CHAPTER X.

DECIMAL FRACTIONS.

140. Definitions.—If any integral unit (as one apple) be divided into 10 equal parts, each of these parts is called one-tenth. If one-tenth be divided into 10 equal parts, each of these parts is called one-hundredth. Similarly, from one-hundredth we form one-thousandth, etc. A set of fractional units so obtained is called a set of decimal units.

A **decimal fraction,** a decimal, is a fraction whose denominator is 10, or 100, or 1000, or some other power of 10.

A **mixed decimal** is a number composed partly of integers and partly of decimals.

141. Advantage in the Use of Decimal Fractions. Notation.—Since each decimal unit is one-tenth as great as the decimal unit which precedes it, a set of decimal fractions can be expressed in a simplified way similar to that used in expressing integral numbers in the decimal scale. This is done by the use of what is called the *decimal point*, and letting the position of each figure to the right of the decimal point determine the size of the decimal unit which this figure represents.

Thus, instead of 29 yds. + $\frac{3}{10}$ yds. + $\frac{7}{100}$ yds. + $\frac{4}{1000}$ yds. + $\frac{5}{10000}$ yds., we write 29.3745 yds.

Thus the labor of writing the denominators of the various fractions is saved, since the denominator of each decimal figure is determined by the decimal point and the number of figures between the decimal point and the figure considered.

Thus, in the above illustration, the unit represented by the figure 4, or *thousandths*, is determined by the decimal point and the two figures 3 and 7 intervening between the decimal point and the 4.

It should be observed that the source of this advantage lies in the fact that each figure is put to several uses. Thus, 3 not only expresses the number of tenths, but it also helps to determine the decimal denomination or local value of 7, 4, and 5, and hence serves four purposes at once.

This economy in representing fractions leads to other advantages in operating with the fractions after they are expressed in the decimal notation.

142. Illustrations of Decimal Fractions.—The most familiar illustration of decimal fractions is found in the money used in the United States. The primary unit, one dollar, is divided into ten equal parts called *dimes*, each dime is divided into ten equal parts called *cents*, and each cent into ten equal parts called *mills*. Thus, 12 dollars, 8 dimes, 6 cents, and 5 mills can be briefly expressed by the aid of the decimal notation as $12.865.

The ease and rapidity with which calculations can be made when money is expressed on a decimal scale will be appreciated by the student when he comes to reckon with money expressed in some other way, as, for instance, by pounds, shillings, and pence, as in English money.

So great are the advantages of subdividing a unit by the decimal method that this method is being applied more and more widely wherever possible. Thus, engineers divide the unit of length, the foot, not into inches, but into tenths and hundredths. Astronomers frequently divide the year decimally, indicating, for instance, April 1, 1879, by 1879.25. They also sometimes divide a degree of longitude decimally, instead of into degrees and minutes, using, for instance, 324.5° for 324° 30′. The United States Treasury Department uses tenths of a foot, pound, etc., instead of the ordinary fractions.

143. Metric System.—An entire system of weights and measures, based on decimal divisions of the fundamental units, has been devised and is in use in all civilized countries except Great Britain and the United States.

A unit of length is taken, called the *meter*, which is divided into tenths called *decimeters;* each decimeter is subdivided into ten equal parts called *centimeters*, etc. Similarly the unit of weight, the *gram*, is divided by the decimal system, as also are the units of area and volume, the *are* and the *stere*. This system of decimal units will doubtless come, in time, to be used by the entire civilized world. See page 326.

144. Notation and Numeration of Decimals.—The positional system of expressing fractions by the aid of the decimal point has been explained in Art. 15. The following table will enable the pupil to give readily the decimal unit which each figure in a decimal represents.

Order of Unit.	Ten millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
Number.	7	3	4	5	1	2	7	8	.	4	6	5	7	8	3	1	0	5
Place.	8th.	7th.	6th.	5th.	4th.	3rd.	2nd.	1st.		1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.
	Integers.									Decimals.								

145. Reading Decimals.—The most convenient way of reading decimals is to express each decimal number in terms of the smallest decimal unit and read the number of such units.

Thus, to read 0.37, instead of reading three tenths and seven hundredths, we express the tenths as thirty hundredths, and read the entire decimal fraction as 37 hundredths. Similarly, the decimal fraction expressed in the above table (Art. 144), viz.: 0.465,783,105, is read 465 millions 783 thousands 105 billionths. Hence, in general, *read the decimal as if it were a whole number, and give it the name of the last decimal place.*

In reading *whole numbers* never use "and," but in reading a *mixed decimal put "and" in place of the decimal point.*

Thus, 462 reads "four hundred sixty-two."
4.062 reads "four *and* sixty-two thousandths."

