MEASURES

102. Measures of Weight. It is curious to note what an important part the grain of wheat or barley has played in the establishment of a unit of weight, both among the ancients and the more modern Europeans. In England, as early as 1266, we find the pennyweight defined as the weight of "32 wheat corns in the midst of the ear"; again about 1600, as "24 barley corns, dry and taken out of the middle of the ear." Still later the artificial grain ($\frac{1}{24}$ pwt. Troy) is defined as "one grain and a half of round dry wheat." The Greeks made four grains of barley equivalent to the keration or carob seed. From this is derived the carat, the measure by which diamonds and pearls are weighed. The grain of barley and the carat have been used by all European countries as the basis of existing weights.

103. Great inconvenience was long experienced from this lack of uniformity, so that Parliament in 1824 passed an act adopting the **Imperial Pound Troy** as the **standard of weight**. It was also enacted that of the 5760 grains contained in the pound Troy, the pound avoirdupois should contain 7000. The **international kilogram** is now the fundamental standard of weight in the United States. The *pound* avoirdupois is defined as $\frac{1}{2.2046}$ kilogram (see § 139). The ounce, grain, etc., are subdivisions of the pound.

104. Avoirdupois Weight

16 drams (dr.)	= 1 ounce (oz.)
16 ounces	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)

112 lb. = 1 long cwt. and 2240 lb. = 1 long ton are used in the customhouse and in weighing coal and iron at the mines.

The *c* in cwt. stands for the Latin word *centum*, a hundred.

Lb. is a contraction of the Latin word *libra*, pound.

Pound is from the Latin word *pondus*, a weight.

Ounce is from the Latin word *uncia*, a twelfth part.

Dram is from the Latin word *drachma*, a handful.

105. Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce Troy
12 ounces Troy	= 1 pound Troy

This weight is used for the precious metals and jewels. The ounce Troy and pound Troy must be carefully distinguished from the ounce and pound avoirdupois. The grain, however, is the same throughout.

437.5 grains = 1 ounce avoirdupois	480 grains = 1 ounce Troy
7000 grains = 1 pound avoirdupois	5760 grains = 1 pound Troy

106. Apothecaries' Weight

20 grains	= 1 scruple (sc. or ℈)
3 scruples	= 1 dram (dr. or ʒ)
8 drams	= 1 ounce (oz. or ℥)
12 ounces	= 1 pound
5760 grains	= 1 pound

This table is used in compounding drugs and medicines. Scruple is from the Latin word *scrupulum*, a small weight.

Of the above measures of weight, avoirdupois is the most generally used.

107. Measures of Length. The ancients usually derived their units of length from some part of the human body. Thus, we find the *fathom* (the distance of the outstretched hands), the *cubit* (the length of the forearm), and later the *ell* (the distance from the elbow to the end of the finger), the *foot* (the length of the human foot), the *span* (the distance between the ends of the thumb and little finger when outstretched), the *palm* (the width of the hand), the *digit* (the breadth of the finger). The Roman foot was subdivided into four palms, and the palm into four digits. The division into inches or *unciæ* (a twelfth part) applied not only to the foot but to anything.

108. For longer measures there was still less uniformity. We find the Hebrew's *half-day's journey*; the Chinese *lih*, the distance a man's voice can be heard upon a clear plain; the Greek *stadium*, probably derived from the length of the race course; the Roman *pace* of five feet; the *furlong*, the length of a furrow. The *mille passus*, a thousand paces, is the origin of the modern *mile*.

109. In 1374 the inch is defined in English law as the length of "three barley corns, round and dry." Later, other arbitrary measures of length were adopted by the government. The **international meter** of length were adopted by the government. The **international meter** is now the fundamental standard of length in the United States. The *yard* is defined as $\frac{3600}{3937}$ of a meter (see § 129). The **foot** and **inch** are subdivisions of this standard yard.

110. Common Measures of Length

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5½ yards or 16½ feet	= 1 rod (rd.)
320 rods or 5280 feet	= 1 mile (mi.)

The furlong, equal to 40 rods, is seldom used.

The fathom, equal to 6 feet, and the knot or geographical mile, equal to one minute of the equatorial circumference of the earth (6080 feet), are sometimes used.

111. Surveyors' Measures of Length

7.92 inches	= 1 link (li.)
100 links	= 1 chain (ch.) = (4 rd.)
80 chains	= 1 mile

112. Measures of Surfaces

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
1 square mile	= 1 section.
36 sections	= 1 township (twp.)

