33. A rectangular block of an alloy is 3.4 m. × 16 dm. × 25 cm. If the sp. gr. of the alloy is 15, and the block contains 6.12 Kg. of brass, what per cent. of it is brass

34. A rectangular vessel, 10 cm. wide and 3 cm. deep, contains 3 Kg. of sea-water (sp. gr. 1.025). How long is the vessel?

35. A mixture is formed by diluting 1 l. of nitric acid with 2 l. of water. If the sp. gr. of the acid is 1.5, what is the weight of the mixture? What % of this weight is nitric acid?

36. It being given that a cu. in. equals 16.39 cu. cm., how many cu. in. of gold (sp. gr. 19.3) weigh 100 Kg.?

37. A tank containing a certain volume of liquid, which weighs .962 as much as an equal volume of water, is emptied at the rate of 25 l. per hour. At the end of 3 hours the tank contains 163.2 Kg. of the liquid. What was the weight in Kg. of the liquid in the tank at first? What was its volume in gallons?

38. What must be the height of a tank whose bottom is 1 m. 2 dm. × 7 dm., and which holds 414.54 Kg. of oil (sp. gr. 1.05)?

39. If 20 cu. cm. of iron weigh as much as 144 cu. cm. of water, what will be the weight in Kg. of an iron cube 21 cm. on each edge? Of a cubic meter of iron?

40. A block of wood is 0.1 m. × 0.15 m. and weighs 3 Kg. Find its 3d dimension (sp. gr. 0.8).

CHAPTER XXI.

ARITHMETICAL HISTORY.

HISTORY OF NUMERATION AND NOTATION.

350. Number Groups.—Arithmetic begins with the making of units into groups, and the dealing with number by some group method. The first grouping of units in almost all savage tribes is done by aid of the fingers, either of one hand or of both hands, or by use of all the fingers and the toes. Hence the first number groups were fives, tens, or twenties.

For instance, one South African tribe uses three persons for numeration purposes, the first to count units on his fingers, the second to count tens, and the third hundreds.

Five is the primary number group among tribes who do not count much beyond twenty, as those in North Siberia, in New Hebrides, and the Esquimaux; twenty was the primary group among the Phœnicians, Basques, Aztecs, and is so among most of the tribes of South America, and some of those of North America. Ten is the usual base among primitive peoples in the rest of the world. However, the Maories of New Zealand use eleven as a base (thus, for them, the symbols 13 would mean one 11 and 3 units, or 14).

The Indo-European races seem to have used twelve as a base to a great extent, owing probably to the fact that two, three, four, and six will all divide it exactly. Twelve is in fact the best practical base, but ten is now too well established to make possible a general change to twelve.

The ancient Babylonians used sixty as a base, for a reason which will be given later.

351. Number Words.—Owing to the difficulty which savage peoples have in forming and using abstract language, the number words used by them do not always correspond to the groups formed by them by the aid of their fingers and toes. For language purposes they seem at first to use smaller groups of units.

Not a few tribes have no number words beyond two; they count "one, two, many." Others have a binary system, in that for four they use "two-two"; for six, "two-two-two," etc.

The Campos of Peru count to three; for four, they say "one and three"; for five, "two and three."

If a tribe has a number word for four, it is almost sure to go one step further and have a word for five, and thus reach a quinary system. Five is often expressed by the word for hand; six as "hand one"; seven as "hand two," etc.; ten as "both hands"; twelve as "two on the foot"; twenty as "the whole man"; sixty as "three men," etc.

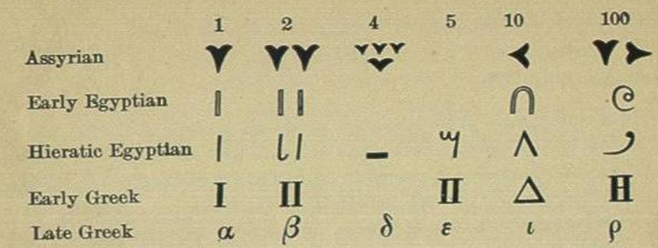
The number words "one, two, three," etc., no doubt originally had similar concrete meanings, but these were early lost, as it was an advantage, when number words were much used, to have purely abstract terms for them.