146. Writing Decimals.—Similarly, to express in figures a decimal which is given in general language, *express the numerator in figures, and then fix the decimal point so that the name of the last figure shall express the denomination of the given decimal.*

Ex. 1. Express in figures "four hundred sixty-two thousandths."

We write 462 and place the decimal point immediately to the left of the 4, since 2 must come in the third or thousandths place, and obtain .462, *Result.*

Observe that four hundred *and* sixty-two thousandths would be written 400.062.

EXERCISE 52.

Express correctly as decimal fractions.

1. Forty-six hundredths.
2. Ten and sixteen hundredths.
3. Seven and fifty-one thousandths.
4. Thirty-six millionths.
5. Two hundred twelve hundred-thousandths.
6. Five and five millionths.
7. Seventy-five and forty-two ten-thousandths.
8. Seven hundred six thousandths.
9. Seven hundred and six thousandths.
10. One thousand five hundred and one tenth.
11. Four hundred and four hundred one thousandths
12. Two hundred forty-one and four hundred twelve millionths.
13. Eighty-nine and ninety-eight hundred-millionths.
14. One thousand and one thousandth.
15. Three thousand and three millionths.
16. Sixteen ten-millionths.
17. $\frac{7}{100}$; $\frac{51}{10000}$; $1\frac{2}{1000}$; $31\frac{701}{1000000}$; $25\frac{405}{1000000}$.
18. $3\frac{91}{1000}$; $7\frac{6}{10}$; $71\frac{17}{100}$; $45\frac{46}{1000000}$; $9\frac{9}{10}$; $9\frac{9}{100}$.

Read the following decimal fractions:

19. 0.78; 1.071; 20.05; 275.572; 0.4758.
20. 0.705; 0.0102; 100.0301; 51.0007; 0.003001.
21. 300.001; .301; 6175.0214; 5001.005001.

147. Primary Processes with Decimals.—The simplicity of the decimal system of fractions is such that certain elementary methods of operating with them arise immediately from the notation.

1. Shifting the decimal point one place to the right increases the value of the decimal tenfold; shifting it two places to the right increases its value one hundredfold, etc.

Thus, 0.063 = 10 × .0063.

For by moving the decimal point one place to the right, each figure in the decimal is made to express a number ten times as great as it did at first.

2. Shifting the decimal point one place to the left decreases the value of the decimal to one-tenth of what it was; two places, to one-hundredth, etc.

Thus, .0055 = 0.055 ÷ 10.

3. Any number of zeroes may be annexed to the right of a decimal without changing its value.

Thus, 0.3 = 0.30 = 0.300, etc.

OPERATIONS WITH DECIMALS.

148. I. **Addition of Decimals.**—If the numbers to be added be so arranged that their decimal points shall be in the same column, all the decimal units of the same order, as tenths, hundredths, etc., will be in the same column, and may be added by columns.

Ex. Add $5.69, $100.257, $37.015.

OPERATION.

$ 5.69
100.257
37.015
$142.962, *Sum.*

EXPLANATION.

Arranging the numbers so that the decimal points are in the same column, we begin at the right hand, or thousandths, column to add. 7 thousandths + 5 thousandths make 12 thousandths, or 1 hundredth and 2 thousandths. Setting down the 2 thousandths, we carry 1 to the hundredths column, and continue the work, "carrying" wherever necessary, just as in the case of the addition of integers in the decimal system.

Hence, in general, to add decimals, *write the numbers so that the decimal points shall be in the same column; begin with the right-hand column and add; place a decimal point between the units and tenths of the result.*

Hence, in the addition of decimal fractions we are saved the labor of reducing fractions to fractions having a common denominator, which is necessary in the addition of common fractions.

EXERCISE 53.

Add:

1.	*2.*	*3.*
$27.05	5.571 inches.	1.0071 square yards.
123.74	93.428 inches.	.0382 square yard.
6.735	.96 inch.	5.917 square yards.
2.045	.407 inch.	41.0328 square yards.
38.7	8.14 inches.	17.51 square yards.

4. $57.13 + $7.15 + $0.61 + $70.09.
5. $125.74 + $307.06 + $51.075 + $6.305.
6. 8.08 + 1.001 + 101.0101 + 3040.1304 + 0.1345.
7. 270.01 + 31.0031 + 0.0073 + 25 + 43.0106 + 4.008.
8. 27.35 mi. + 4.701 mi. + 34.375 mi. + 8.0704 mi.
9. 7.9324 + 79.324 + .079324 + 7932.4 + 0.79324.