113. Measures of Solids

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

The cord, equal to 128 cubic feet, is a rectangular solid 8 feet long, 4 feet wide, and 4 feet high. The common use of the word is, however, a pile of wood 8 feet long and 4 feet high, the width of the pile varying with the length of the stick.

1 cubic yard	= 1 load
24¾ cubic feet	= 1 perch

114. Measures of Money. Originally, among primitive people, buying and selling was carried on by **barter**, or the actual exchange of commodities. The inconveniences arising from transactions of this kind brought about the adoption of a **medium of exchange**, or **money**. Money, usually consisting of gold and silver, was used at a very early period in the world's history. Gold and silver seem at first to have been exchanged for commodities by weight. Business transactions were then still further simplified by the introduction of coins and paper money. Finally, as in the case of weights and measures, governments adopted definite standards of money value.

115. United States Money

10 mills	= 1 cent (ct.)
10 cents	= 1 dime (d.)
10 dimes	= 1 dollar ($)
10 dollars	= 1 eagle (E.)

116. English Money

12 pence (*d.*)	= 1 shilling (*s.*)	= $0.2433
20 shillings	= 1 pound (£)	= $4.8665

117. French Money

10 centimes	= 1 decime	
10 decimes	= 1 franc	= $0.193

118. GERMAN MONEY

100 pfennigs = 1 mark (M.) = $0.238

119. MEASURES OF NUMBER

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (gt. gro.)
Also 24 sheets of paper = 1 quire
20 quires = 1 ream

120. LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.) = 231 cu. in.
31½ gallons = 1 barrel (bbl.)

121. DRY MEASURE

2 pints = 1 quart
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.) = 2150.42 cu. in.

The **Winchester bushel** is the standard measure for dry substances. It is a cylindrical vessel 18½ in. in diameter and 8 in. deep, containing 2150.42 cu. in.

Before the adoption of this and other standards by the English government there was even a greater variety of measures of capacity than of length and weight.

122. Reduction of Compound Numbers. Quantities like 5 mi. 10 rd. 7 yd. 2 ft. and 3 lb. 5 oz. are called *compound numbers*, because they are expressed in several denominations.

Ex. **1.** Reduce 25 yd. 2 ft. 11 in. to inches.

Solution. 1 yd. = 3 ft.
∴ 25 yd. = 25 × 3 ft. = 75 ft.
75 ft. + 2 ft. = 77 ft. 1 ft. = 12 in.
∴ 77 ft. = 77 × 12 in. = 924 in.
924 in. + 11 in. = 935 in.

25	
3	
75	
2	
77	
12	
924	
11	
935	

or

25 yd.	= 900 in.
2 ft.	= 24 in.
11 in.	= 11 in.
∴ 25 yd. 2 ft. 11 in.	= 935 in.

NOTE. The explanation shows that 25 and 77 are the multipliers and 3 ft. and 12 in. the multiplicands; but to shorten the operation, 3 and 12, regarded as abstract numbers, may be used as multipliers, since the product of 25 × 3 = the product of 3 × 25.

Ex. **2.** Reduce 1436 pt. to bushels, pecks, etc.

2	1436	no. of pt.
8	718	no. of qt.
4	89	no. of pk. + 6 qt.
	22	no. of bu. + 1 pk.

Solution. Since there are 2 pt. in 1 qt., in 1436 pt. there are as many quarts as 2 pt. are contained times in 1436 pt., or 718 qt.

Since there are 8 qt. in 1 pk., in 718 qt. there are as many pecks as 8 qt. are contained times in 718 qt., or 89 pk. 6 qt.

Since there are 4 pk. in 1 bu., in 89 pk. there are as many bushels as 4 pk. are contained times in 89 pk., or 22 bu. 1 pk.

∴ 1436 pt. = 22 bu. 1 pk. 6 qt.

EXERCISE 19

1. Reduce 3 A. 5 sq. rd. 12 sq. yd. to square yards.
2. Reduce 11000 sq. rd. to acres.
3. Reduce 2 gt. gro. 5 gro. to dozens.
4. Reduce 972 sheets to reams.
5. Reduce 20 cu. yd. to cubic inches.
6. Reduce 1000 oz. to pounds and ounces (avoirdupois).

