}$ each.
6. 27 lbs. sugar @ $5\frac{3}{4}$ cents per pound.
7. 145 tons of coal @ $\$5\frac{2}{3}$ a ton.
8. $5\frac{5}{8}$ cords of wood @ $\$5\frac{1}{4}$ a cord.
9. $14\frac{1}{2}$ months of board @ $\$18\frac{3}{4}$ a month.
10. $76\frac{3}{4}$ acres of land @ $\$21\frac{7}{10}$ an acre.
11. $103\frac{1}{2}$ acres of land @ $\$61\frac{1}{8}$ an acre.
12. How many square feet in the ceiling of a room $19\frac{1}{8}$ feet long and $8\frac{7}{8}$ feet wide?
13. How many square feet in the walls of a room whose perimeter is $141\frac{1}{3}$ feet and height $17\frac{3}{4}$ feet?
14. If I withdraw from a bank $\frac{3}{8}$ of my deposit, and then $\frac{2}{5}$ of the remainder, what part do I draw the second time? What part of the whole deposit is left?
15. I owned $\frac{3}{4}$ of $\frac{2}{3}$ of a business and sold $\frac{1}{2}$ of my share. What part of the entire enterprise do I still own?
16. If a wagon-wheel $16\frac{1}{4}$ feet in circumference revolves $43\frac{1}{2}$ times in going a certain distance, how many feet in that distance?
17. Of a pole $\frac{1}{6}$ is red, $\frac{2}{3}$ is white, and the rest is black. What part is black?
18. Of another pole $\frac{1}{6}$ is red, $\frac{2}{3}$ of the remainder is white, and the rest is black. What part is black?
19. From a roll of cloth containing $35\frac{3}{4}$ yards, $16\frac{5}{8}$ yards were sold at one time, and, at another, $\frac{4}{5}$ of the remainder. How many yards still remain?

IV. DIVISION OF FRACTIONS.

129. To Divide a Fraction by a Whole Number.—We may divide a number of fractional units just as we divide a number of any other units. Thus, just as
 $12 \text{ dollars} \div 3 = 4 \text{ dollars}$,
 so, $12 \text{ fifteenths (of any unit)} \div 3 = 4 \text{ fifteenths (of this unit)}$
 or, $\frac{12}{15} \div 3 = \frac{4}{5}$, *Quotient.*

Instead of dividing the numerator of the fraction by the divisor, it may be necessary to perform the division by multiplying the denominator by the divisor (see B, Art. 112).

Ex. If 4 yards of calico cost $\frac{7}{8}$ of a dollar, what will 1 yard cost?

$$\frac{\$7}{8} \div 4 = \$\frac{7}{8 \times 4} = \$\frac{7}{32}$$

In this division we diminish the size of the fractional units (from eighths to thirty-seconds), but leave the number of units unchanged.

130. To Divide a Fraction by a Fraction.—If it be required to determine how many times $\frac{7}{10}$ is contained in $\frac{3}{4}$, we may proceed as follows:

$\frac{7}{10}$ is contained in \$1 ten times, hence, $\frac{7}{10}$ is contained in \$1, $\frac{4}{7}$ of 10 times, or $\frac{40}{7}$ times; if $\frac{7}{10}$ is contained in 1 dollar $\frac{40}{7}$ times, it is contained in $\frac{3}{4}$ of a dollar, $\frac{3}{4} \times \frac{40}{7}$ times, or $\frac{15}{7}$ times, or in brief,

$$\frac{3}{4} \div \frac{7}{10} = \frac{3}{4} \times \frac{40}{7} = \frac{30}{7} = \frac{4\frac{2}{7}}{1}, \text{ Quotient.}$$

Similarly, any number of fractional units may be divided by another number of fractional units of the same kind of quantity, or by an abstract number of fractional units. Hence, to divide one fractional number by another, we have the following convenient mechanical rule: *invert the divisor and proceed as in multiplication.*

Ex. Divide $\frac{4}{5}$ by $\frac{6}{15}$.

$$\frac{4}{5} \div \frac{6}{15} = \frac{4}{5} \times \frac{15}{6} = \frac{4}{1}, \text{ Quotient.}$$

131. To Divide one Mixed Number by Another.—First reduce the mixed numbers to improper fractions.

Ex. 1. If one yard of cloth costs $\$2\frac{1}{4}$, how many yards may be bought for $\$40\frac{1}{2}$?

$$40\frac{1}{2} \div 2\frac{1}{4} = \frac{81}{2} \div \frac{5}{4} = \frac{81}{2} \times \frac{4}{5} = 18, \text{ No. of yards.}$$

However, in dividing a large mixed number by an integer, labor may often be saved by first dividing the integral part of the mixed number by the divisor.

Ex. 2. If one ton of coal costs \$6, how many tons may be bought for \$142½?

$$\begin{array}{r} \$6 \overline{) \$142\frac{1}{2}} \\ 23\frac{1}{2}, \text{ No. of tons.} \end{array}$$

Dividing \$6 into \$142½, we obtain 23 for a quotient, with a remainder of 4½. 4½ divided by 6 gives ¾. Hence, the entire quotient is 23¾.

EXERCISE 46.