149. II. **The subtraction of decimals** is similar in method to the addition of decimals; that is, *write the subtrahend under the minuend, so that the decimal points shall be in the same column; begin at the right hand to subtract.*

Ex. At six o'clock the mercury in a certain barometer stood at 39.3 inches; at 10 o'clock the mercury in the same barometer stood at 39.215 inches. How many inches had it fallen?

39.300 inches.
39.215 inches.
.085 inch *Difference.*

EXERCISE 54.

	1.	*2.*	*3.*	*4.*
From	$7.35	$18.196	71.44 inches	65.03
take	3.21	4.75	38.67 inches	47.903

	5.	*6.*	*7.*	*8.*
From	51.7	301.04	19.4003	3.41
take	4.52	79.5281	9.876	2.5807

9. 0.54 − 0.37.	*13.* 0.185 − 0.0917.	*17.* 1. − 0.1.
10. 1.28 − 1.1.	*14.* 0.042 − 0.0318.	*18.* 0.01 − 0.003.
11. 9.53 − 7.99.	*15.* 70.07 − 6.408.	*19.* 10 − .001.
12. 3.01 − 2.714.	*16.* 301.5 − 30.105.	*20.* 2 − 0.010203.

21. Find the difference between $75.08 and $87.85.

22. What is the difference between 3.141592 and 3.142857?

23. From an account of $175.43, a man drew $46.95. How much remained?

24. Upon three days a gentleman deposited in a bank $27.54, $35.97, and $71.16, and on the fourth day withdrew $49.73. How much remained?

25. Find the difference between six hundred twenty-eight thousandths, and four hundred *and* sixty-nine thousandths.

150. III. **Multiplication of Decimals.**—To obtain a method of multiplying one decimal number by another, we shall take an example and work it first by the method of common fractions.

Ex. Multiply 3.372 by 2.28.

Expressing the decimal fractions as common fractions, we have,

$$3.372 \times 2.28 = \frac{3372}{1000} \times \frac{228}{100} = \frac{3372 \times 228}{100000} = \frac{768816}{100000}$$
$$= 7.68816, \textit{ Product.}$$

Hence, *the number of decimal places in the product is equal to the number of zeroes in the two denominators, that is, to the number of decimal places in the multiplier and multiplicand taken together.*

Hence, the above multiplication might have been performed as follows:

```
   3.372
    2.28
   26976
   6744
  6744
  7.68816, Product.
```


Or, in general, *multiply as in whole numbers; point off as many decimal places from the right in the product as there are decimal places in both multiplier and multiplicand taken together, prefixing zeroes to the product if necessary.*

Ex. Multiply 3.0125 by .00104.

```
     3.0125
     .00104
     120500
    30125
   .003133, Product.
```

It is to be observed that as compared with the multiplication of common fractions, in the multiplication of decimals the multiplication of the two denominators is abbreviated into a mere placing of the decimal point in the product.

EXERCISE 55.

	1.	*2.*	*3.*	*4.*	*5.*
Multiply	8.3	3.5	7.6	10.4	.88
	5.	1.2	.05	0.15	.7

6. Multiply each of the following by 4:
1.5; 2.4; 12; 63.5; 23.14; 75.007.

7. Multiply each of the following by 3.6:
2.5; 13; .07; 1.05; 4.005; 1.0008.

8. Multiply each of the following by 2.05:
0.32; 0.036; 10.08; 200.04; 35.1; 71.09.

9. 2 × 0.2.	*11.* 1 × .01.	*13.* 100.1 × 1.001.
10. 20 × .02.	*12.* 100 × .001.	*14.* 3.003 × .03003.

15. 5 × .4 × 30 × 1.8.	*18.* 5.3 × 2.01 × 0.46.
16. 1.1 × 4 × 5.5 × .02.	*19.* 27.5 × 1.6 × .014.
17. 0.24 × 25 × .004 × 10.	*20.* 5.8 × .025 × 1.003.

21. Find the value of 76.235 acres of land at $51.24 an acre.

22. Find the cost of 128.4 yards of cloth at $2.125 a yard.

23. Find the weight of 26.735 cubic yards of earth if one cubic yard weighs 0.76 of a ton.

24. 61.038 tons of hay are worth how much at $20.25 a ton?

25. What is the simplest method of multiplying by 10? By 100? By 1000?

26. What is the simplest way of multiplying by 0.1? By .01? By .001?

151. IV. **Division of Decimals** may be performed directly, but it is of advantage first to multiply the divisor and dividend by such a number (10, 100, 1000, etc.) as will remove the decimal point from the divisor. This will leave the value of the quotient unchanged (See A, Art. 112). The multiplications required are performed by shifting decimal points (See Art. 147).

