2. On Jan. 1st, X. gives Y. four notes as follows: 1st for $700 due in 9 mos., 2d for $850 due in 6 mos., 3d for $400 due in 4 mos., and 4th for $600 due in 7 mos. On what date will a single payment equitably cancel all notes?

3. What is the average date for paying three notes, due, 1st, March 20, $500; 2d, April 25, $600; 3d, June 3, $400?

4. Four notes for $750 each are due respectively Aug. 1, Oct. 8, Nov. 20, and Dec. 5. What is the average date of maturity?

5. Find the equated time of paying $430 due in 8 mos.; $350 due in 9 mos.; $1000 due in 6 mos.

6. Find the equated time of paying following bills: $60 due in 30 days; $100 due in 60 days; $360 due in 90 days; and $250 due in 30 days. They all bear date Oct. 17.

7. Of a debt, $\frac{1}{3}$ is due in 7 mos., $\frac{1}{4}$ in 6 mos., and the rest in a year. Find the equated time that one payment ought to pay it all.

8. Three bills are due as follows: Aug. 5, $365, Oct. 10, $470, Dec. 14, $930. Find the average time of payment.

9. Three notes are due as follows: 1st for $320, June 1st; 2d for $480, Aug. 20; 3d for $520, Oct. 30. I wish to substitute one note for $1320. What should be its day of maturity?

10. A man bought a house for $6400 on 8 months' credit. He paid $2000 at time of purchase; when should the balance be due?

11. A man owes $500 due in 8 mos., $900 due in 6 mos., and $1200 due in a year. After 5 months he pays $1000. When in equity should the remainder be due?

12. A man owes $12000 due in 9 months. If he pays $6800 in 5 months and $2700 in 2 months more, when ought the balance be paid?

13. A certain debt is to be paid $\frac{1}{5}$ down, $\frac{1}{3}$ in 8 months, and $\frac{1}{4}$ in 9 months, and the balance in a year. If the payments are all made in one, when is it equitably due?

CHAPTER XVII.

RATIO AND PROPORTION. PARTNERSHIP.

RATIO.

292. Ratio.—If the quotient of one number divided by another occurs in a problem, it is often of advantage not to perform the division immediately, but to indicate the division for the time being. Thus, in Ex. 2 of Art. 95, the quotient of 200 divided by 9 being indicated for the time being, it was not found necessary to perform the division at all, since 9 ultimately was canceled by a factor of the multiplier, 54.

A **ratio** is the indicated quotient of one number divided by another number of the same kind.

293. The **terms** of a ratio are the quantities whose quotient is indicated. The first of these (the indicated dividend) is called the **antecedent**; the second (the indicated divisor) is called the **consequent**.

Thus, in the ratio 12 to 8, 12 is the antecedent, 8 the consequent.

294. Symbols.—A ratio is usually indicated by the sign, :, between the numbers compared. This sign is probably an abbreviation of ÷, the sign of division.

Thus, the ratio of 12 to 8 is denoted by 12 : 8; it may also be indicated in the fractional form, $\frac{12}{8}$.

295. A **compound ratio** is the product of two simple ratios.

Thus, $\frac{2}{3} \times \frac{5}{7}$, or $\frac{2 \times 5}{3 \times 7}$, is a compound ratio.

It may also be expressed thus $\left\{\begin{matrix} 2:3. \\ 5:7. \end{matrix}\right.$

296. Properties of Ratios.—From the nature of quotients and fractions it is evident that—

(1) *if both antecedent and consequent of a ratio be multiplied or divided by the same number, the ratio is not changed in value;*

(2) *the antecedent equals the product of the ratio by the consequent;*

(3) *the consequent equals the antecedent divided by the ratio.*

EXERCISE 145.

What is the ratio of:

1. 12 to 42?	*4.* $11\frac{2}{3}$ to $12\frac{5}{6}$?	*7.* 5.2 to 7.28?
2. 81 : 135?	*5.* $3\frac{4}{7}$: $8\frac{13}{14}$?	*8.* 12.75 : 16.15?
3. $9\frac{3}{4}$ to $14\frac{5}{6}$?	*6.* $5\frac{4}{9}$: $6\frac{20}{27}$?	*9.* 3.422 : 3.76?

