number is the *square root* of the *square root* of the number. Similarly, the *sixth root* of a number is the *square root* of the *cube root* of the number. The *eighth, ninth, tenth* . . . . . . roots of a number may be found by similar methods.

Ex. Obtain sixth root of 7,529,536.

Extracting the cube root, we obtain 196.

Extracting the square root of 196, we obtain 14 as the sixth root of the original number.

EXERCISE 157.

Find the fourth root of the following:

1. 331776.	*3.* 47458321.	*5.* 1196883216.
2. 4879681.	*4.* 81450625.	*6.* 11574317056.

Find the sixth root of:

7. 148035889.	*8.* 2176782336.

Find the sixth root of the following to 2 places of decimals:

9. 30.	*10.* 55.	*11.* 78.	*12.* 101.

Compute to 2 decimals the values of:

13. $\sqrt[3]{7+\sqrt{5}}$.	*16.* $\sqrt[3]{\sqrt{11}-\sqrt[3]{11}}$.	*19.* $\sqrt{1+\sqrt{2}+\sqrt[3]{3}}$.
14. $\sqrt{10-\sqrt{7}}$.	*17.* $\sqrt[4]{40+\sqrt[3]{40}}$.	*20.* $\sqrt[3]{\sqrt{5}+\sqrt[3]{6}+\sqrt[4]{7}}$.
15. $\sqrt{19-\sqrt[3]{21}}$.	*18.* $\sqrt[6]{\sqrt{50}-\sqrt[3]{17}}$.	*21.* $\sqrt{3\sqrt{10}+7\sqrt{31}}$.

Extract to three decimal places:

22. The square root of 7.0763; of .70763; and of 4.0763.

23. The square root of .387; of .0387; and of .00765.

24. The square root of .938; of .0938; and of .000765.

Extract to two decimal places:

25. The cube root of 6.318; of .6318; of .075.

26. The cube root of .07165; of .007165; of 19.0019.

27. The sixth root of 2.175; of .2175; of .025.

CHAPTER XIX.

MENSURATION.

321. Mensuration is that branch of mathematics which treats of the measurement of lines, surfaces, and volumes.

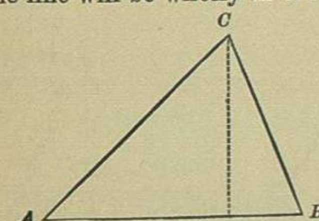
Since *lines* are measured more readily than any other kind of geometrical magnitude, it will be found that, in problems of mensuration, certain lines are usually measured first, and, from the results obtained, the lengths of *other lines*, or required *areas*, or *volumes*, are computed by principles determined by geometry.

It will not be possible to demonstrate fully these principles in the present brief treatment of the subject; but, wherever possible, they will be so presented and illustrated, as to make their truth clear to the pupil and enable him to recall them readily. He should constantly remember, however, that the complete demonstration of the rules and formulas used in this chapter belongs to another branch of mathematics, the subject of Geometry.

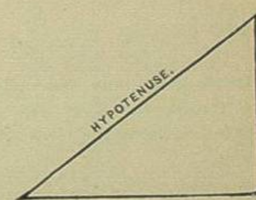
The limitations in the degree of accuracy with which a line can be measured are discussed in Art. 78, which should be reviewed.

I. Mensuration of Lines.

322. Definitions.—A **plane surface** is a surface such that if any two points in it be taken and joined by a straight line, the line will be wholly in the surface.



TRIANGLE.



RIGHT TRIANGLE.

A **triangle** is a portion of a plane surface bounded by three straight lines, as the figure *A B C*.

The **base** of a triangle is the side upon which it is regarded as standing.

The **altitude** of a triangle is the perpendicular distance to the base from the vertex, or point where the other two sides meet.

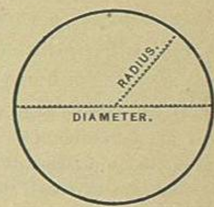
A **right triangle** is a triangle, one of whose angles is a right angle.

The **hypotenuse** of a right triangle is the side opposite the right angle.

