

22. A bin occupying $\frac{2}{3}$ of a cellar which is $20 \times 18 \times 11$ feet is $\frac{3}{4}$ full of coal (ordinary anthracite). Find its approximate value at \$5.25 a ton.
23. How many gallons will a cistern 7 feet cube contain?
24. Water weighs $62\frac{1}{2}$ pounds to the cubic foot. What will the water in a tank containing 1000 gallons weigh?
25. A county in one season built 17 miles of macadamized road 12 feet wide, at the rate of 75 cents per square yard. What was the total cost?
26. If a roll of paper a mile long will just cover $193\frac{3}{4}$ square yards, how wide is the paper? Its width is what decimal of its length?
27. What will be the cost of the palings necessary to inclose a lawn 36×41 yards, if they are 2 inches wide, placed 2 inches apart, and sell for \$2.75 a hundred?
28. How many bushels will a bin hold whose dimensions are $15 \times 11\frac{1}{2} \times 8\frac{3}{4}$ feet? How many gallons?
29. A bin $15 \times 8\frac{1}{2} \times 6\frac{3}{8}$ feet was full of apples, from which enough cider was pressed to fill a tank $9\frac{1}{2} \times 4\frac{1}{2} \times 3$ feet, $\frac{3}{4}$ full. The apples were bought at 36 cents a bushel (approximately) and the cider was sold at 33 cents a gallon. Required the gain.
30. How many square yards in a circular flower-bed 56 feet across?
31. A certain tank contains 1000 gallons. How many bushels would it hold?
32. There are to be laid two cement walks, one on each side of a certain street 2 miles long. The walks are to be 60 in. wide and to cost 40 cts. a sq. yd. What is the total cost?
33. A circular pond 28 yd. across, is surrounded by a path 4 ft. wide. What is the area of the path?
34. My farm is 220 rd. long and 144 rd. wide. It is entirely surrounded with a four-wire fence. The wire cost me half a cent a foot and the posts, which are 11 ft. apart, cost 9 cents each. What did the material cost?
35. What will be the cost of plastering the walls and ceiling of a room 16 ft. 6 in. long \times 11 ft. 4 in. wide \times 10 ft. high @ 36¢ a sq. yd., allowing 15 sq. yd. for doors, etc.?
36. If a cubic foot of water weighs $62\frac{1}{2}$ lbs., what will a barrel of water weigh?
37. How many gallons of water required to weigh a ton?
38. If the diameter of a pail is 6 in., how many square inches in the bottom and lid together?
39. There is an iron cistern made of 3-in. metal plates, and without any top. If the inner dimensions are 8 ft. long, 5 ft. wide, and 10 ft. deep, how many cubic feet of iron in the material?
40. Find the weight of that iron if iron is 7 times as heavy as water.

CHAPTER XIII.

PERCENTAGE.

228. Illustration.—A man has two investments, one of \$800 in real estate, which brings a return of \$48 a year; and another of \$500 in railroad bonds, which brings in \$35 a year. Which is the better investment?

The comparison of the two investments is much facilitated by determining the proceeds of a single hundred dollars in each case, and comparing them.

Thus, if \$800 in real estate brings in \$48 a year.

\$100 " " " \$6 a year.

If \$500 in bonds brings in \$35 a year,

\$100 " " " \$7 a year.

Since \$7 per \$100 invested is a better return than \$6 per \$100, the investment in bonds is relatively more profitable.

229. Value of 100 as a Basis of Comparison.—This use of a standard base, as 100, in making estimates, has two advantages: (1) it facilitates comparisons, as in the above example; (2) it leads in time to an instinctive grasp of the various rates used with reference to one hundred.

Thus, 6 per cent. (6 out of every hundred), 7 per cent., etc., come to have a sharp and definite meaning in the mind, which meaning rises instantly when such words are used.

230. Definitions and Symbols.—Percentage is the process of computing with reference to 100 as a base.

Per cent. (from *per*, by, and *centum*, one hundred) means by or on the hundred. Thus, when a merchant gains 15 per cent., he means that he gains \$15 for every \$100 invested

in goods. If a poultry-raiser lose 8 per cent. of his fowls, he means that he loses 8 fowls out of every hundred that he has.

