$9000, and to a daughter $4000. But upon investigation the estate produced only $20000. How should it be divided equitably?

5. A firm lost in a year $3300. A's stock was $3200, B's was $7100, and C's was $6200. How is the loss to be distributed?

6. A, B, and C go into business with a capital of $12000. From the gain of one year A's share is $1250, B's is $1000, and C's is $750. What was each man's capital?

7. Three persons enter partnership. A puts into it $1600 for 3 months; B $800 for 5 months; and C $900 for 3 months. How should they justly share the profits of $575? The losses of $1035?

8. A pasture is rented by 3 persons for $760. A puts in 7 cows for 5 mos.; B 8 cows for 3 mos.; and C 9 cows for 4 mos. What rent should each pay?

9. Three laborers contracted to dig a trench for $49.50. The first worked 8 days of 7 hours each; the second 10 days of 8 hours each; and the third 14 days of 6 hours each. What should each receive?

10. A entered business with $5000, and in 3 mos. took in B with $4000. After 2 mos. more C entered the firm with $12000. At the end of the year they had gained $8100. How should it be divided equitably?

CHAPTER XVIII.

INVOLUTION AND EVOLUTION.

INVOLUTION.

304. Definitions.—The second power, or **square**, of a number is the number obtained by multiplying a given number by itself.

Thus, $23^2 = 23 \times 23 = 529$.

The **third power**, or **cube**, of a number is the number obtained by using the given number as a factor three times.

Thus, $8^3 = 8 \times 8 \times 8 = 512$.

Let the pupil define **fourth power, fifth power**, etc., of a number and give examples.

Involution is the process of computing any required power of a given number.

305. Memorizing Powers of Small Numbers.—It is important that the pupil calculate and commit to memory the following powers:

Squares of 1, 2, 3, 4 . . . . to 25.
Cubes of 1, 2, 3, 4 . . . . to 12.
Fourth powers of 1, 2, 3, 4, 5, 6.
Fifth powers of 1, 2, 3, 4, 5.
Sixth powers of 1, 2, 3, 4, 5.
Seventh powers of 1, 2, 3.
Eighth, ninth, and **tenth powers** of 1, 2.

306. Methods of Involution.—The powers of numbers may be obtained either by (1) **actual multiplication**, or (2) by the **use of tables**, or (3) by use of **logarithms** (see Art. 86).

It is also useful to be able to separate a number into parts (as tens + units), and form the product by multiplication of these parts. By this means properties of a power are discovered, which can be used in the inverse process of finding the root of a number. See Arts. 309 and 314.

EXERCISE 150.

ORAL.

1. State rapidly the squares of all the numbers up to 20.

2. What is the square of 18? 14? 17? 21? 19? 15? etc.

3. What is the square of $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{3}{7}$? $\frac{5}{8}$? $\frac{14}{23}$? $\frac{17}{15}$?

4. What is the square of .2? .3? .5? .8? .13? .18? 1.6?

5. What is the square of 1.8? 2.1? 2.5? 30? 50? 70? 600?

6. What is the cube of 9? 8? 3? 7? 6? 5? 11?

7. What is the cube of $\frac{2}{5}$? $\frac{5}{3}$? $\frac{4}{9}$? $\frac{6}{7}$? $\frac{8}{11}$? $2\frac{1}{2}$? $3\frac{1}{2}$?

8. What is the cube of .2? .3? .4? 20? 30? 1.2?

9. Tell the value of 8^3, 12^3, 2^4, 3^4, 6^3, 3^5, 2^7, 5^6, 4^3, 4^4, 5^4, 2^8, 3^6, 6^4, 5^5, $(1\frac{3}{5})^3$, $(5\frac{2}{3})^2$, $(2.2)^2$, $(.07)^3$, $(.012)^3$.

EXERCISE 151.

Find the value of:

1. 26^2.	*6.* 1.75^2.	*11.* $(1\frac{1}{3})^3$.	*16.* 15^4.
2. 28^3.	*7.* 23.1^3.	*12.* $(3\frac{1}{2})^2$.	*17.* 23^4.
3. 113^2.	*8.* 31.4^2.	*13.* $(5\frac{2}{3})^3$.	*18.* $(12\frac{1}{2})^4$.
4. 512^2.	*9.* $.038^3$.	*14.* $(4\frac{1}{4})^3$.	*19.* $(3.1\frac{2}{3})^3$.
5. 205^3.	*10.* $.66^3$.	*15.* $(7\frac{1}{7})^3$.	*20.* $(1.02\frac{1}{2})^3$.