Circle and **circumference** are defined in Art. 192.

A **radius** is a line drawn from the center of a circle to any point of the circumference.

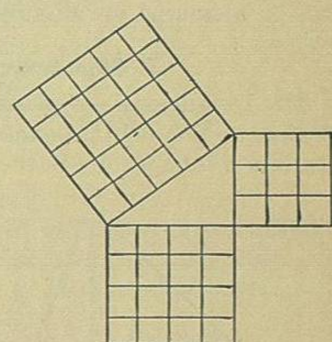
Parallel lines are lines in a plane surface which do not meet, however far they be produced.



CIRCLE.

323. Formulas for mensuration of lines.

1. *The square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides,*



or, denoting the hypotenuse by h, and the other two sides by a and b

$$h^2 = a^2 + b^2,$$
$$\text{and} \quad a^2 = h^2 - b^2,$$
$$\text{whence} \quad h = \sqrt{a^2 + b^2}$$
$$a = \sqrt{h^2 - b^2}.$$

Hence, given any two sides of a right triangle, the third side may be computed without the labor of measuring it.

Ex. Find the hypotenuse of a right triangle of which the two other sides are 50 ft. and 60 ft.

$$\sqrt{50^2 + 60^2} = 78.102+$$
$$\therefore \quad \text{Hypotenuse} = 78.102+ \text{ ft.}$$

2. *Circumference of a circle = diameter × 3.1416 (approx.).*

($\frac{22}{7}$ is sometimes used instead of 3.1416, though it is not quite so accurate).

Hence, also, $\textit{diameter} = \dfrac{\textit{circf.}}{3.1416} = \textit{circf.} \times .3183$ (*approx.*).

Let the pupil measure the diameter and circumference of a silver dollar, and show that the *circf.* = *diam.* × 3.1416 (*approx.*). Let him measure other circles, as a dinner plate, wagon-wheel, etc., similarly.

EXERCISE 158.

In the following examples a and b represent the legs of a right triangle, and c, the hypotenuse.

1. Given $a = 8$, $b = 15$, find c.
2. Given $b = 35$, $c = 37$, find a.
3. Given $c = 29$, $a = 21$, find b.
4. Given $b = 28$, $a = 45$, find c.
5. Given $a = 112$, $c = 113$, find b.
6. Given $c = 73$, $b = 55$, find a.
7. Given $a = 24$, $b = 143$, find c.
8. Given $b = 780$, $c = 901$, find a.
9. Given $c = 1105$, $a = 561$, find b.

Find correctly to 3 decimal places, the remaining side, when:

10. $a = 5$, $b = 8$.
11. $b = 2$, $c = 11$.
12. $c = 43$, $a = 34$.
13. $a = 92$, $b = 65$.

Find the circumference of each circle, when:

14. Radius = 7.
15. Diameter = 46.
16. Radius = $8\frac{1}{2}$.
17. Diameter = $13\frac{1}{3}$.
18. Radius = 74.6.
19. Diam. = 175.4.

Find the diameter of the circle, when:

20. Circumference = 40.
21. Circumference = 375.
22. Circum. = 57.3.
23. Circum. = 103.8.

24. A ladder 25 ft. long stands against the side of a house, and with its foot 7 ft. from the wall. How high is the top of the ladder?

25. A field 156 rds. long and 133 rds. wide is cut by a path running diagonally across it. Find the length of the path.

26. A flag pole was broken 16 ft. from the ground, and the top struck 63 ft. from the foot of the pole. How long was the pole?

27. Two rafters 20.5 ft. long meet at the ridge of a roof 4.5 ft. above the level of the walls. How wide is the house?

28. A ladder 65 ft. long stands in the street; if it fall on one side, it touches a point on that house 16 ft. above the pavement; but on the other side the point it touches is 56 ft. above the pavement. How wide is the street?

29. If the diameter of a pipe is $8\frac{1}{2}$ in., what is its circumference? What is the diameter of another pipe, whose circumference is $8\frac{1}{2}$ in.?