Find the value of the parenthesis in each:

10. 12 : 30 = (?).	*13.* 28 to (?) = $\frac{4}{11}$.	*16.* $7\frac{3}{5}$: (?) = $\frac{4}{9}$.
11. 18 : (?) = $\frac{2}{5}$.	*14.* (?) to 81 = $\frac{4}{9}$.	*17.* (?) : $10\frac{1}{12}$ = $\frac{10}{11}$.
12. (?) : 18 = $\frac{1}{6}$.	*15.* 42 : (?) = $\frac{3}{7}$.	*18.* 4.45 : (?) = $\frac{9}{13}$.

19. If the consequent is 45 and the antecedent is 35, find the ratio.

20. If the antecedent is $14\frac{2}{5}$ and the ratio is $\frac{15}{23}$, find the consequent.

PROPORTION.

297. A **proportion** is an equality between ratios.

By the use of proportion a problem is often solved more readily than by analysis (see Arts. 94, 95), but sometimes the reason for the steps used is not so evident.

A proportion is indicated by placing the symbol, =, or, : :, between the two equal ratios.

Thus, 2 : 3 = 8 : 12, is a proportion.

It may be read in several ways, as "2 is to 3 as 8 is to 12;" or, "2 over 3 equals 8 over 12," or, "the ratio of 2 to 3 equals the ratio of 8 to 12," etc.

298. Terms.—Hence, a proportion contains four terms. The first and last terms are called the **extremes**; the second and third terms are called the **means**.

If the two means are alike, each is called a **mean proportional**, and the last term is called a **third proportional**.

Thus, in 2 : 4 = 4 : 8, 4 is a mean proportional, and 8 is a third proportional.

299. Properties of a Proportion.—The fundamental property of a proportion is that "*the product of the means is equal to the product of the extremes.*"

Thus, in the proportion in Art. 297, 3 × 8 = 2 × 12.

This property is seen to be true for any proportion, since, if $\frac{a}{b}$ and $\frac{c}{d}$ are two equal ratios, and each be multiplied by $b\,d$, we have $a \times d$ and $b \times c$ equal.

It follows at once from the above property that *either extreme equals the product of the means divided by the other extreme.*

What does either mean equal?

To obtain a method for solving problems by proportion, let us consider the following problem.

Ex. 1. If 12 books cost $20, what will 15 books cost?

Solving by analysis, we have

Cost of 1 book = $$\frac{20}{12}$.

Cost of 15 books = $$\frac{20}{12}$ × 15.

To show how this relation may be converted into a proportion we divide each of these equals by $20, and obtain

$$\frac{\text{cost of 15 books}}{\$20 \text{ (or cost of 12 books)}} = \frac{15}{12}.$$

Taking the last ratio first, and writing the denominator first

12 books : 15 books = $20 (cost of 12 books) : cost of 15 books

or, 12 : 15 = $20 : (   ).

Hence, in general, *write the required quantity as the last term of the proportion; use the quantity with which it is compared as the third term; write the ratio which is equal to the ratio of the 3d and 4th terms as the first two terms; solve by using the properties of a proportion.*

Ex. 2. If 20 acres of land produce 320 bushels of wheat, how many acres are needed to produce 400 bushels?

SOLUTION.

20 acres is compared with the unknown number of acres. Hence, we have

$$320 : 400 = 20 : (\ \ ).$$

$$\therefore \text{No. acres required} = \frac{\overset{25}{\not{4}\not{0}\not{0}} \times \not{2}\not{0}}{\underset{\not{1}\not{6}}{\not{3}\not{2}\not{0}}} = 25.$$

EXERCISE 146.

Supply the missing term in each proportion:

1. 3 : 8 :: 6 : (?).	*6.* $3\frac{1}{2} : 4\frac{2}{3} :: 9\frac{3}{7} :$ (?).
2. 12 : 5 :: (?) : 40.	*7.* $41\frac{1}{3} :$ (?) $:: 196\frac{1}{3} : 232\frac{1}{2}$.
3. 8 : (?) :: 5 : 30.	*8.* $12\frac{4}{11} : 10\frac{1}{5} ::$ (?) $: 9\frac{5}{8}$.
4. 16 : 24 :: (?) : 15.	*9.* (?) : 9.75 :: 13.25 : 10.4.
5. (?) : 11 :: 14 : 33.	*10.* 2.76 : 3.45 :: 2.28 : (?).