Divide:

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| 1. $\frac{1\frac{2}{3}}{\frac{2}{3}}$ by 3. | 14. $\frac{7}{2\frac{1}{2}}$ by $1\frac{1}{3}$. | 27. 203½ by 3. |
| 2. $\frac{2\frac{5}{9}}{11}$ by 5. | 15. $\frac{1\frac{5}{8}}{2\frac{3}{4}}$ by $2\frac{1}{2}$. | 28. 496⅔ by 12. |
| 3. $\frac{8}{11}$ by 6. | 16. $\frac{7\frac{5}{8}}{2\frac{3}{4}}$ by $1\frac{2}{3}$. | 29. 35 by 8⅞. |
| 4. $\frac{10\frac{1}{2}}{10\frac{1}{5}}$ by 8. | 17. $1\frac{3}{8}$ by $1\frac{1}{8}$. | 30. 63 by 5⅔. |
| 5. $\frac{10\frac{1}{2}}{10\frac{1}{5}}$ by 24. | 18. $5\frac{1}{4}$ by $4\frac{1}{8}$. | 31. 48 by 10⅔. |
| 6. $\frac{1\frac{6}{8}}{2\frac{3}{8}}$ by 36. | 19. $12\frac{2}{8}$ by $10\frac{5}{8}$. | 32. 63 by $7\frac{7}{11}$. |
| 7. $\frac{4\frac{5}{12}}{1\frac{2}{12}}$ by 27. | 20. $13\frac{3}{4}$ by $15\frac{1}{8}$. | 33. 96 by 14⅔. |
| 8. $\frac{5}{8}$ by 3. | 21. $20\frac{5}{8}$ by $52\frac{1}{2}$. | 34. 17 by $23\frac{1}{4}$. |
| 9. $\frac{4}{5}$ by 5. | 22. $65\frac{1}{10}$ by 3. | 35. 38 by $52\frac{1}{4}$. |
| 10. $\frac{8}{9}$ by 15. | 23. 92⅔ by 6. | 36. 55 by $66\frac{1}{2}$. |
| 11. $\frac{3\frac{1}{11}}{10\frac{1}{11}}$ by 420. | 24. $247\frac{2}{3}$ by 7. | 37. $26\frac{8}{11}$ by $2\frac{2}{3}$. |
| 12. $1\frac{7}{101}$ by 288. | 25. $342\frac{2}{3}$ by 8. | 38. $13\frac{1}{11}$ by $4\frac{7}{11}$. |
| 13. $\frac{4}{5}$ by $2\frac{7}{8}$. | 26. $153\frac{1}{2}$ by 2. | 39. $36\frac{1}{4}$ by $9\frac{5}{8}$. |

V. SIMPLIFICATION OF COMPLEX FRACTIONS.

RECIPROCAL.

132. A complex fraction is a fraction containing a fraction in its numerator or in its denominator, or in both.

Exs. $\frac{5}{\frac{2}{3}}$, $\frac{3\frac{1}{4}}{5\frac{2}{7}}$, $\frac{\frac{2}{3}-\frac{1}{2}}{3\frac{1}{5}}$.

Hence, the quotient of one fraction by another may be indicated as a complex fraction.

Thus, $\frac{3}{4} \div \frac{5}{8}$ may be written $\frac{\frac{3}{4}}{\frac{5}{8}}$.

133. The reciprocal of a number is the quotient obtained by dividing 1 by that number.

Thus, the reciprocal of 3 is $\frac{1}{3}$; of $\frac{2}{3}$ is $\frac{1}{\frac{2}{3}}$ or $\frac{3}{2}$; of $2\frac{3}{4}$ is $\frac{1}{2\frac{3}{4}}$ or $\frac{4}{11}$.

134. Simplification of Complex Fractions.—To simplify a complex fraction, simplify the numerator and denominator; divide the numerator by the denominator.

Ex. 1. Simplify $\frac{3\frac{2}{3}}{4\frac{5}{8}}$.

$$\frac{3\frac{2}{3}}{4\frac{5}{8}} = \frac{11}{\frac{13}{2}} = 11 \times \frac{2}{13} = \frac{22}{13}, \text{ Result.}$$

Ex. 2. Simplify $\frac{5\frac{1}{2} - 3\frac{1}{4}}{\frac{5}{8} \text{ of } \frac{2}{3}}$. $\frac{5\frac{1}{2} - 3\frac{1}{4}}{\frac{5}{8} \text{ of } \frac{2}{3}} = \frac{11 - \frac{3}{4}}{\frac{5}{8} \times \frac{2}{3}} = \frac{3\frac{5}{4} - \frac{20}{8}}{\frac{5}{8}}$

$$\frac{13}{8} = \frac{13}{6} \times \frac{9}{5} = \frac{39}{10} = 3\frac{9}{10}, \text{ Result.}$$

Ex. 3. Simplify $\frac{7}{5 + \frac{2}{1 - \frac{1}{4 - \frac{1}{3}}}}$

$$\begin{aligned} \frac{7}{5 + \frac{2}{1 - \frac{1}{4 - \frac{1}{3}}}} &= \frac{7}{5 + \frac{2}{1 - \frac{1}{\frac{11}{3}}}} = \frac{7}{5 + \frac{2}{1 - \frac{3}{11}}} = \frac{7}{5 + \frac{2}{\frac{8}{11}}} \\ &= \frac{7}{5 + \frac{2 \times 11}{8}} = \frac{7}{5 + \frac{22}{4}} = \frac{7}{5 + \frac{11}{2}} = \frac{7}{\frac{10 + 11}{2}} = \frac{7}{\frac{21}{2}} = \frac{14}{21} = \frac{2}{3}, \text{ Result.} \end{aligned}$$

EXERCISE 47.