30. The diameter of the earth is about 7920 miles. How many miles is it around the earth?

31. A rope is wound spirally around a cylindrical mast 2 ft. in diameter and 60 ft. high, the spires being 1 ft. apart. How long is the rope?

II. Mensuration of Plane Areas.

324. Definitions.—Triangle and circle have already been defined.

An **equilateral triangle** is one which has all its sides equal.

A **quadrilateral** is a portion of a plane bounded by four straight lines.

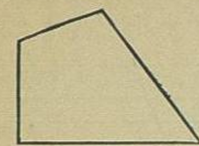
A **parallelogram** is a quadrilateral whose opposite sides are parallel.

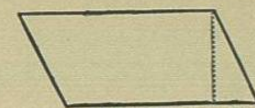
A **trapezoid** is a quadrilateral which has two and only two of its sides parallel.

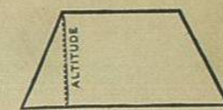
The **altitude** of a parallelogram or trapezoid is the perpendicular distance between parallel sides.

A **rectangle** is a parallelogram whose angles are right angles.

A **square** is a rectangle whose sides are equal.


QUADRILATERAL.


PARALLELOGRAM.



TRAPEZOID.

A **polygon** is a portion of a plane surface bounded by straight lines. A polygon of *three* sides is called a **triangle**; of *four* sides, a **quadrilateral**; of *five* sides, a **pentagon**; of *six* sides, a **hexagon**, etc.

A **regular polygon** is one in which the sides are all equal, and the angles are all equal.

The **perimeter** of a polygon is the sum of the lengths of its sides.

A **unit of area** is a square, each side of which is a linear unit, as a square inch, or a square yard.

The **area of a plane figure** is the *number* of square units which it contains (see also Art. 175).

325. Formulas for areas of plane figures.

1. Area of a **triangle** = *½ base × altitude.*

2. " " **parallelogram** = *base × altitude.*

3. " " **trapezoid** = *½ sum of parallel sides × altitude.*

4. " " **circle** = *radius squared* × 3.1416.

5. " " **circular ring** = $(R^2 - r^2) \times 3.1416$, where R and r are the radii of the two circles.

When the *three sides* of a triangle are given instead of the base and altitude,

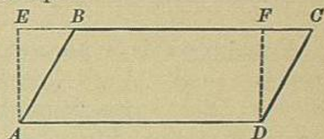
6. Area of a **triangle** $= \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c denote the three sides, and $s = \frac{1}{2}(a+b+c)$.

7. Area of **equilateral triangle** $= \frac{a^2\sqrt{3}}{4}$, where a denotes one of the sides.

It should be clearly understood that when we speak of multiplying one line by another (as the base by the altitude), we mean that the *number* of linear units in one line is to be multiplied by the *number* of linear units in the other line.

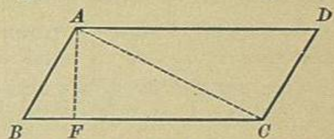
It has been shown (Art. 175) that the product of the number of linear units in the base of a rectangle by the number of linear units in the altitude equals the number of units of area in the rectangle. Thus,

$$7 \text{ in.} \times 5 \text{ in.} = 35 \text{ sq. in.}$$



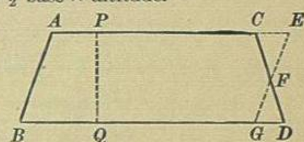
To obtain the area of a parallelogram, it is shown in geometry that the triangle FCD = triangle EAB,

$\therefore$ area $ABCD$ = area of rectangle $AEFD = AD \times DF$ = base $\times$ altitude.