21. $3^3 \times 4^2$.	*24.* $2^3 \times 3^4 \div 6^2$.	*27.* $15^3 \times 45^2$.
22. $4^3 \times 2^4$.	*25.* $6^3 \times 7^2 \div 14^3$.	*28.* $45^3 \div 15^5$.
23. $(\frac{1}{2})^2 \times 5^3$.	*26.* $(2.5)^2 \times (3.5)^2$.	*29.* $7.2^2 \times 7.5^3$.

EVOLUTION.

307. Definitions.—The **square root** of a number is that number which, multiplied by itself, will produce the given number. Thus, 13 is the square root of 169, since $13 \times 13 = 169$.

The **cube root** of a number is that number which, used as a factor three times, will produce the given number. Thus, 8 is the cube root of 512, since $8 \times 8 \times 8 = 512$.

Let the pupil define **fourth root, cube root,** etc.

The student should commit to memory the roots corresponding to the powers mentioned in Art. 305.

Evolution is the process of determining any required root of a given number.

308. The **methods of determining the roots** of numbers are (1) the use of **tables,** when a number has an exact root, or (2) the use of **logarithms,** or (3) the **direct methods** given in the remainder of this chapter, which are independent of tables and logarithms.

As stated in Art. 306, these methods are based on observing how the power of a number is formed when the number is dissected into parts (units and tens) and the product formed by the use of these parts.

SQUARE ROOT.

309. Squaring a Number by Parts.—Since, for example, $47 = 40 + 7$, the square of 47 may be formed thus,

$$\begin{array}{l} 40 + 7 \\ \underline{40 + 7} \\ 40^2 + 40 \times 7 \\ \underline{ + 40 \times 7 + 7^2} \\ 40^2 + 2 \times 40 \times 7 + 7^2 = 1600 + 560 + 49 = 2209. \end{array}$$

Hence, if any number be separated into a number of tens + a number of units, its square will equal (*the square of the tens*) + (*twice the tens* × *the units*) + (*square of the units*), or, denoting the tens by t, and the units by u, $(t + u)^2 = t^2 + 2\,tu + u^2$.

NOTE.—This method of squaring may also be applied to numbers containing three or more figures, and the observed properties employed in extracting the roots of correspondingly large powers.

Thus, $346 = 300 + 46$,
$346^2 = 300^2 + 2 \times 300 \times 46 + 46^2$.

Having found the first and second figures (3 and 4) of a square root by the use of the square in this form, we may then proceed to find the third figure of the root by the use of the square, as if it were in the form,

$$346^2 = (340 + 6)^2$$
$$= 340^2 + 2 \times 340 \times 6 + 6^2.$$

310. Periods.—Since in any given number, as 2209, whose square root is to be extracted, the square of the tens (1600) is not given explicitly, it must be determined indirectly, and its root then extracted. This is done by marking off the figures of the number whose root is to be extracted, into periods of two figures each, beginning at the decimal point, and then determining the largest square number represented in the first period of figures to the left.

For the *square of a number contains twice as many figures as the number itself, or twice as many less one.*

For since

$$1^2 = 1$$
$$10^2 = 100$$
$$100^2 = 10000$$
$$1000^2 = 1000000$$
etc.

it follows that if a number contains one figure, its square is either 1, or lies between 1 and 100, and hence contains one or two figures; if a number contains two digits, its square is either 100, or lies between 100 and 10000, and hence contains three or four digits; similarly, if a number contains three digits, its square contains five or six digits, etc.

Hence, if any number be separated into periods of two figures each, beginning at the decimal point, the *number of periods thus formed will be the same as the number of figures in the square root,* and the square root of the largest square number represented in the left-hand period gives the first figure of the root.

311. Extraction of Square Root.—Ex. 1. Extract the square root of 2209.

OPERATION.			ABBREVIATED FORM.	
	$\overset{\frown}{22}\overset{\frown}{09}.$	$40 + 7$		$\overset{\frown}{22}\overset{\frown}{09}$ | 47
Tens squared (t^2) = 40^2 =	1600			16
$2 \times$ *tens* $= 2 \times 40 = 80$	609		87	609
($2 \times$ *tens* + *units*) $\times$ *units* $= 87 \times 7$	609			609

EXPLANATION.