Ex. 1. Divide .0221 by .013.

If we multiply both divisor and dividend by 1000, that is, shift the decimal point three places to the right in each of them, the value of the quotient will be unchanged and the divisor will be an integer. Hence, we have

OPERATION.

```
13)22.1(1.7, Quotient.
   13
    91
    91
```

Since $22.1 = \frac{221}{10}$, we really divide 221 tenths by 13; hence, the quotient is 17 tenths, or 1.7. Hence, it is necessary in each case to mark off as many decimal places in the quotient as there are decimal places in the dividend, or, in general, *move the decimal point in both divisor and dividend as many places to the right as there are decimal places in the divisor; divide as with integers; mark off as many decimal places from the right in the quotient as there are decimal places in the dividend.*

Ex. 2. Divide .004551 by 1.5.

OPERATION.

```
15).045510(.003034, Quotient.
    45
     51
     45
      60
      60
```

EXPLANATION.

As the divisor, 1.5, contains one decimal place, we move the decimal point one place to the right in both divisor and dividend. This will leave the value of the quotient unchanged. Hence, the quotient of .045510 by 15, or .003034, is the required quotient.

152. Abbreviated Cases.—The student may state for himself the abbreviated ways of dividing a number by 10, 100, 1000, etc.; also by .1, .01, etc.

If the divisor be a whole number ending in one or more zeroes, as, for instance, in dividing 16.45 by 7000, it is more convenient to divide first by 1000 by shifting the decimal point in the dividend three places *to the left*, and then dividing by 7, that being the remaining factor of the divisor. Thus,

```
7).01645
  .00235, Quotient.
```

EXERCISE 56.

Divide:

1. 7.5 by 3.
2. .075 by 5.
3. 3.24 by 18.
4. 25.6 by 32.
5. .0121 by 11.
6. .0513 by 27.
7. 4.185 by 15.
8. 2.4 by .3.
9. 24 by .8.
10. .27 by .9.
11. 3.6 by 1.2.
12. .42 by .14.
13. .063 by .07.
14. .084 by .12.
15. .096 by .008.
16. .007 by .025.
17. .4 by 4.
18. 5 by .5.
19. .06 by 6.
20. 4.5 by 50.
21. 10 by .01.
22. .01 by 100.
23. 25 by .05.
24. .02 by .005.
25. 16.8 by .021.
26. .945 by 1.35.
27. 46.5 by .015.
28. 70.8 by .004.
29. 10.11 by .01011.
30. .228 by 120.
31. 700 by 6.25.
32. .7 by 625.
33. 1.405 by 2810.
34. 4.64 by .145.
35. 1.2915 by .041.
36. 30.622 by 12.2.
37. .203412 by 2.01.
38. 63817.2 by .311.
39. 14.17 by .325.
40. 87.098 by 4.07.
41. 20.202 by .025.
42. 30030.3 by .0375.
43. .00123 by .075.
44. .0456 by .0076.

Divide correctly to four decimal places:

45. 7.101 by 19.
46. 31.76 by 23.
47. 101.5 by 30.7.
48. .0077 by .058.

The pupil should be required to solve an indefinite number of this kind of examples.

RELATION OF DECIMAL FRACTIONS TO COMMON FRACTIONS.

153. I. **To reduce a decimal fraction to an equivalent common fraction**, it is evidently sufficient to *write the decimal fraction as a common fraction and reduce it to its lowest terms.*

Ex. 1. Express .75 as a common fraction.

$$.75 = \tfrac{75}{100} = \tfrac{3}{4}, \textit{ Result.}$$

Ex. Reduce $.56\tfrac{1}{4}$ to a common fraction.

$$.56\tfrac{1}{4} = \frac{56\tfrac{1}{4}}{100} = \frac{\tfrac{225}{4}}{100} = \frac{225}{400} = \frac{9}{16}, \textit{ Result.}$$

Or,

$$.56\tfrac{1}{4} = .5625 = \tfrac{5625}{10000} = \tfrac{9}{16}.$$

EXERCISE 57.