Hence, so many per cent. means so many hundredths.

The symbol, %, is used for the words "per cent." Since *per cent.* means *hundredths*, per cent. may also be indicated by a common or *decimal fraction*, with 100 as a denominator. Thus,

$$6\% = \frac{6}{100} = .06; \quad 30\% = \frac{30}{100} = .30 = .3.$$

The quantities considered in computations in percentage are the *base*, *rate*, *percentage*, and the *amount* or *difference*.

The *base* is the number of which a certain number of hundredths is taken. It is denoted by the symbol, *b*.

The *rate* is the number of hundredths which is taken (or the number of units taken with reference to every hundred units in the base). When expressed decimally it is denoted by *r*.

The *percentage* is the number obtained by taking a certain per cent. of the base (or the number of units taken with reference to all the hundreds in the base). It is denoted by the symbol, *p*.

For example, if a farmer has 800 peach-trees and loses 5 per cent. of them, he loses 5 out of every hundred, or 8×5 (or 40) out of 8 hundred.

$$\begin{aligned} \therefore 800 &= \text{base.} \\ 5\% &= \text{rate.} \\ 40 &= \text{percentage.} \end{aligned}$$

Pupils are likely to confuse the terms *rate per cent.* and *percentage*, because of their similarity in sound. In order to distinguish them, it may be helpful to remember that

rate per cent. means the number by (on, or of) every *single* hundred;
percentage " " " out of *all* the hundreds in the base.

The *amount* is the sum of the base and percentage.

The *difference* is the difference of the base and percentage.

Proceeds is a general term for either amount or difference.

231. I. Given the base and rate, to find the percentage (or proceeds).

Ex. 1. A school of 250 pupils is 60% boys. How many boys are there in the school?

SOLUTION.

The number of boys in each 100 pupils = 60.

The number of hundreds of pupils = $\frac{250}{100}$.

Hence, the entire number of boys in the school

$$\begin{aligned} &= (\text{No. of hundreds of pupils}) \times (\text{No. boys in every hundred}) \\ &= \frac{250}{100} \times 60 = 250 \times \frac{60}{100} = 250 \times .60 = 150. \end{aligned}$$

Abbreviated form of computation.

$$\begin{array}{r} 250, \text{ Base.} \\ \underline{.60, \text{ Rate per cent.}} \\ 150.00, \text{ Percentage.} \end{array}$$

Hence, in general, to find the percentage,

Multiply the base by the rate expressed decimally (or in symbols, $p = b \times r$).

Also, to obtain the proceeds, *multiply the base by 1 plus the rate, or 1 minus the rate, expressed decimally.*

Ex. 2. The population of a town, which contained 18000 people, decreased 5% in a year; what was the population at the end of the year?

OPERATION.

$$\begin{array}{r} 18000, \text{ Base.} \\ \underline{.95, \text{ Final rate per cent.}} \\ 90000 \\ 162000 \\ 17100.00, \text{ Final population.} \end{array}$$

EXPLANATION.

If a number be diminished by 5% of itself, it becomes 95%, or .95 of itself. 95% of 18000 is 17100.

EXERCISE 108.

Find:

- | | |
|--------------------|------------------------------------|
| 1. 3% of 800 boys. | 6. 12% of 530 tons. |
| 2. 25% of 40 days. | 7. 6% of \$325. |
| 3. 10% of 70 lbs. | 8. 80% of \$4500. |
| 4. 18% of \$420. | 9. $4\frac{1}{2}\%$ of 600 pupils. |
| 5. 50% of 90 bu. | 10. $7\frac{1}{4}\%$ of 80 days. |

11. Thirty per cent. of a drove of 240 horses were sold in one day. How many were sold?

12. I invested \$675 for a year at 6%. What amount was due me at the end of the year?

13. Which is the greater, $9\frac{1}{2}\%$ of \$750, or $27\frac{1}{8}\%$ of \$240?

14. Smith had 230 lambs and bought 20% more. How many had he then?

15. Mr. Cox lost 15% of his 640 tons of hay. How many tons did he lose? How many remained?

16. A city of 78000 inhabitants gained 12% in three years. What was its population after this gain?

17. A gentleman's salary of \$3500 was increased 16%. What is his present salary?

18. A house formerly valued at \$8500 decreases 8%. What is the valuation now?

Find the percentage and the proceeds:

- | | |
|----------------------------|---|
| 19. \$7000 gaining 11%. | 22. 15 ft. gaining 20%. |
| 20. 950 pupils losing 16%. | 23. $12\frac{1}{2}$ bu. decreasing 5%. |
| 21. 875 men gaining 24%. | 24. 240 trees increasing $7\frac{1}{2}\%$. |

EXERCISE 109.