Since 2209 contains two periods of two figures each, the root must contain two figures, a tens figure and a units figure. Since the largest square in 2200 is 1600, and the square root of 1600 is 40, the number of tens is 4. Subtracting the square of the tens, 1600, from 2209, the remainder, 609, must be $2 \times$ *tens* $\times$ *units* + (*units*)^2. Since (*units*)^2 is much less than $2 \times$ *tens* $\times$ *units*, much the largest part of 609 must be $2 \times$ *tens* $\times$ *units*, and if 609 be divided by $2 \times$ *tens*, or 80, it will give the units figure or a slightly larger number. We obtain 7 as the approximate quotient, and, by trial, determine that it is the exact number of units, since $87 \times 7 = 609$.

Ex. 2. Extract the square root of 119716.

We have

$$\begin{array}{r|l}
 & \overset{\frown}{11}\overset{\frown}{97}\overset{\frown}{16}. \;|\; 346, \textit{ Root.} \\
 & 9 \\
64 & 297 \\
 & 256 \\
686 & 4116 \\
 & 4116
\end{array}$$

By use of the Note to Art. 309, we determine that 3 is the number of hundreds in the root, and that the number of units is 40 +. Having found the first and second figures by this means, we may then find the third figure by separating the given number, as in the latter part of the note, into tens plus units, thus 340 + units.

Let the student write out a detailed explanation of the entire process of extracting the square root of 119716.

312. Square Root of Decimal Numbers.—If it be required to extract the square root of a decimal number, we may proceed thus, for example:

$$\sqrt{.0225} = \sqrt{\tfrac{225}{10000}} = \tfrac{15}{100} = .15, \textit{ Root.}$$

It is better, however, to put the work in a different form, by marking off the given number into periods of two figures each, beginning at the decimal point. Thus, we have,

$$\begin{array}{r|l}
 & .\overset{\frown}{02}\overset{\frown}{25} \;|\; .15, \textit{ Root.} \\
 & 1 \\
25 & 125 \\
 & 125
\end{array}$$

If necessary, annex a zero to complete the last period of figures to the right. In such cases, however, the root cannot be extracted exactly.

Ex. Extract the square root of 0.369 to 4 decimal places.

$$
\begin{array}{r|l}
 & .36\,90\,00\,00 \;|\; .6074+,\ \textit{Root.} \\
 & 36 \\
1207 & 9000 \\
 & 8449 \\
12144 & 55100 \\
 & 48576
\end{array}
$$

313. Square Root of Common Fractions.—If the denominator of the fraction, whose square root is to be extracted, is a perfect square, extract the root of the numerator and of the denominator separately, and divide the one result by the other.

Ex. $\sqrt{\frac{289}{324}} = \frac{\sqrt{289}}{\sqrt{324}} = \frac{17}{18}.$

If the denominator is not a perfect square, reduce the fraction to a decimal and extract the root of the decimal.

Ex. $\sqrt{\frac{2}{3}} = \sqrt{0.66666666+}$

$$
\begin{array}{r|l}
 & 0.66\,66\,66\,66+ \;|\; 0.8164+,\ \textit{Root.} \\
 & 64 \\
161 & 266 \\
 & 161 \\
1626 & 10566 \\
 & 9756 \\
16324 & 81066 \\
 & 65296
\end{array}
$$

Hence, in general, to extract the square root of a number,

Separate the number into periods of two figures each, beginning at the decimal point;

Find the greatest square in the left-hand period, and set down its root as the first figure of the required root;

Square this figure, subtract the result from the left-hand period, and to the remainder bring down the next period;

Double the root already found for a trial divisor, divide it into the remainder (omitting last figure of the remainder), and annex the quotient obtained, to the root and to the trial divisor.

Multiply the complete divisor by the figure of the root last found, and subtract the result from the remainder;

Proceed in like manner till all the periods of figures have been used.

EXERCISE 152.

Find the square root of:

1. 676.	*4.* 1764.	*7.* 6889.	*10.* 710649.
2. 841.	*5.* 3364.	*8.* 18496.	*11.* 879844.
3. 961.	*6.* 4489.	*9.* 173889.	*12.* 54804409.

13. 64272289.	*15.* 96177249.	*17.* 2181637264.
14. 82646281.	*16.* 1228292209.	*18.* 5416076836.