Reduce each decimal to its equivalent common fraction in its lowest terms:

1. .8.	*8.* .075.	*15.* $.66\tfrac{2}{3}$.	*22.* 1.4.
2. .25.	*9.* .625.	*16.* $.12\tfrac{1}{2}$.	*23.* 2.52.
3. .32.	*10.* .875.	*17.* $.62\tfrac{1}{2}$.	*24.* 5.08.
4. .125.	*11.* .925.	*18.* $.06\tfrac{1}{4}$.	*25.* 6.4124.
5. .275.	*12.* .006.	*19.* $.16\tfrac{2}{3}$.	*26.* 10.11375.
6. .375.	*13.* .015.	*20.* $.14\tfrac{2}{7}$.	*27.* 7.00064.
7. .025.	*14.* $.33\tfrac{1}{3}$.	*21.* $.08\tfrac{1}{3}$.	*28.* 1.0875.

154. II. **To reduce a common fraction to a decimal** we may *regard the numerator of the common fraction as an integer, and divide it by the denominator.*

Ex. 1. Reduce $\tfrac{7}{8}$ to the form of a decimal fraction.

OPERATION.

8)7.000
.875, *Result.*

EXPLANATION.

7 units = 7000 thousandths of a unit; hence, $\tfrac{1}{8}$ of 7000 thousandths is 875 thousandths, or .875.

Ex. 2. Express $\tfrac{2}{3}$ as a decimal fraction.

OPERATION.

3)2.0000
.6666 . . .
or, .6667 —, *Result.*

EXPLANATION.

In this case, no matter how far we continue the division the quotient will not terminate. As it is convenient to terminate the quotient at some place, as the fourth, and as the next figure is 6, or more than half of a unit in the fourth place, we write 7— as the last figure in the quotient, giving .6667 as the quotient.

In general, a decimal which continues to repeat the same figure, or set of figures, is called an *infinite*, or *repeating, decimal.* If, after reducing a fraction to its lowest terms, the denominator contains any factor beside 2 or 5 (which are the only exact divisors of 10, beside 10 and 1), the division of the numerator by the denominator of a common fraction will produce an infinite decimal. Since it has been shown, however (Art. 78), that all figures beyond the sixth and seventh places vanish into insignificance for all practical purposes, it is sufficient ordinarily to let the division terminate at the sixth or seventh place. If the remainder is less than a half, we reject it and annex a plus sign; if equal to or greater than a half, we increase the last figure of the quotient by 1 and annex a minus sign.

155. III. **Comparative value of decimal and common fractions.**

On comparing decimal and common fractions, it will be observed that it is sometimes more advantageous to use one, sometimes the other. In general, decimal fractions have a simpler notation, since their denominators are indicated by a decimal point merely. This leads to special advantages in adding, subtracting, multiplying, and dividing fractions, which have already been pointed out. On the other hand, in particular instances a common fraction is simpler than a decimal fraction, thus $\tfrac{1}{8}$ is simpler than 0.125, and $\tfrac{1}{7}$, than 0.142857142.

In general, we may say that as a system for standard use the decimal system of fractions is superior, but that it is advantageous to supplement it by the use of common fractions in certain special cases. Hence, the tendency in practical life is to use systems of decimals (as the metric system) wherever possible, using common fractions in a supplementary way.

EXERCISE 58.

Reduce each common fraction to an equivalent decimal:

1. $\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{3}{4}, \tfrac{4}{5}, \tfrac{1}{8}, \tfrac{5}{8}, \tfrac{7}{16}, \tfrac{11}{16}, \tfrac{8}{25}, \tfrac{21}{25}, \tfrac{19}{32}, \tfrac{31}{32}$.

2. $\tfrac{17}{20}, \tfrac{29}{20}, 1\tfrac{5}{16}, 4\tfrac{18}{25}, \tfrac{15}{64}, \tfrac{31}{64}, 1\tfrac{45}{64}, 3\tfrac{27}{80}, 1\tfrac{71}{80}$.

Find correctly to five decimal places the value of:

3. $\frac{1}{6}$, $\frac{2}{7}$, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{8}{13}$, $\frac{15}{17}$, $\frac{4}{19}$, $1\frac{1}{15}$, $3\frac{4}{15}$, $5\frac{10}{21}$.

4. $\frac{7}{30}$, $\frac{9}{400}$, $\frac{11}{600}$, $\frac{27}{7000}$, $\frac{37}{9000}$, $1\frac{75}{76}$, $\frac{301}{11000}$.

What decimal fraction is,

5. 20 of 80? Of 50? Of 140?
6. 16 of 48? Of 40? Of 200?
7. 24 of 36? Of 60? Of 192?
8. 36 of 54? Of 96? Of 64?

EXERCISE 59.

Perform the operations indicated:

1. $2.5 + 1\frac{3}{5} + 4.07 + 3\frac{7}{200}$.

2. $5.06 - 4.001 + \frac{7}{16} - .09 + 3.02\frac{2}{3} + \frac{1}{3}$.