As civilization developed, primitive systems of numeration survived along with later systems, and mixed systems resulted. Thus in our use of "pair, brace, couple," the binary system survives along with our decimal system. In our use of score (as in "three score and ten," see also the French "quatre vingt" for eighty), the vigesimal system appears.

352. Number Symbols.—The first number symbols were fingers held up, or pebbles laid aside, or scratches made on some object, as wood or stone.

The Greeks, by using the separate joints of the fingers, could indicate numbers up to 10,000 on the hands. Proceeding from the little finger of the left hand through to the little finger of the right hand, the joints of the first three fingers denoted units; those of the next two fingers denoted tens; of the next two, hundreds; and of the last three, thousands. The Chinese to-day, by using the two sides and the front of each finger-joint as symbols, express 100,000 on the left hand alone.

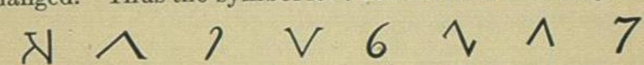
The following are illustrations of early written symbols used for numbers:



Number symbols are combined in these early systems in additive or multiplicative ways (thus, 𒁹𒁹 for "two" is an example of the additive use of the symbol for one; 𒁹➤ of the multiplicative use of symbols).

The Romans also use a subtractive principle in combining symbols, as in IV., IX., etc.

The positional (or exponential) system of written symbols was used to some extent by the Babylonians, but was rediscovered by the Hindoos, and the zero symbol invented, about 400 A. D. The figures 1, 2, 3, 4, etc., were originally the initial letters of the Hindoo words for the corresponding numeral adjectives, but the form of some of them has been much changed. Thus the symbol for 7 has had the following forms:



The Semitic peoples write from right to left, the Chinese from top to bottom, the Aryan peoples from left to right. Similarly, in writing numbers, each of these peoples as a rule follows its own order, putting the symbols for the largest groups first.

353. Higher Number Words.—The Hindoos used a separate name for each order of units; thus, they read 52965378196 as "5 kharva, 2 padwa, 9 vyarbada," etc. This system required the use of an unnecessarily large set of number words.

In modern times, the Italians grouped the digits into periods, sometimes of six, sometimes of three figures. The number given above would at one time have been read by them thus, "52 thousand thousand thousand, 965 thousand thousand, 378 thousand, 196."

The word *million* was invented by the Italians in the fourteenth century, and words *billion, trillion*, etc., by the French about the year 1500.

At that time figures were generally separated into periods of six figures each, hence, billion meant one million million, trillion meant one million billion, etc. These words continue to have these values in England, Germany, and the north of Europe generally. About the year 1750 it became the custom in France to divide figures into periods of three figures each; hence billion came to mean one thousand million, trillion one thousand billion, etc., which is the meaning now assigned to these words in the United States, France, and the south of Europe generally.

HISTORY OF ARITHMETICAL OPERATIONS.

354. Finger Reckoning.—The ancient Greeks, for example, had methods of performing addition, subtraction, etc., by a finger symbolism. The precise methods employed are not now understood, but there is a possibility that they may yet be worked out by a study of Greek monuments and literature. Finger reckoning was also much used in the Middle Ages in the monastic work of calculating the date of Easter, etc.

355. Abacus.—The method of counting by tens early led to the invention of the abacus.

This instrument had many different forms among different peoples, as the Egyptians, Chinese, Greeks, and Romans. The typical form is a rectangular frame containing parallel wires, on each of which are 9 buttons or counters. The counters on the wire to the extreme right (or left) represent units; those on the next wire represent tens, etc.

Let the teacher show the class an abacus, and how addition and subtraction are performed on it. Multiplication is performed by successive additions, and division by successive subtractions. Thus, to multiply 37 by 64,

$$\begin{aligned}37 \times 64 = (30+7)(60+4) &= 30 \times 60 + 30 \times 4 + 7 \times 60 + 7 \times 4 \\ &= 1800 + 120 + 420 + 28.\end{aligned}$$

Frequently a flat board covered with dust was used as an abacus. Lanes or columns were marked out in the dust, and in these pebbles were used as counters. Sometimes grooves were cut in a metallic plate, and movable buttons used in the grooves. Abacus reckoning was modified into reckoning with marks on horizontal lines, called counters. This was used in

England as late as the seventeenth century, and Shakespeare makes allusions to it.*

Addition and subtraction, and even multiplication can be performed rapidly with the abacus, but its use has the serious disadvantage that no record is kept of the steps of the work.