7. Reduce 12 lb. 5 oz. 11 pwt. 20 gr. to grains.

8. Reduce 113 T. 7 cwt. 11 lb. to pounds.

9. Reduce 14763051 lb. to tons.

10. Reduce 5 sq. yd. 3 sq. ft. 91 sq. in. to square inches.

11. Reduce 46218385 sq. in. to acres.

12. Reduce 5 bu. 7 pk. 3 qt. to quarts.

13. Reduce 34372 pt. to pecks.

14. Reduce 21 yd. to a decimal of a mile.

15. Reduce 2 pk. 3 qt. 1 pt. to a decimal of a bushel.

16. Reduce 0.0125 A. + 0.25 sq. rd. to square feet.

17. Reduce 0.01 of a cubic yard to cubic inches.

18. Reduce 43629145 in. to miles.

19. Reduce $\frac{3}{8}$ of a peck to pints.

20. Reduce 2 qt. 1 pt. to a fraction of a peck.

21. Reduce 1 mi. 11 ch. to feet.

123. Addition and Subtraction of Compound Numbers. Compound addition and subtraction is the addition and subtraction of compound numbers of the same kind. The processes differ very little from the corresponding processes in the addition and subtraction of abstract numbers.

Ex. 1. Add 4 lb. 7 oz. (Av.), 3 lb. 4 oz., 12 lb. 10 oz., 9 lb. 5 oz.

4 lb.	7 oz.
3	4
12	10
9	5
29 lb.	10 oz.

Solution. The work is as follows:

5, 15, 19, 26 oz. = 1 lb. 10 oz.
1, 10, 22, 25, 29 lb.

Ex. 2. From 41 lb. 4 oz. (Av.) subtract 29 lb. 8 oz.

The work is as follows:

1 lb., or 16 oz. + 4 oz. = 20 oz.
8 and 12 are 20
1 and 29 and 11 are 41.

41 lb.	4 oz.
29	8
11 lb.	12 oz.

124. Multiplication of Compound Numbers.

Ex. Multiply 5 yd. 2 ft. by 7.

7×2 ft. = 14 ft. = 4 yd. 2 ft.
7×5 yd. = 35 yd.
35 yd. + 4 yd. 2 ft. = 39 yd. 2 ft.

5 yd. 2 ft.
7
39 yd. 2 ft.

125. Division of Compound Numbers. Compound division is of two kinds. The first is the converse of multiplication. In this case the quotient is a compound number of the same kind as the dividend. In the second case the dividend and divisor are both compound numbers of the same kind, and the quotient is an abstract number.

126. The two cases arise from the fact that division may be regarded as the operation of finding one of two factors when the other factor and the product are given.

Thus, 39 yd. 2 ft. is the product of 7 and 5 yd. 2 ft.

$\therefore \frac{39 \text{ yd. } 2 \text{ ft.}}{7} = 5$ yd. 2 ft., or $\frac{39 \text{ yd. } 2 \text{ ft.}}{5 \text{ yd. } 2 \text{ ft.}} = \frac{119 \text{ ft.}}{17 \text{ ft.}} = 7$, the dividend and divisor being reduced to the same denomination before dividing.

Ex. 1. Divide 29 mi. 2 yd. 2 ft. by 8.

Solution. 29 mi. ÷ 8 = 3 mi. + a remainder of 5 mi.
5 mi. = 8800 yd. and 8800 yd. + 2 yd. = 8802 yd.
8802 yd. ÷ 8 = 1100 yd. + a remainder of 2 yd.
2 yd. = 6 ft., and 6 ft. + 2 ft. = 8 ft.
8 ft. ÷ 8 = 1 ft.
∴ 29 mi. 2 yd. 2 ft. ÷ 8 = 3 mi. 1100 yd. 1 ft.

Ex. 2. Divide 139 lb. 8 oz. (Av.) by 4 lb. 8 oz.

Solution. 139 lb. 8 oz. = 2232 oz.
4 lb. 8 oz. = 72 oz.
2232 oz. ÷ 72 oz. = 31.

127. Check. Compound addition and subtraction may be checked in the same way as addition and subtraction of simple numbers. Multiplication may be checked by division, and division by multiplication.