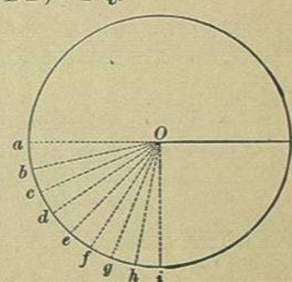
To obtain the area of a triangle, it is shown that triangle ABC = triangle ADC.

$$\therefore \text{ area triangle } ABC = \tfrac{1}{2} \text{ area } ABCD.$$
$$= \tfrac{1}{2} BC \times AF.$$
$$= \tfrac{1}{2} \text{ base} \times \text{altitude}.$$

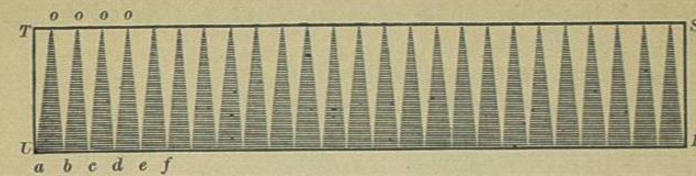


To obtain area of a trapezoid, $ABDC$, we take F, the middle point of CD, and draw EG parallel to AB, and produce AC to meet it at E, and prove triangle ECF = triangle FGD (hence, $CE = GD$).

$$\therefore \text{ area } ABCD = \text{area } ABGE \text{ (a parallelogram)}.$$
$$= BG \times PQ.$$
$$= \tfrac{1}{2}(AC + BD) \times PQ.$$



In order to understand the formula for obtaining the area of a circle, it will be useful to regard the circle as split up into parts as in the figure opposite; and then conceive the parts oab, obc, ocd, etc., as arranged in the figure on the next page.



The smaller the parts into which the circle is divided, the more nearly will their bases, when taken thus, approximate to a straight line, and their areas taken together = $\tfrac{1}{2}$ rectangle $RSTU$, whose base is the circumference of the circle and altitude its radius.

$$\therefore \text{ area of circle} = \tfrac{1}{2} \text{ circumference} \times R$$
$$= \tfrac{1}{2} \times 2\,R \times 3.1416 \times R = R^2 \times 3.1416.$$

NOTE.—The student should carefully observe that the determination of all the above areas is made by first measuring certain straight lines, and computing the area from the lengths obtained. This is much more expeditious than any direct counting of the units of area, which is indeed often impossible.

Ex. Find area of a triangle whose sides are 13, 14, 15.

SOLUTION.

Here $a = 13$, $b = 14$, $c = 15$. Hence, $s = 21$

$s - a = 8$

$s - b = 7$

$s - c = 6$

$$\therefore \text{ area} = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84.$$

EXERCISE 159.

Find the area of:

1. A rectangle 8 yards long and 11 ft. 8 in. wide.
2. The walls and ceiling of a room $17\tfrac{1}{2} \times 16\tfrac{2}{3} \times 8\tfrac{1}{3}$ feet.
3. A parallelogram whose base is 30 yd. and alt. 20 ft.
4. A straight rectangular street 7 mi. long and $2\tfrac{1}{2}$ rd. wide.
5. A page $8\tfrac{2}{3}$ in. long and $4\tfrac{1}{2}$ in. wide.
6. A triangle on base of 18 in. and alt. 15 in.
7. A triangular field whose alt. is 40 yd. and base 45 rd.
8. A trapezoid whose bases are 60 and 75 feet and alt. is 15 yd.

9. A trapezoid whose bases are 3 mi. and 400 rd. respectively, and altitude is 80 rd.

10. A circle whose radius is 6 in.

11. A circle whose diam. is 10 rds.

12. A circle whose circumference is 80 ft.

13. A triangle whose sides are 9, 10, 17 in.

14. A triangle whose sides are 12, 17, 25 ft.

15. A triangle whose sides are 13, 30, 37 yds.

16. A triangle whose sides are 20, 37, 51 rds.

17. A triangle whose sides are 25, 63, 74 mi.

18. An equilateral triangle whose sides are each 5 in.

19. An equilateral triangle whose sides are each 80 rds.

20. A circular ring whose two diameters are 28 and 16 ft.

21. A circular race-track is 3 rds. wide and placed around and just inside a field whose radius is 63 rds. Find area of the track in acres.

22. What is the land in a river-bed worth at $60 an acre, if the river increases from 6 to 60 rds. in width and is 20 miles long? (Trapezoid.)

23. A farm in shape of a triangle whose sides are 140, 143, 157 rods was sold at $85 an acre. Find the value of the farm.