356. Addition and Subtraction.—The Hindoos performed their arithmetical work upon a small board, on which they made large marks by a cane or brush. To save space, they erased a figure as soon as it had done its service. Hence, their methods of operation differed in some respects from those employed at present. The Hindoos usually performed addition and subtraction from left to right, and set the result above the numbers added or subtracted instead of below. Thus, to add 376 and 258, they would arrange the work as at the right, and say "3 and 2 = 5, set down 5 above 3; 7 and 5 = 12, erase 5 and put 6 in its place, set down 2; 8 and 6 = 14, erase 2 and put 3 in its place, and set down 4 above 6."

$$\begin{array}{l} \underline{634}, \textit{ Sum.} \\ 376 \\ 258 \end{array}$$

The Arabs followed the same methods, except that they crossed out figures and wrote others above them, instead of erasing them. Thus, to subtract 275 from 653, they proceeded in the manner indicated at the right.

$$\begin{array}{l} 37 \\ \underline{\cancel{4}\cancel{8}8}, \textit{ Difference.} \\ 653 \\ 275 \end{array}$$

The present method of subtracting from right to left and setting results below came into use in Europe after the year 1200 A. D.

357. Multiplication was performed by the Hindoos in several different ways, of which the following two are the most representative. Ex. Multiply 157 by 62. The multiplier is written below and the product above the multiplicand. Thus, $6 \times 1 = 6$, set down 6 above 1; $6 \times 5 = 30$, erase 6 and put 9 in its place; set down 0 above 5; $6 \times 7 = 42$, etc.

$$\begin{array}{l} 7 \\ \cancel{6} \\ 9\cancel{4}3 \\ \cancel{6}\cancel{0}\cancel{2}4 \\ 157 \\ 62 \end{array}$$

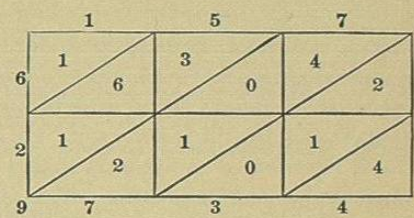
* Othello, act I., scene 1, line 31, "counter-caster." Cymb., V., 4, 167. As You Like It, II., 7, 63.

The Hindoos erased figures so that the result of their work would appear thus:

9734
157
62

The Arabs crossed out figures, leaving the work as given above.

The Hindoos also used a diagonal method of multiplication. The multiplication of 157 by 62 by this method is here given:



The method of multiplication commonly used at present is found in Pacioli (1494 A. D.), but it had been occasionally used long before.

A kind of multiplication called complementary multiplication, or sluggard's rule, was much used in the Middle Ages. In working with it the multiplication table was not needed beyond 5×5, but the method was tedious in operation. The principle on which it was based is as follows: If a and b represent digits, then,

$$a \times b = (10 - a)(10 - b) + 10(a + b - 10).$$

For example, $7 \times 6 = 3 \times 4 + 10(13 - 10) = 12 + 30 = 42$.

358. Division.—In all ancient mathematics we find no idea of a quotient. Division is performed by successive subtractions. After the Hindoos devised our present system of notation, they performed division by writing the divisor below the dividend, and setting down and erasing remainders above.

Ex. Divide 8479 by 36.

36 is contained in 84 twice; $2 \times 3 = 6$, 6 from 8 leaves 2, which set down above 8; $2 \times 6 = 12$, erase 2 and set down 1 above it, 2 from 4 leaves 2, which set down above 4, etc.

$$\begin{array}{l|l} \cancel{1}1 & \\ \cancel{1}3\cancel{4} & \\ \cancel{2}2\cancel{9}9 & \\ 8479 & 235 \\ 36 & \end{array}$$

The quotient is 235, with remainder 19.

As the Hindoos erased figures instead of scratching them, the result of their work would appear as follows:

$$\begin{array}{r|l} 19 & \\ 8479 & 235 \\ 36 & \end{array}$$

This "scratch" or "galley" method continued to be the favorite method of division in Europe till about the year 1700.