ORAL.

- | | | |
|---------------|---------------|--------------------------------|
| 1. 3% of 200. | 5. 25% of 4. | 9. 16% of 50. |
| 2. 7% of 500. | 6. 30% of 5. | 10. 18% of 30. |
| 3. 8% of 40. | 7. 40% of 6. | 11. 45% of 40. |
| 4. 20% of 65. | 8. 25% of 90. | 12. $7\frac{1}{2}\%$ of \$800. |
13. Of 75 examples, a boy did 40%. How many did he solve?
14. A man 60 years old has a son 35% of his age. How old is the son?
15. A certain salary of \$2500 is to be increased 12%. What will it then be?
16. My expenses last year were \$1400, and this year will be 5% more. What will they be this year?
17. The length of a field is 30% more than its width, which is 230 rds. What is its length?
18. The price of a carriage was 8% less than that of a horse which cost \$150. What was the price of the carriage?

19. Lost 5% of an investment of \$8000 and then gained 30% of the remainder in speculation. What had I then?

20. What is 25% of 10? Of $2\frac{3}{8}$? Of $5\frac{1}{4}$? Of 100?

232. Special Cases.—When the rate per cent. is an aliquot part of 100 (see Art. 74), the process of computing the percentage may often be abbreviated.

Thus, $33\frac{1}{3}\% = \frac{1}{3}$, and if it be required to find $33\frac{1}{3}\%$ of a given number, we may substitute the simpler process of taking $\frac{1}{3}$ of the number. Similarly,

$6\frac{1}{2}\% = \frac{1}{16}$.	$20\% = \frac{1}{5}$.	$50\% = \frac{1}{2}$.
$8\frac{1}{3}\% = \frac{1}{12}$.	$25\% = \frac{1}{4}$.	$62\frac{1}{2}\% = \frac{5}{8}$.
$12\frac{1}{2}\% = \frac{1}{8}$.	$33\frac{1}{3}\% = \frac{1}{3}$.	$66\frac{2}{3}\% = \frac{2}{3}$.
$16\frac{2}{3}\% = \frac{1}{6}$.	$37\frac{1}{2}\% = \frac{3}{8}$.	$87\frac{1}{2}\% = \frac{7}{8}$.

Ex. 1. A man owns $12\frac{1}{2}\%$ of a vessel valued at \$24000. What is the value of his share?

$$12\frac{1}{2}\% \text{ of } \$24000 = \$24000 \times \frac{1}{8} = \$3000, \text{ Result.}$$

Care should also be exercised in dealing with cases where the rate % is very small or very large, as $\frac{1}{8}\%$ or 225%.

Thus, $\frac{1}{8}\%$ is $\frac{1}{8}$ of 1% = $.00\frac{1}{8} = .00125$; 225% = 2.25 when expressed decimally.

Ex. 2. A broker bought \$1200 worth of stocks and charged $\frac{1}{8}\%$ commission. What was his commission?

\$1200, Base.
<u>.00$\frac{1}{8}$, Rate.</u>
\$1.50, Percentage or commission.

EXERCISE 110.