24. A barn is 48 feet wide and 90 feet long. At the corner its height is 20 ft., but at the middle the height to the peak is 38 ft. Find (*a*) the area of the end; (*b*) the length of the rafters; and (*c*) the entire exterior surface of the barn.

III. Mensuration of the Surfaces of Solid Figures.

326. Definitions.—A **solid** is that which has length, breadth, and thickness.

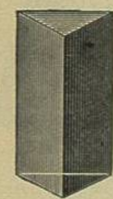
A **prism** is a solid bounded by two equal and parallel polygons called bases, and by parallelograms (which together form the **lateral surface**).

The **altitude** of a prism is the perpendicular distance between the bases.

Prisms are **triangular, quadrangular, pentagonal,** etc.,

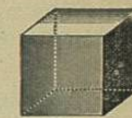
according as their bases are triangles, quadrilaterals, pentagons, etc.

A **regular prism** is one which has regular polygons for its bases.


TRIANGULAR PRISM.


QUADRANGULAR PRISM.


PENTAGONAL PRISM.


CUBE.

A **right prism** is one in which the other faces are perpendicular to the bases.

An ordinary box is a right rectangular prism.

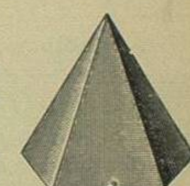
A **cube** is a prism bounded by squares.

A **pyramid** is a solid bounded by a polygon called the base, and by triangles meeting at a point called the vertex.

The triangles which meet at the vertex taken together form the *lateral surface.*

The **altitude** of a pyramid is the perpendicular distance from the vertex to the base.

A pyramid is **triangular, quadrangular, pentagonal,** etc., according as the base is a triangle, quadrilateral, pentagon, etc.


PYRAMID.

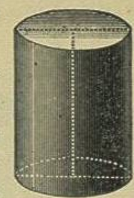
A **regular pyramid** has a regular polygon for its base, and the triangles bounding the pyramid all equal.

The slant height of a regular pyramid is the perpendicular distance from the vertex to one side of the base.

A **cylinder** is a solid formed by the revolution of a rectangle about one of its sides as an axis. Hence, a cylinder has two circles for bases.

A **cone** is a solid formed by the revolution of a right triangle about one of its sides as an axis. Hence, a cone has a circle for its base.

The altitude of a cone is the perpendicular distance from the vertex to the base. The **slant height** is the distance from the vertex to any point in the circumference of the base.

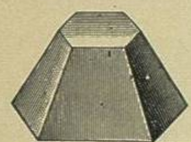


CYLINDER.

CONE.

The **frustum of a pyramid** is the portion of the pyramid intercepted between the base and a plane parallel to the base.

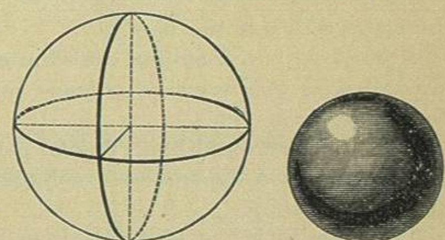
The **frustum of a cone** is the portion of a cone intercepted between the base and a plane parallel to the base.



FRUSTUM OF PYRAMID.

FRUSTUM OF CONE.

A **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within called the center.



The **radius** of a sphere is a line drawn from the center to any point of the surface. The **diameter** is a line passing through the center and terminated by the surface.

327. Formulas for areas of surfaces of solids.

1. *Lateral surface of a* **right prism** = *perimeter of base* × *altitude.*

2. *Convex surface of a* **cylinder** = *circf. of base* × *alt.* = $2\pi RH$ (where $\pi = 3.1416$, R = radius of base, H = altitude).

3. *Lateral surface of a* **regular pyramid** = $\frac{1}{2}$ *perimeter of base* × *slant height.*

4. *Convex surface of a* **cone** = $\frac{1}{2}$ *circf. of base* × *slant height* = πRL (where L = slant height).