Our present method is given by Pacioli (1494), and is called by him the method "giving" (*i. e.*, bringing down one more figure after each subtraction), but he prefers the other method.

359. Factors. Primes.—The Greeks classified numbers in a great variety of ways; for example, as triangular, perfect, defective, excessive, etc.

The distinction of *odd* and *even* is due to Pythogoras (550 B. C.). Euclid discusses the properties of *prime* numbers (300 B. C.). Eratosthenes (200 B. C.) invented the method of determining primes, called the "sieve."

The Hindoos discovered the short way of determining whether a number is divisible by 3 or 9. They used this property of numbers in a method of verifying operations, called "casting out the nines," which is still used to some extent.

HISTORY OF FRACTIONS.

360. Fractions presented great difficulties to early peoples. An early Egyptian MS. (dating 1500 B. C.) shows that the Egyptians used only those fractions which have unity for a numerator.

This MS. gives tables by which other fractions can be reduced to these unit fractions. Thus, $\frac{2}{35} = \frac{1}{30} + \frac{1}{42}$. Fractions were written by writing only the denominator with a dot or special mark over it. By this method, even the addition of fractions was extremely complex.

The Babylonians used only those fractions which have 60 for a denominator (called sexagesimal fractions).

These fractions were expressed by writing the numerator a little to the right of its ordinary position, and omitting the denominator. The use of sexagesimal fractions survives in our present system of dividing a degree into 60 minutes, etc.

The Greeks used both the unit fractions of the Egyptians, and fractions of any numerator or denominator.

They indicated unit fractions by writing only the denominator with an accent, thus, ριδ′ means $\frac{1}{114}$. If the numerator was not unity, they wrote it with an accent, and the denominator twice, with a double accent. Thus, $\iota\delta'\ \kappa\gamma''\ \kappa\gamma''$ means $\frac{14}{23}$.

The Romans used only those fractions which have 12 for a denominator (called duodecimal fractions), and a few others derived from them, as $\frac{1}{24}$, $\frac{1}{48}$, $\frac{1}{72}$, $\frac{1}{144}$, $\frac{1}{288}$.

The addition and subtraction of such fractions present no difficulty, but multiplication and division are extremely complex.

The Hindoos used fractions in general, writing the numerator above the denominator, with no line between. Thus, $\begin{smallmatrix}2\\3\end{smallmatrix}$ means $\frac{2}{3}$, and $\begin{smallmatrix}5\\2\\3\end{smallmatrix}$ means $5\frac{2}{3}$.

Methods of obtaining the L. C. D. of fractions are given by Tartaglia (1556 A. D.).

The method of dividing by a fraction by inverting the divisor and multiplying, is given by Stifel (1544).

361. Decimal Fractions.—Several close approaches were made to decimal fractions before they were finally invented. Thus, in the Middle Ages, the square root of 3 was extracted by annexing six zeroes to the 3, extracting the square root of 3000000, and writing the last three figures of the root over 1000, thus $1\frac{732}{1000}$; but the part of the root $\frac{732}{1000}$ was at once converted into sexagesimal fractions.

Rudolff (1525) divided a number by 1000 by marking off the last three figures with a comma.

Stevinus (Belgium, 1548–1620) was the inventor of decimal fractions. As he had no decimal point, however, his notation was clumsy. Thus he expressed 3.912 either as $\overset{0}{3}\overset{1}{9}\overset{2}{1}\overset{3}{2}$, or $3_{(0)}9_{(1)}1_{(2)}2_{(3)}$, and read it "3 and 9 primes 1 sekonde 2 terzes," etc.

Later 3.912 was written $3\underline{|912}$.

The decimal point was first used by Pitiscus (Germany, 1612).

Decimal fractions at first were used in a very limited way, as in the calculation of interest. They did not come into general use till after the adoption of the metric system (1799).

HISTORY OF COMPOUND QUANTITIES.