11. If 75 acres of land cost $5090.40, what will 175 acres cost? 411 acres?

12. If $5890.50 buys 85 acres, how many acres will $34650 buy? $25410?

13. If 7 horses require a pasture of 18 acres, how large a pasture will 300 horses require?

14. If a pole 12 feet high casts a shadow $5\frac{1}{4}$ feet long, how long will be the shadow from a steeple 144 feet high at the same time?

15. If the shadow from a chimney $46\frac{1}{2}$ feet high is 38 ft. 9 in., what is the height of the tree whose shadow is 111 ft. 3 in.?

16. If $3\frac{2}{3}$ bu. of grain are used to sow $13\frac{3}{4}$ acres, how many bushels will be required to sow 100 acres?

17. If $75 yields $35 interest, how much must be invested to yield $63 interest?

18. If I pay $75.65 for the use of $425, what should be paid for the use of $545 for same time?

19. If 12 men accomplish a certain task in 30 days, how many days will 45 men require?

NOTE.—The pupil should observe that 45 men will not require as long as 12 men do. Hence, we invert the unknown ratio in the proportion. For this reason such proportions are called *inverse proportions*. The solution is arranged thus: 12 men : 45 men : : (? days) : 30 days.

20. If 18 men dig a ditch in 35 days, how long would it take 21 men to do the same? 14 men?

21. If 10 men lay a wall in 48 days, how many men will be needed to lay a similar wall in 30 days?

22. A man borrows $72 for 5 years, and lends $240 in return. How long ought he lend the latter sum to pay for the former loan?

23. If $4500 is borrowed for a certain time at 5%, what sum must be loaned at $4\frac{1}{2}$% the same time, to compensate?

24. If a man can do in 8 days as much work as his son does in 15 days, and the son's wages are $1.60 a day, what pay should the father receive per day?

25. Find a fourth proportional to 7, 17, and 21.

26. Find the number which has to $6\frac{3}{5}$ the same ratio which $11\frac{2}{3}$ has to $3\frac{1}{7}$.

27. Find a third proportional to $3\frac{1}{2}$ and $4\frac{2}{3}$.

28. Find the fourth proportional to 3.81, .056, and 1.67.

29. By a certain pipe a certain cistern can be emptied in $5\frac{3}{8}$ hours. In what time can another cistern $3\frac{1}{3}$ times as large be emptied by a pipe carrying only $\frac{3}{4}$ as much water?

COMPOUND PROPORTION.

300. **A compound proportion** is an equality between a simple ratio and a compound ratio, or between two compound ratios.

$$\text{Exs. } \left\{\begin{matrix} 3 : 6 \\ 10 : 30 \end{matrix}\right\} = 2 : 12, \text{ or } \left\{\begin{matrix} 2 : 6 \\ 14 : 28 \end{matrix}\right\} = \left\{\begin{matrix} 1 : 2 \\ 2 : 6 \end{matrix}\right\}.$$

Ex. 1. If 12 men can earn $180 in 5 days, how much can 16 men earn in 9 days?

SOLUTION.

$180 is compared with the required number of dollars. The ratio which is equal to the ratio, $180 : required No. $, is 12 × 5 days' work : 16 × 9 days' work. Hence,

$$\left\{\begin{matrix} 12:16 \\ 5:9 \end{matrix}\right\} = \$180 : (\quad)$$

$$\text{or } (\quad) = \frac{180 \times 16 \times 9}{12 \times 5} = \$432, \textit{ Result.}$$

Sometimes the terms of ratio vary *inversely*, and must be used accordingly (for instance, the number of days required to do a given piece of work varies inversely as the number of workmen, that is, the *greater* the number of workmen the *fewer* the days).

Ex. 2. If 15 men can dig a ditch 180 rods long in 8 days, how many days will it take 20 men to dig a ditch 300 rods long?

SOLUTION.

The final ratio is, 8 days: required No. days.

The longer the ditch, the greater the number of days required, hence, 180 : 300 is a part of the first ratio equal to the above ratio; but the greater the number of men the fewer the number of days, ∴ 20 : 15 is the other part of the first ratio. Hence,

$$\left\{\begin{matrix} 20:15 \\ 180:300 \end{matrix}\right\} = 8 : (\quad).$$

$$\text{Hence, } (\quad) = \frac{8 \times 15 \times 300}{20 \times 180} = 10, \textit{ No. of days.}$$

If the first set of men had worked 8 hours a day, and the second set 12 hours a day, how would this have affected the solution?