Find:

- | | |
|-------------------------------------|--------------------------------|
| 1. 25% of 44 books. | 8. 100% of 325 years. |
| 2. $16\frac{2}{3}\%$ of 96 lambs. | 9. 120% of 290 tons. |
| 3. $33\frac{1}{3}\%$ of 750 boys. | 10. 215% of 480 cities. |
| 4. $37\frac{1}{2}\%$ of 640 pounds. | 11. $\frac{1}{2}\%$ of \$800. |
| 5. 20% of 36 miles. | 12. $\frac{3}{4}\%$ of \$68. |
| 6. $66\frac{2}{3}\%$ of 450 days. | 13. $\frac{3}{8}\%$ of \$720. |
| 7. 50% of 225 acres. | 14. $\frac{1}{3}\%$ of \$3500. |

15. What is the commission on a sale of \$42000 at $\frac{1}{8}\%$?
16. A city had a population of 8750 some years ago, but has now 240 per cent. of its numbers at that time. What is its present population?
17. A man weighs 270 per cent. of what he did when as a boy he weighed 60 pounds. What is his weight?
18. A gentleman spent $2\frac{1}{8}\%$ of an income of \$2800 in books. How many dollars was that?
19. Mr. Jones owned $\frac{2}{3}$ of a farm worth \$8000, but sold $37\frac{1}{2}\%$ of his share. What part does he still own, and what is its value?
20. Which is greater, $87\frac{1}{2}\%$ of \$250 or $\frac{5}{8}\%$ of \$35000?

EXERCISE 111.

ORAL.

Find:

- | | |
|-----------------------------|--|
| 1. 50% of 90. | 5. $66\frac{2}{3}\%$ of 75. |
| 2. $33\frac{1}{3}\%$ of 66. | 6. $37\frac{1}{2}\%$ of $5\frac{1}{2}$. |
| 3. 75% of 52. | 7. $\frac{1}{2}\%$ of 600. |
| 4. $87\frac{1}{2}\%$ of 96. | 8. 125% of 60. |

233. II. To find what per cent. one number is of another (given the percentage and the base, to find the rate).

Ex. 1. A house which cost \$6000 rents for \$540. What per cent. of the cost is the rent?

SOLUTION.

If \$6000, or 60 hundreds of dollars, bring in \$540,
1 hundred dollars " " $\frac{\$540}{\$6000}$ or \$9.

Hence, \$540 is 9% of \$6000, or the income from the investment is 9%.

The same rate expressed decimally may be obtained by regarding \$540 as a fractional part of \$6000 and reducing till the denominator of the fraction is 100.

Thus, $\frac{\$540}{\$6000} = \frac{9}{100} = .09$.

Hence, \$540 is 9% of \$6000.

Hence, in general, to obtain the rate per cent., divide the percentage by the base—in symbols, $r = \frac{p}{b}$.

Sometimes the *proceeds* and *base* are given instead of the percentage. In this case the percentage may be obtained by subtracting the base from the proceeds.

Ex. 2. A dealer bought a horse for \$60 and sold it for \$75. What was his gain per cent.?

OPERATION.

\$75, Proceeds.

\$60, Base.

\$15, Percentage.

60)15.00(.25, Rate.

120

300

300

EXPLANATION.

Since the horse cost \$60 and was sold for \$75, the gain was the difference between \$75 and \$60, or \$15.

Hence, the problem reduces to finding what per cent.

15 is of 60.

EXERCISE 112.

ORAL.

Express in per cent. with decimals:

- $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{7}{10}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{25}, \frac{3}{10}, \frac{1}{50}, \frac{2}{5}, \frac{1}{100}, \frac{1}{1000}$.
- $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{7}{10}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{25}, \frac{3}{10}, \frac{1}{50}, \frac{2}{5}, \frac{1}{100}, \frac{1}{1000}$.
- What per cent. of 8 is 4? Is 2? Is 1? Is 5?
- What per cent. of 12 is 6? Is 4? Is 8? Is 2?
- What per cent. of 24 is 3? Is 6? 8? 9? 12? 15? 16?
- 12 is what per cent. of 18? Of 24? Of 30? Of 48?
- 15 is what per cent. of 25? 30? 35? 50? 75?
- 18 is what per cent. of 27? 42? 54? 48? 180?

EXERCISE 113.

Find the rate per cent. when,

- Base is \$45 and percentage is \$9.
- Base is \$240 and percentage is \$84.
- Base is 428 bushels and percentage is $160\frac{1}{2}$ bushels.
- Base is 150 days and percentage is 12 days.
- Percentage is $23\frac{1}{4}$ tons and base is 320 tons.
- Percentage is 3250 boys and base is 1400 boys.
- Percentage is \$73.50 and base is \$84.
- Percentage is \$1.40 and base is \$56.