362. The earliest **units of length** were taken from convenient parts of the human body, as the *digit* (a finger breadth), *palm*, *span*, *foot*, *cubit*, *ell*, the *fathom* (the extended arms). These units were convenient, being always at hand, but were not uniform enough when transactions were required to be exact. Later, the length of some natural object, as a *grain of* barley, became the unit of length. Finally, the length of some *piece of metal*, kept in the government archives, was used as a standard.

In very early times (in Egypt, Assyria, Canaan) two principal units of length, the *digit* and the *cubit*, were used.

The *foot* first came into general use in Greece and Rome, and from Rome it spread all over Europe. The Romans divided the foot into 12 "*unciae*" or *inches*.

In the year 1324, English law first defined the length of 3 barley-corns as equal to 1 inch, 12 inches = 1 foot, etc.

363. Of **units of weight**, the *pound* or *libra* originated in Rome, and from Rome was handed down to the various European peoples.

The Romans divided the libra into 12 *unciae* or *ounces* (thus the words ounce and inch each mean one-twelfth). The Greeks at times also divided the pound into 16 parts.

In the Middle Ages it became customary for merchants to make their profits in many cases, not merely by buying goods at one price and selling them at another, but by buying goods according to one kind of a pound (or other measure) and selling them by another, just as coal is now often bought by the long ton and sold by the short ton. In this way many different kinds of pound (and of other measures) arose, each trade or guild often having its own. Thus, the *Troy pound* was one used at a famous fair at the city of Troyes in France. Many changes and customs also arose which are now difficult to trace.

In the year 1266 English law fixed the weight of 32 barley-corns as equal to 1 pennyweight, 20 pennyweights = 1 oz., 12 oz. = 1 lb.

364. Of units of capacity a *bushel* (diminutive of box) measure was kept in the town hall at Winchester, the ancient

Saxon capital of England. This was the standard bushel in England till the year 1826, when the Imperial bushel of 2218.192 cu. in. was adopted by law. The Winchester bushel, however, continues to be standard in the United States.

365. Of **units of value** the *libra*, or pound of silver, was used in the Roman empire. From it are derived the *pound* of Great Britain, the *livre* of France, the *lira* of Italy, etc.

These were all originally of the same value, about $15, but the currency of each country was debased by the government at different times, till in England the unit now has but $\frac{1}{3}$ its original value, in France, $\frac{1}{75}$, etc. *Sterling* means easterling, referring to the coinage of the Hanseatic League, to the east of Great Britain.

366. In units of **time**, the Babylonians divided the day into 24 hours, the hour into 60 minutes, and the minute into 60 seconds. The month is determined approximately by the time it takes the moon to go round the earth, $29\frac{1}{2}$ days.

The Babylonians divided the circle into 360 degrees for convenience in astronomical work, since 360 in a close approximation to the number of days in the year, and then divided the circumference into 6 equal parts of 60 degrees each, because they knew that a radius applied as a chord 6 times exactly completes a circumference. Hence, probably arose the whole system of sexagesimal notation.

367. The **metric system** was adopted in France in 1799. The theory of the system is that the meter is $\frac{1}{10000000}$ of a quadrant of the earth's circumference through Paris, though owing to an error in the calculation it is actually a very small fraction less. Hence, the meter as used must be taken as the distance between two marks on a bar of platinum kept in Paris.

The liter, gram, and other units are derived from the meter in the manner described in Chapter XX.

The metric system has been adopted in all countries of the civilized world except Great Britain and the United States. It is used in such countries as Mexico, Hayti, Congo Free State, etc.

HISTORY OF OTHER TOPICS AND PROCESSES.

368. Percentage and **Interest** were used among the Romans, but these took their modern form among the Italians (especially at Florence, where bookkeeping by double entry was also invented).

Many mistakes in computing discount were made, and the method of true discount was not established till about the year 1700.

Equation of Payments is treated by Tartaglia (1556).

Exchange was developed to its present form among the Dutch.

369. Proportion, or the **Rule of Three,** till early in the nineteenth century, was used to include almost all the operations of arithmetic except the fundamental ones, and that in a very mechanical and superficial way. At one time eleven different kinds of proportion were used. During the nineteenth century an intelligent method of analysis has gradually taken the place of the mechanical "Rule of Three."

Partnership problems occur in Ahmes' treatise (Egypt, 1500 B. C.).