19. 61.7796.	*22.* 1.752976.	*25.* 11.67657241.
20. 6955.56.	*23.* 1419.7824.	*26.* 175351.5625.
21. 0.822649.	*24.* 0.50665924.	*27.* 25.81554481.

Find the square root of each of the following to three decimal places:

28. 40.	*32.* $13\frac{1}{2}$.	*36.* $5\frac{1}{4}$.	*40.* $7\frac{5}{12}$.
29. 8.	*33.* 0.9.	*37.* $3\frac{1}{3}$.	*41.* $26\frac{1}{11}$.
30. 31.	*34.* $\frac{3}{4}$.	*38.* 246.01.	*42.* $36\frac{1}{6}$.
31. 17.2	*35.* $\frac{7}{10}$.	*39.* $30\frac{3}{100}$.	*43.* $100\frac{1}{9}$.

EXERCISE 153.

ORAL.

State the square root of each of the following:

1. 256.	*6.* 121.	*11.* $\frac{64}{225}$.	*16.* .09.
2. 361.	*7.* 324.	*12.* $\frac{49}{100}$.	*17.* 1.21.
3. 400.	*8.* 169.	*13.* $\frac{81}{16}$.	*18.* .0036.
4. 144.	*9.* 529.	*14.* $1\frac{7}{9}$.	*19.* 2500.
5. 289.	*10.* 196.	*15.* $6\frac{1}{4}$.	*20.* $.062\frac{1}{2}$.

CUBE ROOT.

314. Cubing a Number by Parts.—In order to discover a method of extracting the cube root of a number, we separate a number, as 54, into its tens and units, $50 + 4$, and form its cube as follows:

$$
\begin{array}{l}
50 + 4 \\
\underline{50 + 4} \\
50^2 + 50 \times 4 \\
\underline{\qquad 50 \times 4 + 4^2} \\
50^2 + 2 \times 50 \times 4 + 4^2 \\
\underline{50 + 4} \\
50^3 + 2 \times 50^2 \times 4 \qquad + 50 \times 4^2 \\
\underline{\qquad\qquad 50^2 \times 4 + 2 \times 50 \times 4^2 + 4^3} \\
50^3 + 3 \times 50^2 \times 4 + 3 \times 50 \times 4^2 + 4^3 \\
= 125000 + 30000 + 2400 + 64 = 157464.
\end{array}
$$

Hence, if any number be separated into tens + units, its cube will be equal to

(*cube of the tens*) + (*three times the square of the tens times the units*) + (*three times the tens times the square of the units*) + (*cube of the units*), or in symbols

$$(t + u)^3 = t^3 + 3\,t^2 u + 3\,t u^2 + u^3.$$

NOTE.—This method may also be applied in cubing a number which contains three or more figures, and the observed properties employed in extracting the cube root of a correspondingly large number. See Art. 309, Note.

315. Periods.—Since in any given number, as 157464, whose cube root is sought, the cube of the tens is not given explicitly, it must be determined indirectly, and its root then extracted. It is determined by marking off the figures of the number whose root is sought into periods of three figures each, beginning at the decimal point, and then determining the largest cube number represented in the first period of figures to the left.

For the *cube of a number contains three times as many digits* (*less one or two*) *as the number itself*.

For since,

$$
\begin{aligned}
1^3 &= 1 \\
10^3 &= 1000 \\
100^3 &= 1000000 \\
1000^3 &= 1000000000
\end{aligned}
$$

it follows that if a number contains one figure, its cube is either 1, or lies between 1 and 1000, and hence contains one, two, or three digits; if a number contains two digits, its cube is either 1000, or lies between 1000 and 1000000, and hence contains four, five, or six digits; similarly, if a number contains three digits, its cube contains seven, eight, or nine digits, etc.

Hence, if we begin at the decimal point and mark off the digits of any number in periods of 3 figures each, the *number of periods thus formed will be the same as the number of figures in the root.*

316. Extraction of cube root.

Ex. 1. Extract the cube root of 157464.