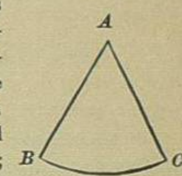
5. *Lateral surface of* **frustum of regular pyramid** = $\frac{1}{2}$ *sum of perimeters of bases* × *slant height.*

6. *Convex surface of* **frustum of cone** = $\frac{1}{2}$ *sum of circumferences of bases* × *slant height* = $\pi (R + r) L$.

7. *Surface of a* **sphere** = 3.1416 × *diameter squared.*

$$= \pi D^2, \text{ or } 4\pi R^2.$$

These formulas (except 7) are derived from those given in Art. 325. Thus the lateral surface of a right prism is composed of rectangles, all having the same altitude; that of a regular pyramid is composed of equal triangles; that of a frustum of a regular pyramid of equal trapezoids. Also the convex surface of a cylinder unrolled forms a rectangle; of a cone forms a portion of a circle, called a sector, as in the Fig. ABC, its area being $\frac{1}{2}\, BC \times AC$, which is determined in the same way that the area of a circle is obtained; the convex surface of a frustum of a cone equals the difference between two sectors.



The student should read at this point the note to Art. 325.

EXERCISE 160.

Find the area of the lateral surface of:

1. A right prism 10 in. high on square base, 3 in. on a side.

2. A right prism 8 ft. high, and on an octagonal base 9 in. on each side.

3. A regular pyramid on a hexagonal base 5 in. on a side, and of slant height of 10 in.

4. A regular pyramid on pentagonal base, 7 ft. on a side, and slant height = 19 yds.

5. A frustum of a triangular pyramid, each side of the lower base being 6 ft., and of the upper base being 5 ft., and with slant height of 8 ft.

6. A cylinder of revolution 7 ft. long, the radius of whose base is 3 ft.

7. A cone of revolution on base of radius 8 ft., and whose slant height is 40 ft.

8. A pipe 18 in. through and a mile long.

9. A cone whose radius is 6 in. and slant height is a yard.

10. The frustum of a cone of revolution, if the radii of the bases are 7 and 17 in. respectively, and the slant height is 20 in.

11. Find area of surface of a sphere whose radius is 3 ft.

12. Find area of surface of a sphere whose diameter is 19 in.

13. At 12¢ a sq. ft., what is the cost of painting a pyramidal spire, whose base is a hexagon of 9 ft. on a side and slant height is 90 ft.?

14. What will it cost to paint a cylindrical water-tower at 20¢ a sq. yd., if the diameter of the tower is 10 ft. and its height is 80 ft.?

15. Compute the cost of gilding a dome in the shape of a hemisphere, whose radius is 18 ft., at $1.75 a sq. yd.

16. A post 40 ft. long, in the shape of the frustum of a cone, is 10 in. thick at one end and 18 in. at the other. Find its *entire* superficial area.

IV. Mensuration of Solids.

328. Definitions.—Beside the definitions given in Art. 326, it should be recalled (see Arts. 178, 179) that a *unit of volume* is a cube, each edge of which is a linear unit, as a cubic inch, or a cubic yard; and that the *volume* of a solid is the number of cubic units which the solid contains. Thus the volume of a room is the *number* of cubic feet which it contains.

329. Formulas for volumes of solids.

1. *Volume of a* **prism** = *area of base* × *altitude.*

2. *Volume of a* **rectangular prism** = *length* × *breadth* × *thickness.*

3. *Volume of a* **cube** = *cube of its edge.*

4. *Volume of a* **cylinder** = *area of base* × *altitude* = $\pi R^2 H$.

5. *Volume of a* **pyramid** = $\frac{1}{3}$ *area of base* × *altitude.*

6. *Volume of a* **cone** = $\frac{1}{3}$ *area of base* × *altitude* = $\frac{1}{3}\pi R^2 H$.

7. *Volume of a* **frustum of pyramid**

= $\frac{1}{3}$ *altitude* × (*sum of areas of bases* + *square root of their product*)

= $\frac{1}{3} H(B + b + \sqrt{Bb})$ (when H = alt., B, b = areas of bases).

8. *Volume of a* **frustum of cone** = same as in 7.