370. Involution and **Evolution** were performed by the Hindoos much as at present.

For other details of the history of arithmetic, the student is referred to Cajori's History of Elementary Mathematics, and to Fink's Brief History of Mathematics (translated by Beman and Smith).

EXERCISE 172.

A MISCELLANEOUS EXERCISE.

1. Find, to 3 decimal places, the number of gallons in a bushel.

2. Divide twelve per cent. of four hundred sixty by two-thirds of seven and two-tenths.

3. Reduce 2.4637 years to lower denominations.

4. What is the value of:

$$37 \times 42 + 86 - (738 - 528 \div 4) + 19 \times 17 - 1300.$$

5. Change $\frac{5}{9}$ acre to lower denominations.

6. Compute: $.01$ of $\frac{3}{4} \times 200 \times .08\frac{2}{5} \div .035$.

7. Subtract the sum of $9\frac{3}{5}$, $8\frac{7}{8}$, $4\frac{9}{10}$ from the sum of $7\frac{5}{12}$, $8\frac{3}{10}$, $10\frac{1}{15}$.

8. Reduce 75 rd. 3 yd. 1 ft. 5 in. to inches.

9. Find $2\frac{2}{3}\%$ of \$295.

10. Compute the interest of \$270 at 4% for 3 yr. 15 days.

11. What will $3\frac{1}{4}$ acres of land cost, if $7\frac{4}{5}$ acres cost \$$655\frac{1}{2}$?

12. Find the loss per cent. when a horse which sold for \$225 cost \$325.

13. Find the H. C. F. and L. C. M. of 473, 516, 559.

14. At $\frac{7}{8}$ dollar each, how many books can be purchased with \$$17\frac{1}{2}$?

15. If a man can mow a lawn in 6 days and his boy can do it in 9 days, how many days will they both require to do it, working together?

16. Change the following fractions to other equivalent fractions having 72 for their denominator:

$$\frac{11}{12},\ \frac{7}{24},\ \frac{8}{9},\ \frac{35}{45},\ \frac{50}{16},\ \frac{21}{56},\ \frac{33}{108},\ \text{and}\ \frac{24}{27}.$$

17. If 9 be added to both terms of the fraction $\frac{13}{16}$, will the value of the fraction be increased or diminished?

18. If the divisor is $\frac{3}{4}$, the quotient $\frac{2}{5}$, and the remainner $\frac{1}{3}$, what was the dividend?

19. How many bushels of corn at \$$\frac{2}{3}$ a bushel will pay for $\frac{4}{5}$ barrel of flour at \$$6\frac{1}{3}$ a barrel?

20. A carpenter worked $23\frac{1}{3}$ days and paid $\frac{3}{7}$ of his earnings for board and other expenses. If he saved \$$53\frac{1}{3}$ in this time, what was his daily wage?

21. A and B can spade a garden in 5 days, but B alone could do it in 7 days. How long would A require?

22. Reduce 5 wk. 3 da. 11 hr. 16 min. to minutes.

23. Find the actual gain if a selling price of \$1320 was a gain of 10%.

24. Add $8\frac{1}{4}$, $10\frac{5}{7}$, $12\frac{2}{3}$, $5\frac{19}{42}$, $9\frac{10}{21}$, $17\frac{5}{6}$.

25. Simplify $(2\frac{3}{4} \times 11\frac{3}{7}) \div (\frac{2}{3}$ of $18\frac{3}{5} \times 1\frac{4}{7})$.

26. Find the least whole number that is exactly divisible by $4\frac{1}{5}$, $3\frac{1}{2}$, $4\frac{2}{3}$.

27. A certain ore contains $8\frac{1}{2}\%$ of metal. How much metal will be obtained from 75 tons of ore?

28. A man lost 10% in selling a carriage for \$234. What should he have sold it for to gain 10%?

29. What is the difference between the $\sqrt[3]{10.01}$ and the $\sqrt{10.01}$ expressed in 3 decimal places?

30. Reduce 5.1735 mi. to lower denominations.

31. If $\frac{11}{12}$ of one line is $\frac{14}{15}$ of another, which line is the greater?

32. After I sold $\frac{5}{8}$ of my apple crop to one man and $\frac{3}{4}$ of the remainder to another there were 186 barrels left. How many barrels were there in the crop?