EXERCISE 147.

1. If 15 men can earn \$360 in 8 days, how much can 7 men earn in 40 days?

2. Five clerks use 50 quires of paper in 16 days. At the same rate, how much paper will 9 clerks use in 15 days?

3. If 8 persons spend \$470 in 5 days, how much will 15 persons spend in 16 days at same rate?

4. If a block of stone 2 ft. × 3 ft. × 4 ft. weigh 1740 lbs., what will a block of like stone 3 × 5 × 7 ft. weigh?

5. If 7 men working 8 hours a day can accomplish a task in 15 days, how many days of 6 hours will 10 men require for the same task?

6. If a cistern $17\frac{1}{2}$ ft. long, $10\frac{1}{2}$ ft. wide, and 13 ft. deep, hold 546 bbl., how many barrels will a cistern hold that is 16 ft. long, 7 ft. wide, and 15 ft. deep?

7. If 22 men can cut 294 cords of wood in 7 days when they work 14 hours a day, how many days will it take 5 men to cut 375 cords, working 10 hours a day?

8. If 25 men dig a ditch 396 feet long in 36 days of 7 hours each, in how many days will 30 men dig a similar ditch 990 feet long, if they work 9 hours a day?

9. If 90 men build a wall 2304 ft. long, 8 ft. wide, and $2\frac{1}{2}$ ft. high in 45 days of $7\frac{1}{2}$ hrs. each, how long a wall 7 ft. wide and 4 ft. high can 125 men build in 35 days of 9 hrs. each?

10. If a slab of marble 9 ft. long, 3 ft. wide, and 4 in. thick weighs 1200 lbs., how much will another similar slab weigh which is 6 ft. long, 2 ft. wide, and 3 in. thick?

11. A certain bin 7 ft. × $2\frac{1}{2}$ ft. and 2 ft. deep contains 28 bushels of grain; what is the depth of a second bin 18 ft. × 1 ft. $10\frac{1}{2}$ in. which contains 120 bu.?

12. If 496 men, in 5 da. of 12 hr. 6 min. each, dig a trench of 5 degrees of hardness, 465 ft. long, 3 ft. 8 in. wide, and 4 ft. 8 in. deep, how many men will be required to dig a trench of 8 degrees of hardness, $168\frac{3}{4}$ ft. long, 7 ft. 6 in. wide, and $2\frac{4}{5}$ ft. deep, in 22 da. of 9 hr. each?

PROPORTIONAL PARTS.

301. Proportional Parts.—It may be required to divide a given number into parts which shall be proportional to a series of given numbers. We may do this either by the use of proportion, or by the use of fractions and fractional units.

Ex. Three men working a mine agree to divide the profits in the proportion of 2, 3, and 4. They make \$2700. What is the share of each?

SOLUTION.

Since 2 + 3 + 4 = 9, we may regard the profits as forming 9 shares, of which the miners get 2, 3, and 4 shares respectively.

9 : 2 = $2700 : share of 1st, $600.
9 : 3 = $2700 : " 2d, $900.
9 : 4 = $2700 : " 3d, $1200.

Or, since $2700 is to be divided into 9 shares,

1 share = $\frac{1}{9}$ of $2700 = $300.
2 shares = $600.
Etc.

EXERCISE 148.

1. Divide 20000 into four parts proportional to 5, 7, 8, 12.

2. Divide 6300 into four parts proportional to 3, 5, 11, 17.

3. Divide 31800 into parts in the relation of 1 : 2 : 3.

4. Divide 3864 into parts in the relation of 2 : 3 : 4 : 5.

5. The weights of three casks aggregate a ton, but their individual weights are as 11 : 13 : 16. Find the weight of each.

6. A father divided his property of $46500 among three sons, in parts proportional to their ages, 17 yr., 20 yr., and 25 yr. How much did each receive?

7. Divide 210 into three parts, which shall be proportional to $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

PARTNERSHIP.

302. Partnership.—When two or more persons can carry on a business to better advantage together than singly, they often unite and form a partnership.

Partnership is the combination of two or more persons as a single firm to carry on business.

The **capital** is the money invested by the different partners in the business.

303. In **simple partnership** all the partners invest their capital for the same length of time.

In **compound partnership** the partners invest their capital for different lengths of time.