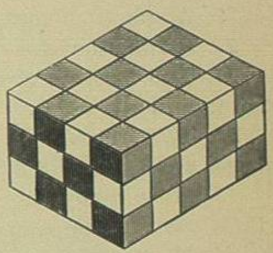
= $\frac{1}{3} H\pi (R^2 + r^2 + Rr)$ where R and r are radii of bases.

9. *Volume of a* **sphere** = $\frac{1}{3}$ *surface* × *radius,*

$$= \tfrac{4}{3}\pi R^3.$$

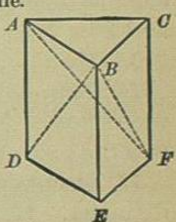
It should be remarked again that the student needs to study solid geometry, in order to understand fully the reasons for these formulas.

It will be of service, however, to recall (see Art. 179) that in a rectangular prism the volume, or number of cubic units, is equal to the number of linear units in the three edges multiplied together.



It is also to be observed that a rectangular prism may be conceived as divided into two equal triangular prisms with equal bases and the same altitude. Hence, the volume of each will equal half the volume of the rectangular prism, or the volume of a triangular prism = *base* × *alt.*

Any other prism may be split up into triangular prisms and its volume obtained by the same rule.



A cylinder may be conceived as determined by a prism with an infinite number of sides.

Any triangular prism, as $ABCDEF$, may be separated into three equivalent pyramids, for $B - DEF = F - ABC$ (or $B - ACF$) = $B - ADF$.

∴ Volume of pyramid $B - DEF = \frac{1}{3}$ prism $ABCDEF$.

The sphere may be regarded as an aggregate of very small pyramids with their common vertex at the center of the sphere, and the sum of their bases approximating to the surface of the sphere. Hence, the volume of the sum of the volumes of the pyramids will be determined by $\frac{1}{3}$ the product of the surface of the sphere $\times$ its radius.

The student should again read the note to Art. 325, and state how it applies to the mensuration of volumes.

EXERCISE 161.

Find the volume of:

1. A prism on base of 10 sq. ft. and whose height is 12 ft.

2. A rectangular prism whose dimensions are $8 \times 9 \times 10$ ft.

3. A room whose dimensions are 15 ft. 3 in. $\times$ 13 ft. 4 in. $\times$ 11 ft. 6 in.

4. A 12-in. cube. A 13-ft. cube.

5. A pyramid 9 ft. high whose base is 100 sq. ft.

6. A pyramid whose alt. is 28 ft. and base is 60 sq. yds.

7. A cylinder whose radius is 9 in. and alt. is 10 ft.

8. A piece of wire $\frac{1}{2}$ in. thick and 75 yds. long.

9. A sphere of radius 5 in. One of radius 7 ft.

10. A sphere whose diameter is 11 in.

11. The frustum of a pyramid whose bases are 32 and 50 sq. ft. and alt. is 9 ft.

12. The frustum of a cone 6 ft. high, the radii of whose bases are 6 ft. and 8 ft.

13. How many cubic feet of water in a cylindrical water-tank 10 ft. in diameter and 80 ft. high? How many gallons?

14. How many cu. in. in a glass shaped in the frustum of a cone $3\frac{1}{2}$ in. high, if the diameters of the base and top are 2 and 3 in. respectively?

15. How much larger is a 4-inch cube than a 4-in. sphere?

16. From a 7-ft. cube of granite the greatest possible sphere was cut out. How many cu. ft. of stone were removed? What was the area of the surface of the sphere?

17. Supposing a drop of water to be a sphere having $\frac{1}{5}$ in. diameter. How many drops of rain in a cylindrical pail 20 in. deep and 8 in. in diameter? How many such drops in a gallon?

18. If a bushel-measure in form of a cylinder is 18 inches in diameter, how deep is it? If it is 18 inches deep, what is its diameter?

19. Into a cylindrical water-tank 13 ft. in diameter and standing on end, an iron globe 10 ft. in diameter is sunk. How far will the surface of the water rise?

20. A heap of wheat in shape of a cone is 8 ft. deep and the diameter of the base is 15 feet. How many bushels in the heap?