Simplify:

- | | | |
|--|---|---|
| 1. $\frac{\frac{5}{6}}{1\frac{9}{10}}$ | 6. $\frac{11\frac{1}{2}}{41\frac{1}{11}}$ | 11. $\frac{\frac{2}{3} \text{ of } 5\frac{1}{4}}{\frac{5}{8} + \frac{3}{4}}$ |
| 2. $\frac{1\frac{2}{3}}{3\frac{1}{8}}$ | 7. $\frac{9\frac{3}{8}}{9\frac{9}{10}}$ | 12. $\frac{\frac{2}{3} \text{ of } 9\frac{1}{6}}{1\frac{1}{3} - \frac{2}{3}}$ |
| 3. $\frac{2\frac{1}{5}}{4\frac{9}{10}}$ | 8. $\frac{101\frac{7}{8}}{13\frac{7}{8}}$ | 13. $\frac{11\frac{2}{3}}{4\frac{1}{8} - 3\frac{3}{4}}$ |
| 4. $\frac{7\frac{3}{8}}{8\frac{5}{8}}$ | 9. $\frac{17}{10\frac{1}{5}}$ | 14. $\frac{6\frac{1}{2} + 8\frac{1}{2}}{10\frac{3}{8} - 4\frac{3}{8}}$ |
| 5. $\frac{10\frac{2}{3}}{3\frac{1}{10}}$ | 10. $\frac{25\frac{5}{6}}{93}$ | 15. $\frac{8\frac{1}{2}}{7 + 4\frac{1}{4}}$ |

$$16. \frac{2\frac{5}{8} - 1\frac{5}{8}}{8\frac{1}{8} - 7\frac{5}{8}} \quad 17. \frac{\frac{4}{7} \text{ of } 9\frac{4}{5}}{9\frac{3}{10} - 5\frac{1}{5}} \quad 18. \frac{\frac{7}{13} \times 4\frac{9}{11} \times 4\frac{3}{4}}{3\frac{1}{3} - 2\frac{1}{2} + 1\frac{2}{3}}$$

19. Divide the sum of $7\frac{3}{5}$ and $9\frac{2}{7}$ by their difference.
 20. Divide the sum of $4\frac{1}{6}$, $3\frac{2}{3}$, $6\frac{1}{10}$ by their product.
 21. Multiply the sum of $\frac{1}{8}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{5}{9}$, $\frac{1}{18}$ by the least, and divide the product by the greatest of the five fractions.

What part of:

22. $6\frac{1}{4}$ is $2\frac{1}{2}$?
 23. $7\frac{1}{3}$ is $5\frac{1}{2}$?
 24. $4\frac{1}{3}$ is $3\frac{1}{2}$?
 25. $12\frac{5}{8}$ is $1\frac{3}{4}$?
 26. $14\frac{7}{10}$ is $4\frac{1}{5}$?
 27. 140 is $134\frac{1}{2}$?
 28. $3\frac{1}{11}$ is what part of $5\frac{9}{22}$?
 29. 6 is what part of $12\frac{3}{4}$?

Find the value of:

$$30. \frac{1\frac{1}{2}}{9} + \frac{3\frac{1}{4}}{26} + \frac{25}{4\frac{2}{3}} \quad 33. \frac{4\frac{2}{3}}{1\frac{1}{6}} - \frac{2\frac{1}{8}}{5\frac{2}{3}} - \frac{3\frac{5}{8}}{8\frac{1}{6}}$$

$$31. \frac{2}{7} \text{ of } 9\frac{4}{5} - 3\frac{4}{10} \quad \frac{4}{7} \text{ of } 8\frac{5}{9} - \frac{7\frac{3}{8} - 2\frac{3}{8}}{11\frac{3}{10}}$$

$$32. 7\frac{1}{5} - \frac{4}{9} \text{ of } 8\frac{1}{7} \quad 34. \frac{4\frac{2}{3}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{2} + 1\frac{5}{6}}$$

Simplify:

$$35. 2 + \frac{1}{3 + \frac{1}{4}} \quad 39. 3 - \frac{1}{10 - \frac{5}{3 - \frac{3}{1 + \frac{1}{4}}}}$$

$$36. 3 + \frac{2}{5 - \frac{1}{3}} \quad 40. 9 + \frac{8}{7 - \frac{5}{3 - \frac{6}{2 + \frac{3}{4}}}}$$

$$37. 5 - \frac{6}{2 + \frac{4}{3 - \frac{1}{3}}} \quad 41. 1 + \frac{3}{2 + \frac{5}{4 - \frac{3}{4 + \frac{1}{2}}}}$$

$$38. 1 + \frac{10}{2 - \frac{5}{3 + \frac{1}{2}}}$$

$$42. 6 - \frac{8}{1 + \frac{1}{2 + \frac{1}{1 + \frac{2}{3}}}} \quad 43. 5 + \frac{7}{4 + \frac{3}{2 + \frac{8}{3 + \frac{1}{4}}}}$$

VI. G. C. D. AND L. C. M. OF FRACTIONS.

135. G. C. D. of Fractions.—In order to find the G. C. D. of two or more fractions, the simplest method is to *reduce the fractions to their lowest terms (mixed numbers to improper fractions); reduce the fractions thus obtained to their least common denominator; find the G. C. D. of the numerators, and set the result over the common denominator.*

This is equivalent to expressing the fractional quantities in terms of the same fractional unit, and finding their G. C. D. in this form.

Ex. Find the G. C. D. of $1\frac{1}{3}$ and $1\frac{2}{3}$.

$1\frac{1}{3}$ and $1\frac{2}{3}$ reduce to $\frac{4}{3}$ and $\frac{5}{3}$, and when reduced to their common denominator are $\frac{8}{6}$ and $\frac{10}{6}$.