33. If a man can repair $\frac{5}{6}$ of a bridge in 10 days and his brother can repair $\frac{2}{3}$ of it in 6 days, how long would it require them both to repair the entire bridge, working together?

34. If the circumference of a wheel is 3 yd. 11 in., how many revolutions will it make in going a mile?

35. A man spent $\frac{1}{3}$, $\frac{1}{7}$, $\frac{2}{5}$ of his money and had \$2613 left. How many dollars did he spend altogether?

36. Of the earth's surface $39.87\frac{1}{2}\%$ lies in the Torrid Zone, and $25.91\frac{1}{2}\%$ lies in the Temperate Zones. What part lies in each Frigid Zone?

37. A tank's inside dimensions are 3 ft. 4 in. by 2 ft. 6 in. by 1 ft. 10 in. How many gallons of water will it contain?

38. How many bushels of grain will it hold?

39. Reduce 8 mi. 5 yd. 4 in. to inches.

40. Find the interest of \$916.50 for 4 yr. 6 mo. 24 da. at 4%.

41. Write a list of the prime numbers between 200 and 300.

42. A merchant marks goods 20% above cost and sells them $12\frac{1}{2}\%$ below marking price. What is his per cent. of gain?

43. Two cities have longitude $90° 15' 16''$ W. and $30° 20' 14''$ E. respectively. What is their difference in time?

44. If a man buys stock at 40% discount and every three months receives a dividend of 2% on the par value of the stock, what annual rate of interest does he receive on his investment?

45. What sum of money put at interest for 7 yr. 6 mo. 12 da. at $3\frac{1}{2}\%$ will gain \$1392.16 interest?

46. The expenses of a town for a year are \$7324 and the balance in treasury is \$696. There are 6862 polls to be assessed at \$0.25 each, and taxable property amounting to \$1965000. Besides the town tax there is a county tax of $1\frac{1}{4}$ mills and a state tax of $\frac{3}{4}$ mill on every dollar of taxable property. Mr. A. pays for 2 polls and has property worth \$28970. Find his total tax.

47. How many rails will be required to fence a field 5456 yd. long and $4051\frac{2}{3}$ yd. wide, provided the fences are all straight, all 6 rails high, and the rails of equal length, and the longest that can be used without cutting any?

48. What is the smallest sum of money with which I can purchase either chairs at \$8 each, or desks at \$24, or tables at \$52, or couches at \$72?

49. A farmer planted $\frac{3}{8}$ acre on Monday, $\frac{2}{7}$ acre on Tuesday, $\frac{2}{5}$ acre on Wednesday, $\frac{1}{4}$ acre on Thursday, $\frac{4}{15}$ acre on Friday, and the rest of his 2-acre lot on Saturday. Find on which day he planted the most ground and on which day the least.

50. A owned $\frac{5}{7}$ of a store and sold $\frac{2}{3}$ of his share to B, who sold $\frac{3}{5}$ of what he bought to C. C, in turn, sold $\frac{4}{9}$ of his purchase to D. What part of the entire store did each then own?

51. If I paid \$40 an acre for some land, how much must I ask for it, that I may abate 25% from my asking price and still gain 30% on the cost?

52. Divide $\dfrac{\frac{3}{4} \times (\frac{4}{5})^2 \text{ of } 4\frac{1}{8}}{\frac{2}{3} \text{ of } (2\frac{1}{4})^3 \times (2\frac{1}{2})^4}$ by $\dfrac{(\frac{4}{7})^2 \times 8\frac{3}{4}}{\frac{10}{11} \text{ of } (1\frac{2}{3})^3 \times (3\frac{3}{4})^3}$.

53. The front wheel of a wagon was 11 ft. in circumference and the rear wheel was 13 ft. A screw in the tire of each was uppermost when the wagon started, and when it stopped the same screws were uppermost again for the 633d time. How many miles had the vehicle traveled?

54. A real estate agent sold $5\frac{5}{9}$ acres at $\$138\frac{3}{5}$ each; $12\frac{5}{8}$ acres at $\$118\frac{1}{8}$ each; and $20\frac{11}{12}$ acres at \$123.60 each. Find the number of acres sold, the aggregate price, and the average price per acre.