OPERATION.

$$
\begin{array}{rrr|l}
 & & \overset{\frown}{157}\overset{\frown}{464} & 50 + 4 \\
t^3 = 50^3 = & & 125000 & \\
3\,t^2 = 3 \times 50^2 = & 7500 & 32464 & \\
3\,t u = 3 \times 50 \times 4 = & 600 & & \\
u^2 = 4^2 = & 16 & & \\
u \times (3\,t^2 + 3\,t u + u^2) = & 4 \times 8116 = & 32464 &
\end{array}
$$

ABBREVIATED FORM OF OPERATION.

$$
\begin{array}{rr|l}
 & \overset{\frown}{157}\overset{\frown}{464} & 54,\ \textit{Root.} \\
 & 125 & \\
3 \times 50^2 = 7500 & 32464 & \\
3 \times 50 \times 4 = 600 & & \\
4^2 = 16 & & \\
8116 & 32464 &
\end{array}
$$

EXPLANATION.

Since 157464 contains two periods of three figures each, the cube root must contain two figures, a tens figure and a units figure. Since the largest cube in 157,000 is 125,000, and the cube root of 125,000 is 50, the number of tens is 5.

Subtracting the cube of the tens 125,000 from 157464, the remainder, 32464, must be $3 \times tens^2 \times units + 3 \times tens \times units^2 + units^3$, and since $3 \times tens^2 \times units$ is much the largest part of the remainder, if the remainder be divided by $3 \times tens^2$, or 7500, it will give the *units* figure or a slightly larger number as the quotient. Dividing we obtain 4 as the approximate quotient, and on trial find that it is the exact number of units, since,

$$(3 \times 50^2 + 3 \times 50 \times 4 + 4^2) \times 4 = 32464.$$

By use of the Note of Art. 314, the same method may be used in extracting the cube root of a number of more than two periods.

Ex. 2. Extract the cube root of 8,627,738.651.

OPERATION.

	$\overset{\frown}{8627}\ \overset{\frown}{738}.\overset{\frown}{651}$ | 205.1, *Root.*
	8
$3 \times (200)^2 = 120000$	627 738
$3 \times (200 \times 5) = 3000$	
$5^2 = 25$	
123025	615125
$3 \times (2050)^2 = 12607500$	12613651
$3 \times (2050 \times 1) = 6150$	
$1^2 = 1$	
12613651	12613651

Hence, in general, to extract the cube root of a number,

Separate the number into periods of three figures each, beginning at the decimal point;

Find the greatest cube in the left-hand period, and set down its cube root as the first figure of the required root;

Cube this figure, and subtract the result from the left-hand period, and annex the next period of figures to the remainder;

Take three times the square of the root already found as a trial divisor; divide the remainder by it, and set down the quotient as the next figure of the root;

Complete the trial divisor by adding to it three times the product of the first figure of the root with zero annexed, multiplied by the last figure, and the square of the last figure;

Multiply this complete divisor by the figure of the root last found, and subtract the result from the remainder;

Proceed in like manner till all the periods have been used.

EXERCISE 154.

Find the cube root of:

1. 19683.	*5.* 592704.	*9.* 119823157.
2. 97336.	*6.* 1906624.	*10.* 317214568.
3. 195112.	*7.* 31855013.	*11.* 371694959.
4. 250047.	*8.* 155720872.	*12.* 794022776.

13. 114.084125.	*18.* 5900304.943.
14. .270840023.	*19.* 14.154926059.
15. 487443.403.	*20.* 28877.930432.
16. 529.475129.	*21.* 185.485563927.
17. .773620632.	*22.* .494538357312.

Find the cube root to three decimal places:

23. 6.	*27.* 100.	*31.* $64\frac{2}{3}$.
24. 12.	*28.* $19\frac{1}{3}$.	*32.* 512.9.
25. 29.	*29.* $80\frac{3}{5}$.	*33.* 51.29.
26. 4.5.	*30.* 28.	*34.* $10\frac{4}{11}$.

EXERCISE 155.

ORAL.

State the cube root of:

1. 125; 64; 216; 729; 1000; 512; 1728.

2. 343; 27; 1331; $\frac{8}{27}$; $\frac{64}{125}$; $3\frac{3}{8}$; $\frac{1}{64}$; .008.

3. .001; .064; .001728; 1.728; 8000; 27000.