The G. C. D. of 20 and 24 is 4.

Hence, the G. C. D. of $1\frac{1}{3}$ and $1\frac{2}{3}$ is $\frac{4}{6}$, Result.

136. L. C. M. of Fractions.—To find the L. C. M. of two or more fractions, we proceed similarly, thus, *reduce the given fractions to their L. C. D.; find the L. C. M. of the numerators of the fractions thus obtained, and set the result over the common denominator.*

Ex. Find the L. C. M. of $1\frac{1}{3}$ and $1\frac{2}{3}$.

These fractions reduce to $\frac{4}{3}$ and $\frac{5}{3}$; the L. C. M. of 20 and 24 is 120; hence, the L. C. M. of $1\frac{1}{3}$ and $1\frac{2}{3}$ is $\frac{120}{6}$, or 20.

EXERCISE 48.

Find the G. C. D. of

1. $\frac{4}{5}$ and $\frac{6}{7}$.
 2. $\frac{5}{8}$ and $\frac{7}{10}$.
 3. $\frac{8}{11}$ and $\frac{9}{10}$.
 4. $4\frac{1}{3}$ and $1\frac{1}{4}$.
 5. $5\frac{1}{8}$ and $7\frac{1}{12}$.

Find the L. C. M. of

6. $\frac{1}{12}$, $\frac{2}{15}$, $\frac{3}{8}$.
 7. $1\frac{1}{3}$, $2\frac{1}{6}$, $3\frac{1}{2}$.
 8. $4\frac{1}{3}$, $6\frac{2}{3}$, $2\frac{1}{12}$.
 9. $2\frac{2}{3}$, $3\frac{1}{15}$, $4\frac{1}{3}$.
 10. $17\frac{1}{2}$, $16\frac{1}{3}$, $5\frac{1}{12}$.

11. $\frac{2}{3}$ and $\frac{1}{4}$.
 12. $\frac{1}{15}$ and $\frac{2}{3}$.
 13. $\frac{4}{9}$ and $\frac{5}{10}$.
 14. $2\frac{1}{3}$ and $3\frac{2}{3}$.
 15. $5\frac{1}{3}$ and $3\frac{1}{3}$.

16. $\frac{1}{15}$, $\frac{2}{3}$, $\frac{7}{8}$.
 17. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{2}$.
 18. $\frac{7}{10}$, $1\frac{2}{3}$, $1\frac{1}{4}$.
 19. $3\frac{2}{3}$, $1\frac{1}{3}$, $11\frac{1}{12}$.
 20. $4\frac{1}{3}$, $6\frac{1}{3}$, $2\frac{1}{12}$.

VII. ANALYSIS INVOLVING FRACTIONS.

137. I. Given the value of a number of integral units, to find the value of another number of units.—We may proceed in the same way as in Art. 95, where only integers are involved. In all cases it is important to save labor by the use of cancellation wherever possible.

Ex. 1. If 7 chickens cost $\$4\frac{3}{8}$, what will 16 chickens cost?

Cost of 7 chickens = $\$4\frac{3}{8}$ or $\$3\frac{5}{8}$.

Cost of 1 chicken = $\frac{1}{7}$ of $\$3\frac{5}{8}$ = $\$3\frac{5}{56}$.

Hence, cost of 16 chickens = 16 times $\$3\frac{5}{56}$ or $\$10$.

It may be that the value of a number of fractional units is required.

Ex. 2. If 10 acres of land cost $\$1124$, what will $5\frac{3}{4}$ acres cost?

Cost of 10 acres = $\$1124$.

Cost of 1 acre = $\$112\frac{4}{10}$.

Cost of $5\frac{3}{4}$ acres = $\$112\frac{4}{10} \times 2\frac{3}{4}$ = $\$646\frac{3}{10}$, *Result*.

EXERCISE 49.

1. If 3 pounds of candy cost $8\frac{1}{4}$ cents, what will 8 pounds cost at the same rate? $3\frac{2}{3}$ pounds?
2. If 7 pairs of boots cost $\$23\frac{5}{8}$, what will be the cost of 12 pairs?
3. How many yards of cloth will be required for 16 coats, if 11 coats can be cut from $34\frac{5}{8}$ yards?
4. How many tons of hay will a horse require in 365 days, if he eats $1\frac{9}{10}$ tons in 133 days?
5. When $\$355$ will buy 15 acres of land, what are $8\frac{1}{2}$ acres worth?
6. A bar of metal 5 feet long weighs $35\frac{5}{8}$ pounds. What will a similar bar $3\frac{3}{4}$ feet long weigh?
7. If a load of 40 bushels of lime weigh 3210 pounds, what will be the weight of a like load containing $72\frac{2}{3}$ bushels?

8. If there are $404\frac{1}{4}$ cubic inches in 7 quarts of milk, how many cubic inches in $42\frac{2}{3}$ quarts?

9. In 15 links there are $118\frac{1}{2}$ inches. How many inches in $41\frac{1}{3}$ links?

10. If $\$57$ buy 9 rolls of cloth, how many rolls will $\$53\frac{1}{2}$ buy?

11. If 7 loads of lumber cost $\$95\frac{1}{2}$, how many loads can be bought with $\$162\frac{1}{2}$?

138. II. Given the value of a number of fractional units, to find the value of a another number of other units.

In this case the process consists in brief in *finding the value of a single fractional unit, then finding the value of a single integral unit, then finding the value of any number of other units.*

Ex. 1. The value of $\frac{2}{3}$ of a steamboat is $\$12000$. What is the value of the entire steamboat?