9. Cost is \$236 and gain is \$29.50.
 10. Gain is \$0.98 and cost is \$1.40.
 11. Gain and cost are each \$75.
 12. Gain is $\frac{1}{3}$ cost; $\frac{2}{3}$ cost; $\frac{5}{8}$ cost.
 13. Cost is \$2350 and gain is \$173.90.
 14. Cost is \$85 and loss is \$6.80.
 15. Cost is \$125 and loss is \$22.50.
 16. Loss is \$2.76 and cost is \$7.36.
 17. Amount is \$800 and base is \$640.
 18. Amount is \$4397.61 and base is \$4261.25.
 19. Amount is \$4420 and percentage is \$420.
 20. Amount is \$470.61 and percentage is \$209.16.
 21. Selling price is \$120 and gain is \$30.
 22. Selling price is \$60 and cost is \$48.
 23. Selling price is \$10.83 and cost is \$9.50.
 24. Selling price is \$1462.50 and loss is \$1037.50.
 25. Selling price \$642.60 and cost is \$765.
 26. Base is \$900, percentage is \$4.50.
 27. Amount is \$7110, percentage is \$47.40.
 28. Selling price is \$136, cost is \$134.98.
 29. Cost is \$30, gain is 25 cts.
 30. Loss is 36 cents, selling price is \$62.64.
 31. Cost is \$180, loss is \$1.

What per cent. of

- | | | |
|---------------------------------------|--|--|
| 32. $\frac{5}{8}$ is $\frac{1}{2}$? | 35. $3\frac{1}{2}$ is $1\frac{1}{2}$? | 38. 15 is $12\frac{1}{2}$? |
| 33. $\frac{3}{4}$ is $\frac{1}{2}$? | 36. $5\frac{1}{4}$ is $3\frac{1}{2}$? | 39. $10\frac{2}{3}$ is $9\frac{1}{3}$? |
| 34. $\frac{5}{8}$ is $\frac{3}{16}$? | 37. $8\frac{3}{4}$ is 7? | 40. $21\frac{1}{4}$ is $1\frac{1}{16}$? |

41. In a school of 320 pupils, 176 are boys. What per cent. of the school is boys? What per cent. is girls?
 42. From a farm containing 146 acres, the owner sold 124.1 acres. What per cent. of the farm did he sell?
 43. For threshing a crop of grain amounting to 872 bu. 2 pk., the thresher took 209 bu. $1\frac{3}{4}$ pk. as his pay. What per cent. of the crop did he take?

44. Of a regiment numbering 980 men, 147 were sick and the rest able-bodied. What per cent. of the regiment entered battle?
 45. If 271 men were killed in battle and all of the sick recovered, what per cent. of the whole regiment (1000 men) would be able to enter another fight?
 46. A house which cost \$3500 was sold for \$4060. What was the gain per cent.?
 47. A pencil which cost 5 ct. sold for 6 ct. What was the gain per cent.?
 48. Which investment returns the greater per cent., \$40 in a bicycle which sold for \$50, or 3 ct. in a newspaper which sold for 4 ct.?
 49. A man has \$4500 with which to speculate. Will he do better buying cattle at \$10 and selling at \$18 a head, or buying railroad stock at \$75 and selling it at \$132? What will be his gain per cent. in the better investment?

234. III. To find a number from a given per cent. of it (given the rate and percentage, to find the base).

Ex. 1. \$30 is 12% of what number of dollars?

SOLUTION.

$$\begin{aligned} 12\% \text{ of the required number} &= \$30. \\ \therefore 1\% \text{ of the required number} &= \$\frac{30}{12}. \\ \therefore 100\% \text{ of the number, or the number itself} &= \$\frac{30}{12} \times 100 = \$250. \end{aligned}$$

$$\text{Or, more briefly, } \frac{\$30}{.12} = \$250,$$

$$(\text{since } \$\frac{30}{12} \times 100 = \$\frac{30}{12} \times \frac{1}{100} = \$\frac{30}{.12} \times \frac{1}{.01} = \$\frac{30}{.12} = \$250.)$$

Ex. 2. A house is sold for \$4800 at a gain of 20%. What was the cost of the house?