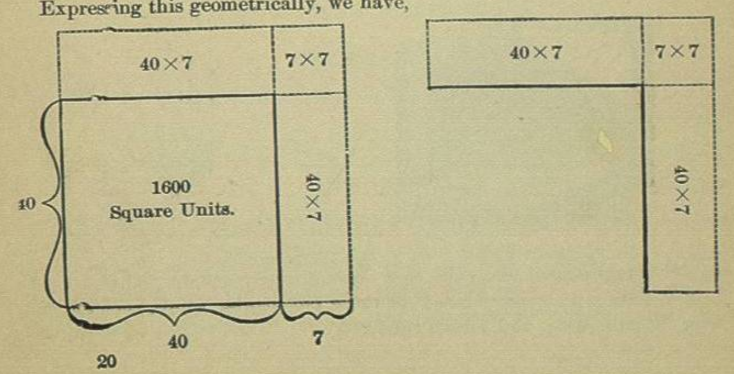
Perhaps no single mental aid to further mathematical study—excepting only the multiplication table—is of more constant benefit than a thorough familiarity with the perfect squares, cubes, and fourth powers of small numbers and the corresponding roots of these powers. Therefore the pupil should pause here until they are most carefully fastened in memory.

OTHER METHODS.

317. Geometrical illustration of square root.

By Art. 309, $47^2 = (40 + 7)^2 = 40^2 + 2 \times (40 \times 7) + 7^2 = 2209.$

Expressing this geometrically, we have,



or 40^2 is represented by a square 40 units of length on a side;
$2 \times (40 \times 7)$ by two rectangular strips, each 40 units long and 7 wide;
7^2 by a small square 7 units on a side.

In extracting the square root of 2209, the square of the tens, 1600 square units, is first removed, leaving a surface of 609 square units.

Much the largest part of this remaining surface is the two equal rectangles. Hence, dividing the area 609 by the combined length of these rectangles, 2×40 or 80, gives the width approximately, or 7.

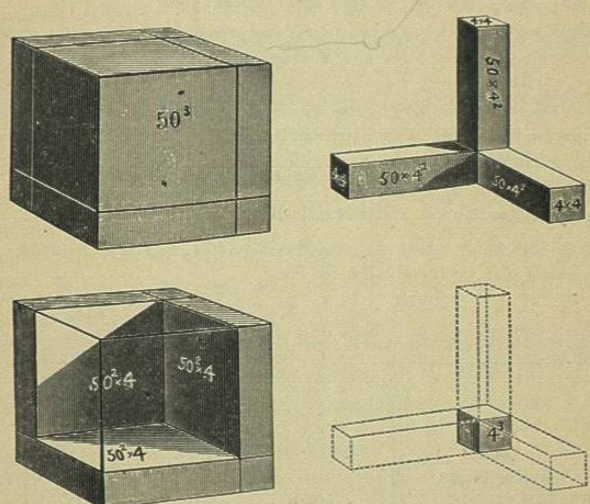
If this width is correct, the entire length of the three figures remaining after the large square (1600) is removed is $40 + 40 + 7$, or 87.

$87 \times 7 = 609$, the remaining area, hence, 7 is the correct width, or the second figure of the root.

318. Geometrical illustration of cube root.

$$54^3 = (50 + 4)^3 = 50^3 + 3 \times (50^2 \times 4) + 3 \times (50 \times 4^2) + 4^3.$$

Expressing this geometrically, we have,



or, 50^3 is represented by a cube, each edge of which contains 50 linear units;
$3 \times (50^2 \times 4)$ is represented by three rectangular solids, each 50 units long, 50 units wide, and 4 units thick:

$3 \times (50 \times 4^2)$ by 3 other solids, each 50 units long, 4 units wide, and 4 units thick;
3^3 by a small cube, each edge of which is 3 units.

Hence, in extracting the cube root of 157,464, the cube of the tens, 125,000 is first removed, leaving a volume of 32464 cubic units. Much the largest part of this is the 3 solids whose bases may be taken as 50×50 each, or $3 \times 50 \times 50$, or 7500 in all.

Hence, dividing the remaining volume, 32464, by 7500, gives the thickness of them approximately as 4.

If this thickness is correct, the sum of the bases of all the remaining solids (after the large cube, 125000, is removed) is

$$3 \times 50^2 + 3 \times 50 \times 4 + 4^2, \text{ or } 8116.$$

But $8116 \times 3 = 23877$, the remaining volume.

Hence, 4 is the correct thickness, or the second figure of the root.

319. Factorial Method of Extracting Roots.—If a number be separated into its prime factors, and each of these factors occurs an even number of times, the *square* root of the number may be obtained by multiplying together all the factors half the number of times they each occur; if each factor occurs three or a multiple of three times, the *cube* root may be obtained by multiplying together all the factors one-third of the number of times which each occurs, etc.