Value of $\frac{2}{3}$ of the vessel = $\$12000$.

Value of $\frac{1}{3}$ of the vessel = $\$6000$.

Value of $\frac{3}{3}$, or the whole of the vessel = $\$30000$, *Result*.

Ex. 2. If $\frac{3}{4}$ of an acre of land is worth $\$72$, what is the value of $\frac{5}{12}$ of an acre?

Value of $\frac{3}{4}$ acre = $\$72$.

Value of $\frac{1}{4}$ acre = $\$24$.

Value of $\frac{1}{4}$ of an acre, or of 1 acre = $\$96$.

\therefore Value of $\frac{5}{12}$ acre = $\$96 \times \frac{5}{12}$ = $\$40$, *Result*.

Ex. 3. A farmer sold $\frac{1}{3}$ of his flock of chickens, then $\frac{2}{3}$ of the remainder, and found that he had 20 chickens left. How many chickens did he have originally?

$1 - \frac{1}{3} = \frac{2}{3}$, the part of the flock left after the first sale.

$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, the part disposed of in the second sale.

$\frac{2}{3} - \frac{4}{9} = \frac{2}{9}$, the part of flock left unsold.

If $\frac{2}{9}$ of the flock = 20 chickens,

$\frac{9}{9}$, or the whole of the flock = 120 chickens, *Result*.

EXERCISE 50.

1. If $\frac{3}{4}$ of a book contains 234 pages, how many pages in the entire book?
2. If $\frac{4}{5}$ of a gentleman's salary is \$3800, what is the whole salary? What is $\frac{3}{4}$ of it?
3. When a man owning $\frac{2}{3}$ of a vessel sells his portion for \$67318, what is the value of the rest?
4. Nine-tenths of a certain journey is 4770 miles; how long is the entire journey?
5. If $2\frac{2}{3}$ yards of cloth cost \$11 $\frac{1}{2}$, what will $7\frac{2}{3}$ yards of the same cloth cost?
6. If $5\frac{1}{2}$ boxes of soap cost \$52, what will be the cost of $4\frac{4}{5}$ boxes?
7. When $\frac{3}{4}$ of a mile of fence can be built for \$17 $\frac{1}{3}$, what will $4\frac{1}{3}$ miles cost?
8. If $\frac{4}{11}$ of a gallon contains 84 cubic inches, what will $\frac{13}{14}$ of a gallon contain?
9. If \$25 $\frac{1}{2}$ purchase $3\frac{1}{2}$ cords of wood, how many cords will \$184 $\frac{1}{2}$ secure?
10. If 75 hairs of a certain length weigh $3\frac{1}{2}$ drams, how many hairs of the same size will be required to weigh $21\frac{3}{4}$ drams?
11. When \$57 $\frac{3}{4}$ will buy $5\frac{1}{4}$ acres of land, what are $7\frac{3}{4}$ acres worth?
12. A bar of iron $4\frac{2}{3}$ feet long weighs $26\frac{1}{4}$ pounds. What will a similar bar of iron $11\frac{1}{3}$ feet long weigh?
13. If for $7\frac{1}{3}$ days of labor a man receive \$6 $\frac{2}{3}$, what will be due him for $\frac{2}{3}$ of 1 day?
14. If \$454 $\frac{3}{10}$ will buy $11\frac{1}{2}$ acres of land, how many dollars will be required to buy $4\frac{2}{3}$ acres?
15. A boy loses $\frac{1}{4}$ of his marbles and gives $\frac{1}{8}$ away. He still has 20. How many had he at first?
16. After selling $\frac{2}{3}$ of my farm and giving $\frac{1}{8}$ to my son, I have 220 acres left. How many acres did I sell?
17. One day I read $\frac{1}{3}$ of the pages of a book; the next day

$\frac{1}{3}$, and the next day $\frac{2}{10}$. There still remained 60 pages. How many pages in the book?

18. A pole is $\frac{1}{4}$ white, $\frac{2}{7}$ red, $\frac{5}{14}$ blue, and the rest, which is 12 feet, is in the ground. How long is the pole and how many feet are above ground?

19. A lad loses $\frac{2}{3}$ of his marbles and then gives $\frac{1}{3}$ of the remainder away. He finds that 12 remain. How many had he at first? How many did he give away?

20. After selling $\frac{2}{7}$ of my farm, I gave $\frac{3}{8}$ of the remainder to my son and have 142 acres left. How many acres had I at first? How many did I give to my son?

21. One day I read $\frac{1}{8}$ of a book; the next, $\frac{1}{2}$ of the remainder and had 155 pages left. How many pages in that book?

22. A gentleman left $\frac{1}{3}$ of his property to his wife; $\frac{1}{4}$ of the remainder to his son; $\frac{1}{4}$ of what still remained to his daughter, who received \$1575. What was the value of the estate?

139. Synopsis of Principles Relating to Fractions.—It will be a useful exercise for the pupil to collect and tabulate the essential principles relating to fractions. Thus, in outline,

FIRST PRINCIPLES OF FRACTIONS.

- A. If the numerator and denominator of a fraction be both multiplied, or both divided, by the same number, the value of the fraction is not changed.
- B. Multiplying the denominator of a fraction by a given number divides the value of the fraction by that number.
- C. Dividing the denominator of a fraction by a given number multiplies the value of the fraction by the same number.

TRANSFORMATIONS OF FRACTIONS.

- I. To reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the sum over the denominator.
- II. To reduce an improper fraction to a mixed number, divide the numerator by the denominator, and to the quotient annex the remainder placed over the denominator.