SOLUTION.

$$\begin{aligned} \text{Making the cost of the house the base,} & \\ 120\% \text{ of the cost of the house} &= \$4800. \\ \therefore 1\% \text{ of the cost of the house} &= \frac{\$4800}{120} = \$40. \\ \therefore 100\% \text{ or the whole of the cost of the house} &= \$40 \times 100 = \$4000. \\ \text{Or more briefly, cost of house} &= \frac{\$4800}{1.20} = \$4000. \end{aligned}$$

Hence, in general, to find the base, either proceed by analysis, that is, obtain 1 per cent. of the base by dividing the percentage by the per cent., and multiply the result by 100, in order to obtain 100 per cent. of the base; or, divide the percentage by the rate, expressed decimally (in symbols $b = \frac{P}{r}$).

The amount or difference being given to find the base, divide the amount by 1 plus the rate; or, divide the difference by 1 minus the rate, i. e., base = $\begin{cases} \text{amount} \div (1 + \text{rate}). \\ \text{difference} \div (1 - \text{rate}). \end{cases}$

EXERCISE 114.

Find the base if it is given that:

1. Percentage is \$120 and rate is 6%.
2. Percentage is 760 bu. and rate is 5%.
3. Percentage is 115 gal. and rate is $2\frac{1}{2}\%$.
4. Rate is 4% and percentage is 550 da.
5. Rate is $3\frac{1}{2}\%$ and percentage is \$553.
6. Rate is 23% and percentage is \$27.83.
7. Proceeds are \$1086.40 and gain is 12%.
8. Proceeds are 23 tons and loss is 20%.
9. Rate of loss is 13% and proceeds are \$453.27.
10. Gain is 18% and proceeds are \$372.88.
11. 140 is 8% of what number?
12. 169 is $3\frac{1}{4}\%$ of what number?

Of what number is

- | | | |
|--------------|---------------|-------------|
| 13. 30, 5%? | 15. 725, 25%? | 17. 7, 4%? |
| 14. 120, 6%? | 16. 486, 30%? | 18. 17, 5%? |

Find the quantity of which:

- | | |
|---------------------|--|
| 19. 48 bu. is 20%. | 25. 97 is $12\frac{1}{2}\%$. |
| 20. 75 da. is 25%. | 26. 123 is $66\frac{2}{3}\%$. |
| 21. 145 tons is 5%. | 27. 429 is $37\frac{1}{2}\%$. |
| 22. 67 mi. is 40%. | 28. 45 is $3\frac{1}{2}\%$. |
| 23. \$73.35 is 15%. | 29. 19 is $\frac{1}{2}\%$. |
| 24. \$1476 is 48%. | 30. $31\frac{1}{4}$ is $\frac{3}{8}\%$. |

What quantity increased by:

- | | |
|--------------------------------|--|
| 31. 9% of itself is \$54.50? | 34. 15% of itself is \$11.04? |
| 32. 27% of itself is \$439.42? | 35. $33\frac{1}{3}\%$ of itself is \$114.48? |
| 33. 60% of itself is 12? | 36. $8\frac{2}{3}\%$ of itself is 1.163403? |

What quantity diminished by:

- | | |
|--|--|
| 37. 25% of itself is 96? | 40. $16\frac{2}{3}\%$ of itself is $76\frac{1}{2}$? |
| 38. 30% of itself is 175 da.? | 41. 19% of itself is $6\frac{3}{4}$ mi.? |
| 39. $28\frac{1}{2}\%$ of itself is \$167.31? | 42. 88% of itself is 0.3? |

Find cost if:

43. Selling price is \$50 and gain is 25%.
44. Selling price is \$308.14 and gain $8\frac{1}{2}\%$.
45. Selling price is \$7.14 and loss is $33\frac{1}{3}\%$.
46. Selling price is \$286.02 and loss is $9\frac{1}{5}\%$.
47. A city whose population is now 74250, has gained 10% in a year. What was its population a year ago?
48. If during a year I spend 45% of my earnings, and have \$1100 saved, what was my income?
49. A boy weighing 90 lbs. gained 20% during the last year. What was his weight a year ago? What will be his weight a year hence at the same rate per cent. of growth?
50. A pencil which sold for 8 cents brought 60% gain. What did it cost?
51. A grocer sold a score of eggs for 55 cents, thereby gaining 10%. What did a dozen eggs cost him?
52. A farmer lost 12% of his lambs by death, and 23% by theft, and there were 585 lambs remaining. How many had he at first?
53. A housekeeper wishes to use $124\frac{1}{4}$ yards of muslin and knows it will shrink $2\frac{1}{2}\%$. How many yards should she buy?
54. I sold 2 horses for \$60 each; on the one I gained 20%, and on the other I lost 20%. Did I gain or lose on them both together? How much?