21. A regular pyramid is 40 ft. high and stands on an equilateral triangle for base whose sides are each 6 ft. Find its volume. Find its slant height and lateral area.

V. Lines, Areas, and Volumes of Similar Figures.

330. Definitions.—**Similar surfaces** are those which have the same shape. Thus, any two squares are similar plane figures.

Similar solids are solids which have the same shape. Thus, any two cubes, or two spheres, are similar.

331. Properties of Similar Figures.—In any two similar figures

I. *Any two* **corresponding lines** *have the same ratio as any other two corresponding lines;*

II. *The* **areas** *of any two similar figures are to each other as the* **squares** *of any two corresponding lines;*

III. *The* **volumes** *of any two similar solids are to each other as the* **cubes** *of any two corresponding lines.*

Let the pupil illustrate these principles by drawing two squares with edges 2 in. and 5 in. respectively, and comparing their areas; and by drawing, or forming, two cubes with edges 2 in. and 5 in. respectively, and determining the number of cubic inches in each figure.

It is to be observed that the comparison of surfaces and solids of the same shape is made to depend again on the measurement of straight lines and computations from them (see Art. 325, note).

Ex. If a pipe 1 in. in diameter discharges 50 gal. in a minute, how much will a pipe 2 in. in diameter discharge?

The quantity discharged by a pipe is in proportion to the area of the section of the pipe, and hence, in proportion to the square of its diameter. Hence,

$$1^2 : 2^2 = 50 : (\quad)$$

$$\text{or No. gals. required} = \frac{50 \times 4}{1} = 200.$$

EXERCISE 162.

1. One of two similar triangles contains 135 sq. in. If its base is 15 in., what is the area of the other whose base is 18 in.?

2. Two sides of a polygon are 27 and 32 inches. In a similar polygon the less of the two corresponding sides is 18 in. What is the length of the other?

3. A polygon whose base is 12 ft. contains 62 sq. ft. What is the area of a similar polygon whose base is 42 ft.?

4. If the area of a circle, whose radius is 5 in., is 78.54 sq. in., find the area of a circle whose radius is 7 in. Prove your answer correct.

5. If a cylinder whose alt. is 8 ft. has a convex surface of 44 sq. ft., what is the convex surface of a similar cylinder whose alt. is 20 ft.?

6. The volume of a solid is 52 cu. in. and one side is 4 in. Find the volume of a similar solid if a corresponding side is 6 in.

7. The volume of a solid is 400 cu. ft. and one side is 12 ft. Find the volume of a similar solid if the corresponding side is 21 ft.

8. If the sides of two squares are as 2 : 3, what is the ratio of their areas? If the edges of two cubes are as 3 : 5, what is the ratio of their volumes?

9. Two spheres have radii equal to 7 and 9 inches respectively. What is the ratio of their circumferences? Of the areas of their surfaces? Of their volumes?

10. A man whose coal-bin is of a certain size, builds another having each dimension twice as great. How much lumber would he require compared with the first bin? How many times as much coal will it hold?

11. The volumes of two similar solids are 297 cu. in. and 704 cu. in. If the shortest side of the less is 3 in., what is the shortest side of the other?

12. The areas of two similar triangles are 324 and 1444 sq. ft. respectively. If the base of the greater is 14 ft., what is the base of the less?

13. If there are 300 yards in a 4-in. ball of yarn, how many yards will there be in a 6-in. ball? In a 2-in. ball?

14. If it costs $250 to paint a certain house, how much will it cost to paint another, all of whose dimensions are double those of the first?

15. If the planet Jupiter has 11 times the diameter of the earth, how do their surfaces compare? How do their volumes compare?

16. How many rods in the radius of a circle twice as large as another which contains 160 sq. rds.?

17. What is the ratio of the depths of similar quart and peck measures? A peck and bushel measure?

18. If a grindstone 18 in. in diameter costs $4, what ought another cost having the same thickness but 24 in. in diameter?

19. If a 3-in. roll of butter is worth 60 cents, what is a 5-in. roll worth?

20. If a person 5 ft. 6 in. tall ought to weigh 150 lbs., what should a person 6 ft. tall weigh?