III. and IV., Etc., Etc.

PROCESSES WITH FRACTIONS.

Etc., Etc.

EXERCISE 51.

REVIEW.

Which is the greater and how much?

$$1. \frac{2}{3} \text{ of } 5\frac{1}{4} \text{ or } \frac{2}{3} \text{ of } 4\frac{5}{8}?$$

$$2. \frac{2}{3} \text{ of } 11\frac{1}{2} \text{ or } 7\frac{1}{2} - 3\frac{3}{4}?$$

Find the sum of:

$$3. \frac{5}{15} + \frac{1}{15} + \frac{2}{15} + \frac{3}{15}$$

$$5. 6\frac{1}{2} + 7\frac{3}{4} + 5\frac{1}{2} + 4\frac{3}{4}$$

$$4. 21\frac{1}{2} + 4\frac{3}{4} + 2\frac{3}{4} + 10\frac{1}{5}$$

$$6. 11\frac{1}{4} + 3\frac{1}{2} + 4\frac{1}{2} + 1\frac{1}{2} + 42\frac{3}{4}$$

$$7. 7\frac{3}{8} + 4\frac{1}{2} + 1\frac{3}{4} + \frac{7}{16} + 7\frac{3}{8} + 4\frac{3}{8}$$

$$8. 5\frac{3}{8} + 4\frac{3}{8} + 2\frac{1}{4} + 17\frac{1}{16} + 23\frac{1}{8} + \frac{1}{16}$$

$$9. 1\frac{1}{2} + \frac{1}{4} + 3\frac{1}{8} + 7\frac{3}{8} + \frac{1}{2} + 5\frac{1}{2} + \frac{1}{2}$$

Find the value of each:

$$10. 7\frac{3}{11} - 4\frac{5}{11}$$

$$18. 7\frac{1}{11} - 5\frac{1}{11}$$

$$11. 5 - 3\frac{1}{2}$$

$$19. 8\frac{1}{11} - 1\frac{4}{11}$$

$$12. 6\frac{1}{2} - 5\frac{1}{2}$$

$$20. \frac{3}{4} - \frac{1}{4}$$

$$13. 10 - 8\frac{7}{10}$$

$$21. 5\frac{2}{10} - 1\frac{3}{10}$$

$$14. 18\frac{7}{10} - 9\frac{1}{10}$$

$$22. 1\frac{3}{8} - \frac{5}{8} + 7\frac{3}{8} - 3\frac{7}{8}$$

$$15. \frac{1}{3} - \frac{1}{6}$$

$$23. 9\frac{3}{4} + 11\frac{7}{10} - 8\frac{4}{15} - 3\frac{1}{2}$$

$$16. 3\frac{7}{12} - 1\frac{1}{12}$$

$$24. 15\frac{1}{4} - 6\frac{2}{4} - 1\frac{5}{4} - 4\frac{1}{4}$$

$$17. 1\frac{1}{18} - \frac{2}{18}$$

$$25. 19 - 4\frac{2}{3} - 6\frac{1}{10} - 2\frac{2}{3} - 5\frac{1}{4} + \frac{1}{2}$$

$$26. 15\frac{1}{2} + 14\frac{3}{11} - 4\frac{1}{2} + 1\frac{7}{10} - 3\frac{1}{2} + \frac{7}{10}$$

$$27. 4\frac{1}{2} - 1\frac{3}{4} + (1\frac{1}{2} - \frac{1}{2})$$

$$28. 6\frac{1}{2} - (3\frac{1}{2} - 1\frac{1}{2}) + 1\frac{1}{2}$$

$$30. (8\frac{1}{2} - 3\frac{1}{2}) - (9\frac{3}{4} - 7\frac{1}{4})$$

$$29. 11 + (4\frac{1}{2} + 1\frac{3}{8}) - 12\frac{7}{10}$$

$$31. 15\frac{1}{4} - (1\frac{3}{8} + 7\frac{1}{8} - 5\frac{3}{8})$$

Multiply together:

$$32. 7\frac{1}{2} \times 1\frac{2}{3}$$

$$34. 9\frac{1}{4} \times 1\frac{1}{4}$$

$$36. 9\frac{3}{4} \times 5\frac{5}{12} \times 6\frac{1}{15}$$

$$33. 5\frac{1}{4} \times 4\frac{3}{4}$$

$$35. \frac{9}{11} \times 4\frac{2}{3} \times 1\frac{1}{2}$$

$$37. 15\frac{3}{4} \times 13\frac{1}{4} \times 1\frac{1}{5}$$

Divide:

$$38. 9\frac{3}{4} \text{ by } 5\frac{3}{4}$$

$$40. 16\frac{1}{2} \text{ by } 6\frac{1}{4}$$

$$42. 19\frac{1}{2} \text{ by } (5\frac{1}{2} \times 3\frac{3}{4})$$

$$39. 10\frac{1}{2} \text{ by } 11\frac{1}{2}$$

$$41. \frac{2}{3} \text{ of } 19\frac{1}{2} \text{ by } 3\frac{3}{4}$$

$$43. 15\frac{5}{11} \times 5\frac{2}{3} \text{ by } 5\frac{1}{4}$$

What part of:

$$44. 2\frac{3}{4} \text{ is } \frac{1}{2}?$$

$$47. \frac{1}{3} \text{ of } 6\frac{3}{4} \text{ is } \frac{2}{3} \text{ of } \frac{7}{8}?$$

$$45. 7\frac{1}{2} \text{ is } 1\frac{2}{3}?$$

$$48. \frac{7}{12} \text{ of } 7\frac{1}{2} \text{ is } \frac{1}{12} \text{ of } 4\frac{4}{5}?$$

$$46. 10\frac{1}{2} \text{ is } \frac{1}{4}?$$

$$49. \frac{9}{11} \text{ of } 15\frac{2}{3} \text{ of } 7\frac{1}{2} \text{ is } 19\frac{1}{2}?$$