3. $.0001 + 1\frac{3}{5000} - \frac{5}{32} + 1.03\frac{1}{2} - .07\frac{3}{8}$.

4. $5.4 + .05\frac{2}{5} - .0054 - 5.00\frac{4}{9} + 7.125$.

5. From six hundred and seven thousandths take six hundred seven thousandths.

6. Subtract nine hundred forty and seventy-six millionths from ten hundred twenty and eight tenths.

7. Multiply 7.0032 by $5\frac{3}{80}$.

8. Multiply .00075 by $1.03\frac{3}{4}$.

9. Multiply 2.10007 by .1072.

10. Multiply 12.35 by $.005\frac{1}{4}$.

Divide

11. .7644 by .0052.
12. .00169 by 2.6.
13. 2890 by .085.
14. .002 by 500.
15. 501 by .0003.
16. 5 by .00025.
17. .0001 by 1000.
18. 1000 by .00001.
19. 7.3 by 8000.
20. 40.1 by .00125.

21. If the divisor is 4.153, the remainder .02375, and the quotient 4.25, what is the dividend?

22. If $\frac{2}{3}$ of a bushel of corn be worth $\frac{3}{7}$ of a bushel of wheat, and wheat be worth $1.40 a bushel, how many bushels of corn can be bought for $27?

23. $\frac{9}{13}$ of $\frac{17}{50}$ of 56 times what number equals 50.4?

APPLICATIONS OF THE DECIMAL SYSTEM.

156. Decimal Systems of Money.—Owing to the advantages which arise from the decimal method of representing units and parts of a unit, decimal systems of money have come to be used in all civilized countries except Great Britain. Thus, in France, the **franc** is divided into 100 equal parts called **centimes**; in Germany, the **mark** is divided into 100 equal parts, called **pfennige**, etc.

This general adoption of decimal systems of money is due to the fact that money and its units are used more often and reckoned with more extensively than any other system of units, as, for instance, those of length, weight, etc. Hence, the aggregate of economies which result from the use of a decimal scale for units of money is greater than it would be in the case of any other class of units as those of length, weight, etc.

157. United States Money.—The primary unit in the system of money in use in the United States is the dollar. The other units used in connection with the dollar, and their relation to each other, are shown in the following table:

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar or $1.
10 dollars = 1 eagle.

By means of the decimal system, all the other units of United States money may be expressed as dollars or fractions of a dollar. Thus, 7 eagles, 8 dollars, 4 dimes, 5 cents, and 3 mills are most conveniently *written* as $78.453. Such sums are, however, most conveniently *read* in terms of two units, *dollars* and *cents*. Mills are used only for purposes of computation; if in any result the number of mills is less than 5, it is rejected; if it is 5 or more than 5, it is reckoned as 1 cent.

Thus, $78.453 is read as 78 dollars, 45 cents, 3 mills; or rejecting the mills, 78 dollars, 45 cents.

158. Aliquot Parts of a Dollar.—Operations with United States money are often much facilitated by remembering the number of units which form aliquot parts of a dollar (See Art. 74).

Thus,
$6\frac{1}{4}$ cents = $\frac{1}{16}$ of \$1.
$8\frac{1}{3}$ cents = $\frac{1}{12}$ of \$1.
$12\frac{1}{2}$ cents = $\frac{1}{8}$ of \$1.
$16\frac{2}{3}$ cents = $\frac{1}{6}$ of \$1.

The student may supply the remaining aliquot parts of **\$1.**

Ex. What is the cost of 18 pounds of sugar at $6\frac{1}{4}$ cents a pound?

$$18 \times 6\tfrac{1}{4} \text{ cents} = 18 \times \$\tfrac{1}{16} = \$\tfrac{9}{8}.$$
$$= \$1.125.$$
$$= \$1.13, \textit{ Result.}$$

EXERCISE 60.

Add—

1. \$27.315 + \$15.05 + \$70.145 + \$48.76.

2. Thirteen dollars and thirty-two cents, seven dollars and nineteen cents, forty dollars and ninety-six cents.

3. Twelve dollars and eighty cents, seventy-five cents, eight dollars and three cents, fifty dollars and ten cents.

4. Eleven dollars and seven cents, six cents, nineteen dollars, sixty dollars and nine cents.

5. From \$100 take \$30.25.

6. Take \$25.08 from \$30.41.

7. A man gave \$50 in payment of three items of \$10.76, \$16.13 and \$21.05. How much was due him in return?

8. A farmer bought $65\frac{3}{4}$ acres of land at \$51.50 an acre. What was the amount paid?

9. What will $5\frac{2}{3}$ yards of carpet cost at \$1.75 per yard? At \$2.05 per yard?

10. At $12\frac{1}{2}$ cents a line, what will it cost to insert in a newspaper an advertisement of 17 lines? Of 45 lines?