Ex. Extract the square root of 324.

$$\begin{aligned} \text{Since } 324 &= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \\ &= 2^2 \times 3^4 \\ \sqrt{324} &= 2 \times 3^2 = 18, \textit{ Root.} \end{aligned}$$

EXERCISE 156.

Find the cube root of the following:

1. 13824.	*4.* 110592.	*7.* 884736.
2. 46656.	*5.* 250047.	*8.* 2460375.
3. 74088.	*6.* 421875.	*9.* 4251528.

320. Higher Roots Obtained by Successive Extractions.—From the meaning of an exponent it follows that the square of the square of a number gives the fourth power of the number. Hence, reversing the process, the *fourth root* of a number is the *square root* of the *square root* of the number. Similarly, the *sixth root* of a number is the *square root* of the *cube root* of the number. The *eighth, ninth, tenth* . . . . . . roots of a number may be found by similar methods.

Ex. Obtain sixth root of 7,529,536.

Extracting the cube root, we obtain 196.

Extracting the square root of 196, we obtain 14 as the sixth root of the original number.

EXERCISE 157.

Find the fourth root of the following:

1. 331776.	*3.* 47458321.	*5.* 1196883216.
2. 4879681.	*4.* 81450625.	*6.* 11574317056.

Find the sixth root of:

7. 148035889.	*8.* 2176782336.

Find the sixth root of the following to 2 places of decimals:

9. 30.	*10.* 55.	*11.* 78.	*12.* 101.

Compute to 2 decimals the values of:

13. $\sqrt[3]{7+\sqrt{5}}$.	*16.* $\sqrt[3]{\sqrt{11}-\sqrt[3]{11}}$.	*19.* $\sqrt{1+\sqrt{2}+\sqrt[3]{3}}$.
14. $\sqrt{10-\sqrt{7}}$.	*17.* $\sqrt[4]{40+\sqrt[3]{40}}$.	*20.* $\sqrt[3]{\sqrt{5}+\sqrt[3]{6}+\sqrt[4]{7}}$.
15. $\sqrt{19-\sqrt[3]{21}}$.	*18.* $\sqrt[6]{\sqrt{50}-\sqrt[3]{17}}$.	*21.* $\sqrt{3\sqrt{10}+7\sqrt{31}}$.

Extract to three decimal places:

22. The square root of 7.0763; of .70763; and of 4.0763.
23. The square root of .387; of .0387; and of .00765.
24. The square root of .938; of .0938; and of .000765.

Extract to two decimal places:

25. The cube root of 6.318; of .6318; of .075.
26. The cube root of .07165; of .007165; of 19.0019.
27. The sixth root of 2.175; of .2175; of .025.

CHAPTER XIX.

MENSURATION.

321. Mensuration is that branch of mathematics which treats of the measurement of lines, surfaces, and volumes.

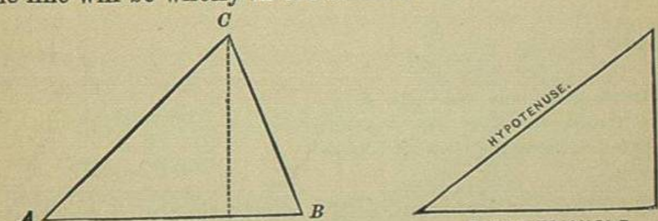
Since *lines* are measured more readily than any other kind of geometrical magnitude, it will be found that, in problems of mensuration, certain lines are usually measured first, and, from the results obtained, the lengths of *other lines*, or required *areas*, or *volumes*, are computed by principles determined by geometry.

It will not be possible to demonstrate fully these principles in the present brief treatment of the subject; but, wherever possible, they will be so presented and illustrated, as to make their truth clear to the pupil and enable him to recall them readily. He should constantly remember, however, that the complete demonstration of the rules and formulas used in this chapter belongs to another branch of mathematics, the subject of Geometry.

The limitations in the degree of accuracy with which a line can be measured are discussed in Art. 78, which should be reviewed.

I. Mensuration of Lines.

322. Definitions.—A **plane surface** is a surface such that if any two points in it be taken and joined by a straight line, the line will be wholly in the surface.



TRIANGLE. RIGHT TRIANGLE.

A **triangle** is a portion of a plane surface bounded by three straight lines, as the figure *A B C*.