55. If I buy a lot and it increases in value each year at the rate of 50% over the value of the previous year for 5 years and then is worth \$9000, how much did it cost?

56. What is the value of

$$\{6\tfrac{1}{4} \times (\tfrac{2}{5})^3 + \tfrac{2}{3} \text{ of } \tfrac{3}{4}\} \div \{18\tfrac{1}{5} - 14\tfrac{1}{8} + \tfrac{7}{24} \div (\tfrac{5}{6})^2 \times \tfrac{4}{7}\}\ ?$$

57. A farmer having a triangular piece of land, the sides of which are 481 ft., 1144 ft., and 1469 ft., wishes to enclose it with a fence having panels of the greatest possible uniform length. What will be the length of each panel?

58. What number is that from which if $11\frac{2}{3}$ be subtracted, $\frac{3}{5}$ of the remainder is $110\frac{4}{5}$?

59. A woman at her death left her son \$11640, which was $\frac{1}{2}$ of $\frac{3}{4}$ of her wealth. He at his death left $\frac{5}{6}$ of his portion to his daughter. What part of her grandmother's estate did the daughter receive? (Compute this fractional part two distinct ways.)

60. An agent wishing to sell a house and lot asked 40% more than it cost. But he finally sold it for 20% less than his asking price, thereby gaining \$4896. How much did the house and lot cost? What was his asking price? What was the selling price?

61. Reduce 3 lb. 8 oz. 19 pwt. 6 gr. to grains.

62. Which is the greater, $\sqrt{50}$ or $\sqrt[3]{346}$?

63. Divide 38 mi. 100 rd. 5 yd. 2 ft. by 6.

64. A man bought wood for $\$287\frac{3}{7}$ and coal for $\$384\frac{5}{9}$ and oil for $\$76\frac{5}{12}$. He sold the wood for $\$327\frac{4}{5}$ and the coal for $\$375\frac{1}{6}$ and the oil for $\$88\frac{9}{10}$. How much did he gain on all?

65. A merchant bought 3 hhd. of molasses, each containing 63 gal., at 40 ¢ a gal. and paid \$6.75 for freight and cartage. Allowing 4% for waste and 5% of sales for bad debts and 2% of the remainder for collecting, what must he charge per gallon in order to make 27% on the entire cost?

66. Find the G. C. D. of 2538, 4089, 4324.

67. If cloth $1\frac{1}{2}$ yd. wide require $8\frac{2}{3}$ yd. in length for a suit of clothes, how many yd. in length will cloth $\frac{2}{3}$ yd. wide require for same suit?

68. Reduce 207958 in. to higher denominations.

69. Collect: $\dfrac{0.7 \text{ of } 4.5 \times 6.8}{0.17 \text{ of } 4.2 \times 9} + \dfrac{3.9 \times 5.7 \times 1.\dot{6}}{0.64 \text{ of } 1.9 \times 13}$.

70. Of a certain ore 6% is iron and 45% is rock, the rest being conglomerate. In a car of ore weighing 20 tons, how much is iron and how much rock?

71. Compute the compound interest on \$2560 for 3yr. 5 mo. 10 da. at 3%.

72. Find the G. C. D. of 2680, 1541, 2211.

73. A man owning 80% of a store sold $\frac{2}{3}$ of his share for $\$6781\frac{2}{5}$; what was the value of the entire store at same rate?

74. Change 16 wk. 5 da. 9 hr. 40 min. 20 sec. to seconds.

75. A square field contains 3 acres. Find the length of each side in rods.

76. What is the edge of a cubical box that will just contain a bushel? Of another that will exactly contain a gallon?

77. If $\frac{5}{6}$ lb. of sugar be worth $\frac{1}{4}$ lb. of butter, and butter be worth $\$\frac{3}{8}$ per pound, how many pounds of sugar will \$75 buy?

78. I gained $33\frac{1}{3}$% in selling a gray horse, and with the money bought a black horse which I sold for \$120, losing 20%. On the two horses did I gain or lose? What per cent.?

79. A can do a piece of work in 15 days; A and B together can do it in 10 days; A and C can do it in 6 days. How many days will B and C require, working together?