55. The distance between two stops was $42\frac{3}{4}$ miles, or $7\frac{1}{2}\%$ of the entire journey. What was the length of the journey?

56. The gain was \$39.60, or 20% of the cost. What was the cost?

57. The loss was \$76.40, or 80% of the cost. What was the cost?

58. In a transaction a man gained 8%, but actually gained \$1256. How much did he invest?

EXERCISE 115.

ORAL.

1. If percentage is \$75 and rate is 3%, what is the base?
2. What is the base when rate is 8%, and percentage is 24 bu.?
3. \$24 is 20% more than what number of dollars?
4. 27 is 10% less than what number?
5. Gain is \$40, or 5%. What was the cost?
6. Loss was \$30, or 15%. What was the cost?
7. Selling price was \$84, and loss was 30%. What was cost?
8. Gained 20% when I sold for \$60. What was cost?
9. A knife selling at 70 cents yielded 40% gain. Find cost.
10. If percentage is 63 bu. and rate is 70%, find base.
11. If proceeds are 525 tons and rate is 5% gain, find base.
12. What was the base if rate was 16% and percentage \$16?

235. Algebraic Treatment of Percentage.—If the student is familiar with the first principles of algebra, the treatment of percentage may be simplified by their use. All three of its cases may be reduced to a single formula.

Thus, since $p = b r$, if any two of the three quantities, p , b , r , are known, the third may be found by substituting for the two known quantities and solving the resulting equation.

Ex. 1. What per cent. is 16 of 64?

Here, $p = 16$, $b = 64$.

Substituting for p and b in $p = b r$

we obtain $16 = 64 r$

$\therefore r = \frac{1}{4} = .25$, Rate.

The above formula may also be made to cover cases where the proceeds occur instead of the percentage.

In such cases let p = proceeds,

r = final rate, i. e., $1 +$ rate, or $1 -$ rate,

b = base.

Ex. 2. A property is sold for \$3360 at a gain of 12%. What was its cost?

Here, proceeds (or p) = \$3360,

final rate (r) = 1.12.

Substituting for p and r in $p = b r$, $\$3360 = 1.12 r$

$$r = \frac{\$3360}{1.12} = \$3000, \text{ Base.}$$

EXERCISE 116.

GENERAL REVIEW.

1. Find 7% of 1456 feet. Of \$351.70. Of $9\frac{1}{2}$.
2. 24 is what per cent. of 36? Of 64? Of 480? Of 18?
3. 30 is what per cent. of 50? Of 120? Of 24? Of 18?
4. 42 is 6% of what number? 8% of what? 21% of what?
5. 48 is 18% of what? $33\frac{1}{2}\%$ of what? 120% of what?
6. Find $101\frac{1}{2}\%$ of 64 acres. Of $7\frac{3}{4}$ miles.

In the following group of examples, two of these three quantities are given, and the problem in each case consists in ascertaining the missing one.

	Base.	Percentage.	Rate.
7.	\$236	\$34.22	?
8.	?	\$12.41	$21\frac{1}{4}\%$.
9.	736 pounds.	?	$2\frac{1}{3}\%$.
10.	$58\frac{3}{4}$ bushels.	8.76 bushels.	?
11.	?	\$355	$\frac{1}{2}\%$.
12.	\$460	?	$1\frac{1}{4}\%$.
13.	$427\frac{3}{4}$	$534\frac{1}{2}$?
14.	?	750.527	$105\frac{1}{2}\%$.
15.	\$896	?	$96\frac{3}{4}\%$.