EXERCISE 20

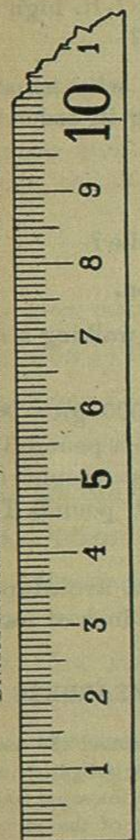
1. How many inches are there in 1 mi. 3 ch.?
2. Add 14 lb. 3 oz., 5 lb. 7 oz., 31 lb. 11 oz.
3. From 17 cu. yd. 11 cu. in. subtract 5 cu. yd. 5 cu. ft.
4. From 11 bu. 1 pk. subtract 4 bu. 5 qt.
5. Multiply 30 A. 11 sq. rd. by 10.
6. Divide 159 A. 29.5 sq. rd. by 2.
7. How many bags containing 2 bu. 1 pk. each can be filled from a bin of wheat containing 256 bu. 2 pk.?
8. How many revolutions will a bicycle wheel 7 ft. 4 in. in circumference make in traveling 25 mi.?
9. How many times can a bushel measure be filled from a bin 8 ft. square and 6 ft. deep? Will there be a remainder?
10. How many gallons of water will a tank 4 ft. 7 in. by 2 ft. 11 in. by 1 ft. 3 in. contain?
11. How many times is 7 ft. 6 in. contained in 195 mi. 280 rd.?

12. How much coal is there in three carloads of 38 T. 3 cwt. 41 lb., 29 T. 7 cwt. 5 lb., 32 T. 17 cwt. 70 lb.?
13. What must be the length of a shed 7 ft. high and 9 ft. wide to contain 50 cd. of 16 in. wood?
14. If a ton of coal occupies 36 cu. ft., what must be the depth of a bin 6 ft. wide by $7\frac{1}{2}$ ft. long in order that it may contain 10 T.?
15. Divide 320 rd. 4 yd. by 10 rd. 2 yd.
16. How many feet are there in $\frac{5}{8}$ of a mile?
17. Reduce 17 pt. to a decimal of a gallon.
18. How many steps does a man take in walking a mile if he advances 2 ft. 10 in. each step?
19. The pound avoirdupois contains 7000 gr. Find the greatest weight that will measure both a pound Troy and a pound avoirdupois. Find the least weight that can be expressed without fractions in both pounds Troy and pounds avoirdupois.
20. A cubic foot of water weighs 1000 oz. avoirdupois. Find the number of grains Troy in a cubic inch of water.

METRIC SYSTEM OF WEIGHTS AND MEASURES

128. Late in the eighteenth century France invented the metric system of weights and measures, but it was not made obligatory until 1837. Previous to this time there existed in France the same lack of uniformity in forming multiples and submultiples of the units of measure as exists in our system at the present time. The metric system is now in use in most civilized countries except the United States and England. It was legalized by Congress in the United States in 1866, but has not been generally adopted. In scientific work the system is quite generally used in all countries.

129. The unit of length is the **meter**. This is the fundamental unit, because from it every other unit of measure or weight is derived; hence the name **metric system**. The meter is theoretically one ten-millionth part of the distance of the pole from the equator. Though an error has since been discovered in the measurement of the distance, the meter has not been changed, and a rod of platinum 39.37 inches in length, deposited in the archives at Paris, is called the standard meter.



1 Decimeter of the Meter Rule
Divided into centimeters, half centimeters and millimeters.

130. The unit of capacity is called the **liter**. It is a cube whose edge is 0.1 of a meter.

131. The unit of weight is the **gram**. The gram is the weight of a cube of distilled water at maximum density, whose edge is 0.01 of a meter.

132. The above units of measure, together with the following prefixes, should be carefully memorized, because from them the whole metric system can be built up.

133. The Latin prefixes, **deci**, **centi**, **milli**, denote respectively 0.1, 0.01, 0.001 of the unit. The Greek prefix **micro** is used to denote 0.000001 of a unit. Thus, decimeter means 0.1 of a meter, and centigram means 0.01 of a gram.

134. The Greek prefixes, **deca**, **hecto**, **kilo**, **myria**, denote respectively 10, 100, 1000, 10000 times the unit. Thus, kilometer means 1000 meters, and hectoliter means 100 liters.

135. In general, nothing beyond practice in arithmetical operations would be gained in reducing from the metric system to our system. Occasionally, however, such reductions are necessary, hence, a few of the common equivalents are given in the tables.