11. How many chairs at \$2.25 each can be bought with \$29.25? With \$83.25?

12. How many acres of land worth \$71.30 an acre can be bought with \$2032.05?

13. With pads worth 12 cents each, a boy gets all he can for \$4.92. How many did he buy?

14. A certain kind of stock is worth $\$27\frac{1}{2}$ a share. How many shares can be bought for \$1347.50?

15. A man paid \$26.25 for 5 tons of coal. What would $17\frac{2}{5}$ tons cost at the same rate?

16. There are $272\frac{1}{4}$ square feet in a square rod. What will be the cost of $20\frac{2}{3}$ square rods of land at $4\frac{1}{2}$ cents a square foot?

17. A dealer bought 3 barrels each containing 31.5 gallons of oil at the rate of $45\frac{1}{2}$ cents a gallon. He sold it at $51\frac{1}{4}$ cents a gallon. How much was his gain? (Solve this problem by two methods.)

18. A miller filled 125 barrels of flour at a cost of \$7.75 each. He then sold 90 of them for \$8.30 each, and the remainder at \$6.35 each. What was his total gain?

19. A farmer spent on a crop of grain \$55.40 for seed, \$1.75 each, for 20 days of labor, and \$62.35 for rent. How much would he gain if the crop yielded 625.30 bushels which sold for 84 cents apiece?

Solve by the method of aliquot parts of \$1.

20. 27 lbs. × \$0.25.
21. 48 lbs. × $\$0.37\frac{1}{2}$.
22. 70 oz. × $\$0.33\frac{1}{3}$.
23. 66 oz. × \$0.60.
24. 94 ft. × $\$0.66\frac{2}{3}$.
25. 68 bu. × $\$0.87\frac{1}{2}$.
26. 95 qt. × $\$0.12\frac{1}{2}$.
27. 83 gal. × \$0.75.
28. 77 in. × $\$0.66\frac{2}{3}$.
29. 80 lbs. × $\$0.66\frac{2}{3}$.
30. 54 lbs. × $\$0.16\frac{2}{3}$.
31. 77 pt. × $\$0.37\frac{1}{2}$.
32. 46 pk. × $\$0.33\frac{1}{3}$.
33. 48 bu. × \$0.25.
34. 61 lbs. × $\$0.12\frac{1}{2}$.
35. 47 gal. × \$0.20.
36. 38 rolls × $\$.16\frac{2}{3}$.
37. 91 ft. × \$0.75.

159. Business Forms in the Use of U. S. Money, Accounts, Bills, Etc.—By the aid of certain abbreviations, and by systematic methods of arranging items, the advantages which arise from the use of a decimal system of money are further increased.

Ex. James Smith bought of Mitchell, Fletcher & Co., 17 pounds of coffee at 38 cents a pound; 75 pounds of sugar at $4\frac{1}{2}$ cents a pound; 20 pounds of oatmeal at 4 cents a pound; and 4 pounds of tea at 80 cents a pound. What is the entire cost of his purchases?

If the purchases be arranged as a *bill*, it is much easier to inspect them at a glance, to verify each item, to make corrections where necessary, and to determine of whom the purchase was made, by whom, whether it is receipted, etc. Thus,

Philadelphia, May 8, 1901.

James Smith,

Bought of Mitchell, Fletcher & Co.

17 lbs. coffee	@ 38¢	$6	46
75 ″ sugar	@ $4\frac{1}{2}$¢	3	38
20 ″ oatmeal	@ 4¢		80
4 ″ tea	@ 80¢	3	20
		$13	84

160. Business Terms and Abbreviations.—**Price** is the value of 1 unit of quantity; **cost** is the value of the entire number of units used or bought. Thus, the *price* of 1 pound of coffee is 38 cents; the *cost* of 17 pounds is $6.46.

A **bill** is a written statement showing the price, quantity, and cost of each item, and the aggregate cost of all the items.

How is a bill **receipted**? What is a **debtor**? A **creditor**?

Let the student determine the meaning of the following abbreviations used in connection with bills and accounts:

@	Cr.	Dr.	Pay't
a/c	Bal.	Per	Rec'd
	No. or #	E. and O. E.	

EXERCISE 61.

Find the amount of each of the following bills:

1.