Simplify:

$$50. \frac{5\frac{1}{2}}{3\frac{1}{2}}$$

$$53. \frac{17\frac{7}{8}}{44\frac{9}{14}}$$

$$56. 6\frac{1}{4} \times \frac{3}{4\frac{1}{2}}$$

$$51. \frac{3\frac{2}{11}}{9\frac{1}{2}}$$

$$54. \frac{7 - 3\frac{1}{2}}{32\frac{3}{8}}$$

$$57. \frac{7\frac{1}{2}}{1\frac{3}{8}} + \frac{4\frac{3}{8}}{9\frac{1}{10}}$$

$$52. \frac{7\frac{7}{10}}{9\frac{1}{4}}$$

$$55. \frac{71\frac{1}{2}}{17\frac{1}{2} + 2\frac{3}{8}}$$

$$58. \frac{4\frac{1}{8}}{5\frac{1}{2}} + \frac{8\frac{1}{2}}{11\frac{1}{2}}$$

$$59. \frac{27}{37\frac{1}{2}} \times \frac{87\frac{3}{4}}{98\frac{3}{8}} \times \frac{7}{2\frac{1}{2}} \times \frac{89\frac{5}{11}}{128}$$

$$60. \frac{3}{8} \times \frac{17}{96} \times \frac{76}{93\frac{1}{2}} \times \frac{44}{1\frac{1}{7}} \times \frac{1}{11}$$

$$61. \frac{2}{3} \text{ of } 7\frac{1}{2} + \frac{5}{8} \text{ of } 9\frac{5}{11} - \frac{1}{11} \text{ of } 7\frac{1}{4}$$

$$62. \frac{7}{10} \text{ of } 6\frac{3}{8} - \frac{2}{3} \text{ of } 9\frac{1}{4} + \frac{1}{5} \text{ of } 4\frac{1}{6} + 3$$

$$63. 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4}}}$$

$$67. 3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{4}}}}$$

$$64. 2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4}}}$$

$$68. 4 + \frac{1}{2 + \frac{1}{3 + \frac{7}{6 - \frac{2}{3}}}}$$

$$65. 3 + \frac{1}{4 + \frac{1}{1 + \frac{1}{4}}}$$

$$69. 2 - \frac{4}{2 + \frac{3}{3 + \frac{1}{5 - \frac{6}{1 + \frac{1}{2}}}}}$$

$$66. 1 + \frac{1}{6 + \frac{1}{2 + \frac{1}{6 + \frac{1}{2}}}}$$

70. If one man earn \$2 $\frac{1}{2}$ in one day, what will 70 men earn in 10 $\frac{3}{4}$ days?71. If the dividend is 41 $\frac{1}{2}$ and the quotient 21 $\frac{6}{13}$, what is the divisor?72. If the product is 89 $\frac{3}{8}$ and the multiplicand is 9 $\frac{5}{8}$, find the multiplier.73. If the dividend is 54 $\frac{1}{10}$, the quotient 11 $\frac{2}{5}$, and the remainder 2 $\frac{1}{2}$, find the divisor.74. When the divisor is 8 $\frac{1}{2}$, the quotient 9 $\frac{1}{2}$, and the remainder 4 $\frac{1}{2}$, what is the dividend?75. What must $\frac{7}{11}$ of 6 $\frac{1}{11}$ be multiplied by to produce $\frac{3}{8}$ of 8 $\frac{1}{4}$ of 6 $\frac{1}{15}$?76. Three men, A, B, C, agree to do a piece of work for \$100, sharing equally. But upon completion it is found that A has done $\frac{2}{5}$ of it, and B $\frac{1}{10}$ of it. What part did C do, and how much money ought he have?77. If $\frac{3}{11}$ of a piece of work is done in 22 $\frac{1}{2}$ days, how much of it will be done in 26 $\frac{3}{4}$ days?78. The sum of $\frac{2}{3}$ and $\frac{1}{4}$ of a certain number is 170 $\frac{3}{10}$. What is the number?