136. Measures of Length

10 millimeters $^{(mm)}$	= 1 centimeter
10 centimeters $^{(cm)}$	= 1 decimeter
10 decimeters $^{(dm)}$	= 1 meter = 39.37 in.
10 meters $^{(m)}$	= 1 decameter
10 decameters $^{(Dm)}$	= 1 hectometer
10 hectometers $^{(Hm)}$	= 1 kilometer
10 kilometers $^{(Km)}$	= 1 myriamoter $^{(Mm)}$

137. Square Measure

100 square millimeters $^{(m^2)}$	= 1 square centimeter
100 square centimeters $^{(cm^2)}$	= 1 square decimeter
100 square decimeters $^{(dm^2)}$	= 1 square meter
100 square meters $^{(m^2)}$	= 1 square decameter
100 square hectometers $^{(Hm^2)}$	= 1 square kilometer $^{(Km^2)}$

This table may be extended by squaring each unit of length for the corresponding unit of square measure. The denominations given in the table are the only ones in common use.

In measuring land, the square decameter is called the are, the square hectometer, the **hectare** = 2.47 acres, and the square meter, the centare.

138. Cubic Measure

1000 cubic millimeters $^{(mm^3)}$	= 1 cubic centimeter
1000 cubic centimeters $^{(cm^3)}$	= 1 cubic decimeter
1000 cubic decimeters $^{(dm^3)}$	= 1 cubic meter $^{(m^3)}$

This table may be extended by cubing each unit of length for the corresponding unit of cubic measure. The denominations given in the table are the only ones in common use.

The cubic meter is used in measuring wood, and is called the **stere.**

139. Measures of Weight

10 milligrams $^{(mg)}$	= 1 centigram
10 centigrams $^{(cg)}$	= 1 decigram
10 decigrams $^{(dg)}$	= 1 gram
10 grams $^{(g)}$	= 1 decagram
10 decagrams $^{(Dg)}$	= 1 hectogram
10 hectograms $^{(Hg)}$	= 1 kilogram = 2.2 lb.
10 kilograms $^{(Kg)}$	= 1 myriagram
10 myriagrams $^{(Mg)}$	= 1 quintal
10 quintals $^{(Q)}$	= 1 tonneau $^{(T)}$

The **metric ton** or **tonneau** is the weight of one cubic meter of distilled water = 2204.62 pounds.

140. Measures of Capacity

10 milliliters $^{(ml)}$	= 1 centiliter
10 centiliters $^{(cl)}$	= 1 deciliter
10 deciliters $^{(dl)}$	= 1 liter = 1 qt. nearly
10 liters $^{(l)}$	= 1 decaliter
10 decaliters $^{(Dl)}$	= 1 hectoliter = 2.837 bu.
10 hectoliters $^{(Hl)}$	= 1 kiloliter $^{(Kl)}$

EXERCISE 21

1. What is the weight of a liter of water? Give the result in grams.

2. What is the weight of a cubic centimeter of water? of a cubic meter?

3. What is the weight of 15^{Hl} of water?

4. Find the sum of 21.14^{m^3}, 321^{l} and 1.25^{dl}. Give the result in liters.

5. Find in hectares and ares the area of a field 450^{m} long and 200^{m} wide.

6. If gold is 19.36 times as heavy as water, find in kilograms the weight of a bar of gold 10^{cm} long, 30^{mm} wide and 25^{mm} thick.

7. How many square millimeters are there in a square centimeter? in two meters square?

8. Reduce 240064^{mm} to kilometers, etc.

9. Reduce 3463^{ca} to hectares, etc.

10. If $15^{Kg}\ 7^{g}$ of beef cost 26 francs $37\frac{1}{4}$ centimes, find the cost per kilogram.

11. How many sacks will be necessary to hold $1245^{Hl}\ 6^{Dl}$ of wheat if each sack holds $1^{Hl}\ 20^{l}$?

12. What decimal of a decagram is $6^{g}\ 4^{cg}$?

13. How much wheat is contained in 1396 sacks, each of which contains $1^{Hl}\ 35^{l}$?

14. If the distance from the equator to the pole is 1000 myriameters, how many meters are there in a degree?

15. What will be the price of $47^{Ha}\ 5^{a}\ 65^{ca}$ of land at 89.76 francs per are?

16. Mercury is 13.598 times as heavy as water. Find the weight of 567.859^{cm^3} of mercury.

17. If a man steps 80^{cm} at each step, how many steps will he take in walking 10^{Km}?

18. Olive oil is 0.914 as heavy as water. Find the cost of a hectoliter at 3 francs a kilogram.

19. A piece of land 1236 meters square sold for 240 francs per hectare. How much did the land bring?

20. A person bought $\frac{3}{4}$ of a piece of land containing 2^{Ha} 15^{a} at 45 francs an are; he sold $\frac{2}{5}$ of what he bought for 5000 francs. How much did he gain?