To 6 lbs. nails	@ $5\frac{1}{2}$¢	$
″ 12 ft. zinc	@ $7\frac{1}{2}$¢	
″ 15 doz. screws	@ $4\frac{2}{3}$¢	
″ 40 squares tin	@ 17¢	

2.

To $8\frac{1}{2}$ yds. ribbon	@ $32\frac{1}{2}$¢	$
″ 15 ″ flannel	@ 65¢	
″ 38 ″ calico	@ $7\frac{1}{2}$¢	
″ 16 ″ cloth	@ $1.10	

3.

To 10 doz. eggs	@ 18¢	$
″ 30 lbs. sugar	@ $5\frac{1}{2}$¢	
″ 5 ″ tea	@ 76¢	
″ $18\frac{1}{2}$ ″ cheese	@ 16¢	
″ $\frac{3}{4}$ ″ pepper	@ 30¢	

4.

To 7 bushels potatoes	@ 63¢	$
″ 8 qts. beans	@ $10\frac{1}{2}$¢	
″ 15 ″ tomatoes	@ $3\frac{1}{2}$¢	
″ 25 gal. oil	@ $9\frac{1}{2}$¢	
″ 11 loaves bread	@ 8¢	

5.

37 yds. Brussels carpet	@ $1.65	$
14 ″ Axminster ″	@ 2.85	
$41\frac{1}{2}$ ″ Ingrain filling	@ .75	
9 small rugs	@ 2.75	
45 step pads	@ $.27\frac{1}{2}$	

6. *Philadelphia, Sept. 18, 1900.*

MRS. FLETCHER EDWARDS,

BOUGHT OF FINLEY ACKER & CO.

		$
26 jugs syrup	@ 30¢	
7 lbs. coffee	@ 28¢	
9 " rice	@ $9\frac{1}{2}$¢	
45 " oatmeal	@ $4\frac{1}{2}$¢	
30 bu. potatoes	@ 48¢	

7. *New York City, Dec. 5, 1900.*

MRS. MARION RUTLEDGE,

BOUGHT OF JOHN WANAMAKER.

		$
61 yds. muslin	@ $11\frac{1}{2}$¢	
55 " table linen	@ $1.25	
23 " silk	@ 1.15	
10 " lining	@ 23¢	
31 " carpet	@ $1.50	

8. MR. THOMAS D. KEYSER,

BOUGHT OF MESSRS. R. L. MYERS & CO.

		$
76 algebras	@ $1.25	
68 grammars	@ 1.40	
83 readers	@ 1.35	
95 Bibles	@ .90	
57 arithmetics	@ .85	

161. Articles Bought and Sold by the Hundred or Thousand.—Some of the advantages which come from the use of the decimal scale are obtained by selling articles by the hundred (or **C**), or thousand (or **M**).

Ex. 1. What will be the cost of 3760 shingles at $7 a thousand (or $7 per M).

The number of thousands is determined by moving the decimal point of 3760 three places to the left, giving 3.76 thousands in the above example.

If one thousand shingles cost $7, the cost of 3.76 thousands will be 3.76 times $7, or $26.32, *Cost.*

Similarly, the number of *tons* in a given number of pounds may be obtained by moving the decimal point three places to the left and dividing by 2.

Ex. 2. Find the cost of 13567 pounds of coal at $6 a ton.

$$13567 \text{ pounds} = \frac{13.567}{2} \text{ tons.}$$

If 1 ton cost $6, the cost of $\frac{13.567}{2}$ tons $= \$\frac{13.567}{\not{2}} \times \overset{3}{\not{6}} = \$40.701$, *Cost.*

EXERCISE 62.

1. Find the cost of 55260 cubic feet of gas at $1.40 per M.

2. Find the cost of 75490 bricks at $8.25 a thousand.

3. A coal dealer supplies a tinsmith with 7565 pounds of coal at $5.75 a ton, and the tinsmith roofs the coal-dealer's house with 156 pounds of tin at $14.15 a C. Which owes the other, and how much?

4. What will it cost to set the type of a book containing 560 pages of 1115 ems each, at 60 cents per thousand ems?

5. Required the cost of 83410 pounds of coal at $5.38 a ton, and 47380 shingles at $5.55 a thousand.

6. I borrowed $7500 for a year, at the rate of $4.50 per hundred. What must I pay for the loan?

7. I lent $31500 for a year at the rate of $5.25 per hundred. What do I receive for the loan?

162. Further Use of Base 100 in Percentage, Interest, Etc.—So great are the advantages of the decimal base in business and other computations and comparisons that the use of this base is developed into special subjects called Percentage, Interest, etc.