16. What per cent. of an hour is a minute? Of a peck is a quart? Of a week is a day? Of a gallon is a quart? Of an inch is a foot? Of a yard is a foot? Of a yard is an inch? Of a day is an hour?

17. Out of an examination of 9 problems, a boy solved $7\frac{1}{2}$ correctly. What should be his per cent. grade?

18. A man owning $\frac{3}{4}$ of a ship sold $33\frac{1}{2}\%$ of his share for \$9200. What was the whole vessel worth?

19. A field was 48×35 rods, and the owner increased each dimension 40% of itself. By how many acres and what per cent. did he increase the field?

20. A carpenter built a house at an expense of \$5200, and sold it for \$7098. What was the gain %?

21. A wholesale grocer buys coffee at 30 cents and sells it at 36 cents a pound. The local grocer buys at 36 and sells at 45 cents. What per cent. does each make? What per cent. would be the wholesale grocer's gain if he sold directly to the consumer for 45 cents?

22. Owning 30% of an office building, a man sold 25% of his share and then valued the balance at \$9000. What was the entire building worth at this valuation?

23. What per cent. of 8 bushels, 3 pecks, 4 quarts is 4 bushels, 6.9 quarts?

24. Find the value of 80% of 60 acres, 125 square rods, 20 square yards at \$60 an acre.

25. Find $\frac{1}{2}$ % of 90. Of 200. Of 8756. Of $\frac{1}{2}$.

26. Find $\frac{2}{3}$ % of 900. Of 4980. Of $\frac{2}{15}$. Of 1.6.

27. Write decimally, and as a common fraction; ten per cent.; four-fifths per cent.; one-half of one per cent.; one hundred twelve per cent.; eight per cent.; three-tenths per cent.; and eleven hundredths per cent.

28. Write as per cent.; $\frac{3}{10}$; $\frac{1}{4}$; $\frac{5}{8}$; $\frac{1}{2}$; $\frac{7}{10}$; $1\frac{1}{2}$; 6; $1\frac{10}{100}$; $1\frac{7}{10}$; $\frac{2}{3}$; $1\frac{1}{10}$; $\frac{3}{8}$.

29. On an examination a boy got 460 credits out of a possible 500. What should his grade be, expressed in per cent.?

30. A watch was sold for \$190 at a loss of 24%. What should it have been sold for to obtain a gain of 5%?

31. A speculator bought 500 shares of railroad stock at \$68 a share and sold it at \$85 a share. What was his gain %?

32. If a teacher's salary is \$2400 and he pays in a year, 15% of it for board, 4% for room, 3% for clothes, 8% for incidentals, and gives his mother a quarter of his salary, how much is left for saving?

33. A man owned $\frac{1}{3}$ of a hotel and sold $12\frac{1}{2}$ % of his share for \$7540. At the same rate, what is the value of the hotel?

34. 707.84 is 12% more than what number?

35. If the cost was $\frac{2}{3}$ of the selling price, what is the gain %? Prove your answer in the case where the cost is \$420.

36. What per cent. of the year 1904 are the Sundays? Of the year 1903?

CHAPTER XIV.

APPLICATIONS OF PERCENTAGE.

236. Applications of Percentage.—The method of reckoning with reference to 100 as a standard or base has so many advantages that it is widely used in many different departments of practical life.

The computations in these different applications are alike, in that, *first*, they all use 100 as a base, and, *second*, they are all concerned with the three quantities, *base*, *rate*, and *percentage*.

The various applications of percentage, however, differ from the general subject and from each other in that (1) different special names are assigned to one or more of the quantities used (thus the percentage is sometimes called *commission*, or *tax*, or *profit*, etc.); or (2), the base may be determined in some peculiar way; or (3) certain special standard rates are used.

In all cases, however, it will be found that the three quantities, base, rate, and percentage, appear in some form, and that two of them are given to find the remaining one.

PROFIT AND LOSS.

237. The subject of Profit and Loss differs very slightly from the general subject of percentage.

Profit or *loss* is the name given the percentage, profit being the excess of money received over that expended, and loss being the excess of money expended over that received.

The student should carefully note that the *base* is the money paid out or invested (not the money received).

Ex. A man sold his horse for \$60, which was a loss of 20%. What did the horse cost him?