79. The difference between $\frac{3}{7}$ and $\frac{7}{8}$ of a number is $26\frac{4}{7}$. Find the number.
80. A school of 150 pupils has only $\frac{2}{3}$ as many boys as girls. How many boys are there in the school?
81. There are 126 green and white balls in a box, but the number of green balls is $\frac{2}{3}$ the number of white ones. How many are there of each?
82. A farmer put his 1000 sheep into two pastures, and in one pasture he put $\frac{2}{3}$ as many as in the other. How many sheep are there in each?
83. After a man has walked $4\frac{1}{2}$ hours on a journey of $31\frac{1}{2}$ miles, he finds that he has traveled $\frac{1}{3}$ of the number of miles remaining. How fast is he walking?
84. How many tons of ore must be taken from the mine, so that after a loss of $\frac{2}{3}$ in roasting, and $\frac{1}{10}$ of the remainder in smelting, there may be 210 tons of pure metal left?
85. A farmer sowed $\frac{1}{3}$ of a field in oats, $\frac{2}{5}$ of the remainder in buck-wheat, and planted $\frac{1}{4}$ of what was left in potatoes; there still remained 8 acres for grass. How many acres in the whole field?
86. There are 5 farms marked A, B, C, D, and E, respectively. Farm A contains $253\frac{1}{2}$ acres; farm B contains half as much as A; farm C is half as large as B, and so on to E. How many in E, and how many in them all together?
87. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $6\frac{1}{2}$ by $\frac{1}{3}$ of $\frac{2}{5}$ of $7\frac{1}{2}$.
88. From $27\frac{9}{10}$ acres I sell to one man $5\frac{5}{8}$ acres at $\$70\frac{3}{4}$ an acre; to another man $7\frac{1}{2}$ acres at $\$85\frac{3}{4}$ an acre; and to a third man the remainder at $\$92\frac{3}{4}$ an acre. Find proceeds of entire tract.
89. In exchange for $7\frac{1}{4}$ dozen eggs at $23\frac{1}{2}$ cents a dozen, and $15\frac{1}{4}$ pounds of butter at $27\frac{1}{2}$ cents a pound, a man takes oats at $5\frac{5}{8}$ cents a quart. How many quarts will he receive?
90. A merchant bought 3 pieces of silk for $\$655\frac{1}{2}$. The first contained $30\frac{7}{10}$ yards, the second $42\frac{1}{2}$ yards, and the third $47\frac{3}{8}$ yards. He wishes to sell the silk so as to gain $\frac{1}{3}$ of the cost. At what price must he sell it per yard? Find selling price of each piece.
91. A man has $\frac{1}{3}$ of his property invested in real estate, $\frac{2}{5}$ of the remainder in stocks, $\frac{1}{4}$ of what is still remaining in machinery, and the residue, which is $\$3500$, in the bank. What is the value of his entire property?
92. Simplify $\frac{2}{3}$ of $\frac{4\frac{5}{12}}{12\frac{1}{4}}$ of $\frac{3\frac{4}{11}}{11\frac{3}{4}} \div 3\frac{3}{11}$.
93. Divide $1\frac{1}{3} \times \frac{3\frac{3}{8}}{\frac{5}{8} \times 1\frac{2}{3}}$ by $\frac{7\frac{3}{8} - 4\frac{1}{2}}{1\frac{1}{4} \div \frac{6}{25}} \times \frac{7\frac{2}{3}}{31\frac{2}{3}}$.
94. The sum of $\frac{3\frac{3}{8} \times \frac{5}{8}}{4\frac{2}{3} \div 9\frac{3}{4}}$ and $\frac{\frac{1}{2} \text{ of } 1\frac{7}{8}}{\frac{1}{3} \text{ of } 2\frac{1}{4}}$ is how many times their difference?

95. The product of 3 numbers is $453\frac{1}{2}$; two of them are $5\frac{5}{8}$ and $11\frac{1}{4}$; find the third.
96. If $\frac{2}{3}$ of a ton of coal cost $\frac{2}{3}$ of $\$9\frac{3}{8}$, what will $\frac{1}{10}$ of a ton cost? What will $9\frac{3}{10}$ tons cost?
97. If 6 be added to both terms of the fraction $\frac{2}{1\frac{1}{2}}$, is the fraction increased or diminished, and how much?
98. Same question, if 6 be subtracted from both terms of same fraction.
99. There are 3 numbers, the least of which is $7\frac{7}{11}$. The second is $3\frac{3}{4}$ times as large as the first, and the third $3\frac{3}{4}$ times as large as the second. Find their sum.
100. A man sold $\frac{2}{3}$ of his farm to one neighbor and $\frac{5}{11}$ of it to another. There remained 90 acres. How many acres in the farm at the beginning?
101. If $3\frac{1}{2}$ yards of silk cost $\$10\frac{1}{2}$, what will $\frac{2}{3}$ of $8\frac{3}{4}$ yards cost at the same rate?
102. If a man saw $\frac{3}{4}$ of $4\frac{3}{4}$ cords of wood in a day, how many cords will he saw in $5\frac{3}{4}$ days?
103. If a man walk $28\frac{1}{2}$ miles in one day, how many days will he require to walk $177\frac{3}{4}$ miles?
104. Find the cost of $8\frac{3}{4}$ yards of carpet when $3\frac{1}{4}$ yards cost $\$10\frac{3}{4}$.
105. If $\frac{2}{3}$ of a ton of coal cost $\$6$, how many tons can be bought for $\$67\frac{1}{4}$?
106. If 4 be subtracted from both terms of the fraction $\frac{1\frac{1}{2}}{\frac{1}{2}}$, is its value increased or diminished, and how much?
107. A horse and cow were bought for $\$160$, and the cow cost $\frac{2}{3}$ as much as the horse. Find the cost of each.
108. The sum of $\frac{2}{3}$ and $\frac{1}{3}$ of a certain number is $388\frac{3}{4}$. Find the number. Find the difference between $\frac{2}{3}$ and $\frac{1}{3}$ of it.
109. If a man can do a piece of work in 12 days and a boy can do it in 18 days, what part can the man do in one day? In 5 days? What part can the boy do in one day? In 8 days? What part can they both do in one day? In 4 days? How many days will they require to do it all, working together?
110. If a man can mow a field in 15 days and his son can do it in 27 days, answer the same seven questions about them.
111. Mr. A. can dig a certain ditch in 6 days, Mr. B. in 10 days, and Mr. C. in 15 days. Find the number of days required by each pair of men working together. Also the number required by all three together.
112. One pipe, X, can empty a cistern in 8 hours; Y, in 9 hours; Z, in 12 hours. The cistern is full and all pipes are open, how long will be required for them to empty the cistern?