21. A spring furnishes 5^{l} of water in 2 min. How long will it take the spring to fill a vessel holding $32\frac{3}{5}$ liters?

22. Three fountains furnish $3\frac{1}{2}^{l}$, $2\frac{2}{3}^{l}$ and $7\frac{3}{4}^{l}$ of water each minute respectively. The three together fill a tank in 2 hr. and 43 min. How many hectoliters of water does the tank contain?

23. If 8^{a} of land are bought for 19200 francs and sold for 25.20 francs per square meter, how much is gained by the transaction?

24. If sea water is 1.026 times as heavy as distilled water and olive oil is 0.914 as heavy, how much more than an equal volume of olive oil will a hectoliter of sea water weigh?

141. Measures of Angles and Time. The sexagesimal division of numbers is undoubtedly of Babylonian origin. The Babylonian priests in their astronomical work reckoned the year as 360 days. They supposed the sun to revolve around the earth once each year and hence divided the circumference of the circle into 360 parts, each of which represented the apparent daily path of the sun. They

probably knew the construction of the regular hexagon by applying the radius to the circumference six times. It was then natural to take one of the 60 parts thus cut off as a unit and to further subdivide this unit into 60 equal parts, and so on, according to their method of sexagesimal fractions. This is the origin of our degree, minute and second. The names *minutes* and *seconds* are taken from the Latin *partes minutæ primæ* and *partes minutæ secundæ*.

142. The principal measures of time are the day and the year. The day is the average time in which the earth revolves on its axis. The division of the day into 24 hours, of the hour into 60 minutes, and of the minute into 60 seconds is probably due to the Babylonians. The solar year is the time in which the earth travels once around the sun. It contains 365.2426 days.

143. In B.C. 46 Julius Cæsar reformed the calendar and decreed that there should be three successive years of 365 days followed by a year of 366 days to account for the difference of 0.2426 of a day between the year of 365 days and the solar year. The difference between four years of 365 days and four years of 365.2426 days is only 0.9704 of a day, so that if a whole day is added every fourth year there is added 0.0296 of a day too much. In 1582 Pope Gregory XIII corrected this by striking 10 days from the year, — calling Oct. 5th Oct. 15th, — and he decreed that three leap years were to be omitted in every four hundred years. Every year whose number is divisible by four is a leap year unless it is a year ending a century, as 1900, when it is a leap year only if divisible by 400. 1800, 1900, 2100, are not leap years, but 1600, 2000, 2400, are. The Gregorian calendar was adopted at once in Roman Catholic countries and in England in 1752. At the same time in England the beginning of the year was changed from March 25 to Jan. 1. Russia and some other countries still use the Julian calendar. Since 1582 they have had three more leap years (1700, 1800 and 1900) than countries using the Gregorian calendar, and hence are now 13 days behind other countries. What we call Jan. 23 is Jan. 10 with them.

144. Dates given according to the Julian calendar are called Old Style (O. S.) and dates according to the Gregorian calendar are called New Style (N. S.)

LONGITUDE AND TIME

145. Longitude is distance east or west of the **prime meridian.** The meridian of the Royal Observatory at Greenwich, England, is the prime meridian generally adopted, and longitude is reckoned east or west 180° from that meridian.

Since the earth makes one complete revolution on its axis in 24 hours, each place in its surface passes through 360° in that time. Hence:

360° of longitude correspond to 24 hr. of time.
1° of longitude corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.
1′ of longitude corresponds to $\frac{1}{60}$ of 4 min., or 4 sec.
1″ of longitude corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

And

24 hr. of time correspond to 360° of longitude.
1 hr. of time corresponds to $\frac{1}{24}$ of 360°, or 15°.
1 min. of time corresponds to $\frac{1}{60}$ of 15°, or 15′.
1 sec. of time corresponds to $\frac{1}{60}$ of 15′, or 15″.

Ex. 1. The difference in time between two places is 2 hr. 25 min. 13 sec. What is the difference in longitude?