Ex. 1. Three men, A, B, and C, are in business together, and gain $3600. A's capital is $2000; B's is $4000; and C's is $6000. What is each one's share of the profits?

SOLUTION.

Entire capital = $2000 + $4000 + $6000 = $12000.

A's share of the capital = $\frac{2000}{12000}$ = $\frac{1}{6}$.
B's " " " = $\frac{4000}{12000}$ = $\frac{1}{3}$.
C's " " " = $\frac{6000}{12000}$ = $\frac{1}{2}$.

Hence, A's share of the gain = $\frac{1}{6}$ of $3600 = $600.
B's " " " = $\frac{1}{3}$ of $3600 = $1200.
C's " " " = $\frac{1}{2}$ of $3600 = $1800.

Ex. 2. A, B, and C form a partnership. A puts in $500 for 8 months, B $600 for 9 months, and C $800 for 12 months. They gain $3040. What share of this belongs to each partner?

SOLUTION.

$500 invested for 8 mo. is the same as $4000 for 1 mo.
$600 " " 9 " " " " " $5400 " 1 "
$800 " " 12 " " " " " $9600 " 1 "
Entire capital = $19000 " 1 "

Hence, A's share of the gain = $\frac{4000}{19000}$ or $\frac{20}{95}$ of $3040 = $640.
B's " " " = $\frac{5400}{19000}$ or $\frac{27}{95}$ " " = $864.
C's " " " = $\frac{9600}{19000}$ or $\frac{48}{95}$ " " = $1536.

EXERCISE 149.

1. Three men contract for a pasture, into which A puts 7 horses, B puts 9, and C 13. Of the rent, which was $17.40, how much ought each pay?

2. Into a partnership A places $3200, B $4100, and C $1700. How should a gain of $4500 be divided?

3. Three men gain $500 after investing $560, $640, and $800 respectively. How do they share the profit?

4. By a will a man leaves to his widow $12000, to a son

$9000, and to a daughter $4000. But upon investigation the estate produced only $20000. How should it be divided equitably?

5. A firm lost in a year $3300. A's stock was $3200, B's was $7100, and C's was $6200. How is the loss to be distributed?

6. A, B, and C go into business with a capital of $12000. From the gain of one year A's share is $1250, B's is $1000, and C's is $750. What was each man's capital?

7. Three persons enter partnership. A puts into it $1600 for 3 months; B $800 for 5 months; and C $900 for 3 months. How should they justly share the profits of $575? The losses of $1035?

8. A pasture is rented by 3 persons for $760. A puts in 7 cows for 5 mos.; B 8 cows for 3 mos.; and C 9 cows for 4 mos. What rent should each pay?

9. Three laborers contracted to dig a trench for $49.50. The first worked 8 days of 7 hours each; the second 10 days of 8 hours each; and the third 14 days of 6 hours each. What should each receive?

10. A entered business with $5000, and in 3 mos. took in B with $4000. After 2 mos. more C entered the firm with $12000. At the end of the year they had gained $8100. How should it be divided equitably?

CHAPTER XVIII.

INVOLUTION AND EVOLUTION.

INVOLUTION.

304. Definitions.—The **second power**, or **square**, of a number is the number obtained by multiplying a given number by itself.

Thus, $23^2 = 23 \times 23 = 529$.

The **third power**, or **cube**, of a number is the number obtained by using the given number as a factor three times.

Thus, $8^3 = 8 \times 8 \times 8 = 512$.

Let the pupil define **fourth power, fifth power,** etc., of a number and give examples.

Involution is the process of computing any required power of a given number.

305. Memorizing Powers of Small Numbers.—It is important that the pupil calculate and commit to memory the following powers:

Squares of 1, 2, 3, 4 . . . . to 25.
Cubes of 1, 2, 3, 4 . . . . to 12.
Fourth powers of 1, 2, 3, 4, 5, 6.
Fifth powers of 1, 2, 3, 4, 5.
Sixth powers of 1, 2, 3, 4, 5.
Seventh powers of 1, 2, 3.
Eighth, ninth, and **tenth powers** of 1, 2.

306. Methods of Involution.—The powers of numbers may be obtained either by (1) **actual multiplication**, or (2) by the **use of tables**, or (3) by use of **logarithms** (see Art. 86).