Solution. $2 \times 15° = 30°$,

$25 \times 15' = 375' = 6° 15'$,

$13 \times 15'' = 195'' = 3' 15''$,

$30° + 6° 15' + 3' 15'' = 36° 18' 15''$.

Check by reducing 36° 18′ 15″ to hours, minutes, seconds, as in the following example.

Ex. 2. The difference in longitude of two places is 46° 32′ 45″. What is the difference in time?

Solution. $46 \times \frac{1}{15}$ hr. = 3 hr. 4 min.

32×4 sec. = 128 sec. = 2 min. 8 sec.

$45 \times \frac{1}{15}$ sec. = 3 sec.

3 hr. 4 min. + 2 min. 8 sec. + 3 sec. = 3 hr. 6 min. 11 sec.

Check by reducing 3 hr. 6 min. 11 sec. to degrees, minutes and seconds as in *Ex.* 1.

EXERCISE 22

1. In what direction does the sun appear to move as the earth revolves on its axis?

2. How many degrees pass under the sun's rays in 5 hr.?

3. When it is noon at Chicago, what time is it at a place 15° 15′ east of Chicago? 45° 30′ 45″ west?

4. What is the difference in longitude between two places, the difference in time being 1 hr. 4 min.?

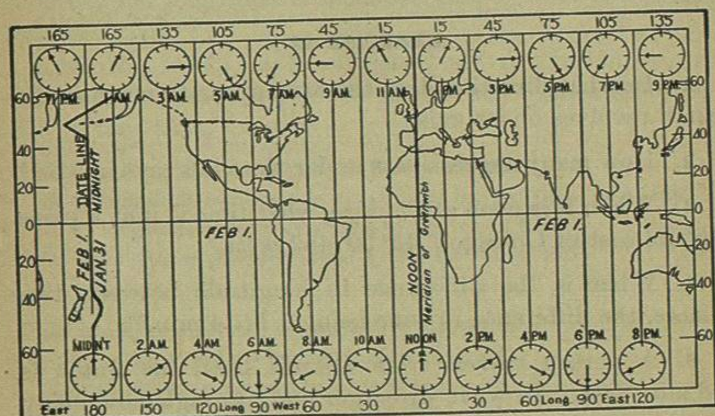
5. A person travels from Detroit until his watch is 45 min. fast. In what direction and through how many degrees has he traveled?

6. What is the difference in time between two places whose longitudes are 75° and 60°?

7. When it is 9 A.M. local time at Washington, it is 8 hr. 7 min. 4 sec. at St. Louis; the longitude of Washington being 77° 1′ W., what is the longitude of St. Louis?

146. International Date Line. Suppose that two men, starting from the prime meridian on Monday noon, travel the one eastward and the other westward, each traveling just as fast as the earth rotates. The man who goes west as fast as the earth turns east keeps exactly beneath the sun all the time; and it seems to him to be still Monday noon when he reaches his starting point again twenty-four hours later. He has *lost* a day in his reckoning by traveling westward around the earth.

The other man travels eastward over the earth as fast as the earth itself turns eastward, and therefore he moves away from the sun twice as fast as the prime meridian does. After twelve hours' travel he reaches the meridian of 180°, but twelve hours rotation has carried this meridian beneath the sun, and so the traveler reaches it at noon. In twenty-four hours the man reaches his starting point on the prime



meridian, but twenty-four hours' rotation has brought this meridian beneath the sun again, so the traveler reaches it on the second noon after his start; he therefore supposes it to be Wednesday noon, though really it is but twenty-four hours after Monday noon. He has *gained* a day in his reckoning by traveling eastward around the earth. To correct such errors in their dates, navigators usually *add* a day to their reckoning when they sail westward across the meridian of 180°, and *subtract* a day when they cross it to the eastward. The line where the adjustment is made, corresponding in general with the meridian of 180°, is called the international date line.

The map represents the earth when it is noon Feb. 1 at Greenwich.

It is, therefore, one hour earlier in the day for each 15° west of Greenwich and one hour later in the day for each 15° east of Greenwich. Hence 180° west of Greenwich it is midnight of Jan. 31, and 180° east of Greenwich it is midnight of Feb. 1.

When it is 6 A.M. Feb. 1 at Greenwich, at 90° W. it is midnight of Jan. 31, and at 90° E. it is noon of Feb. 1; at 180° W. it is 6 P.M. Jan. 31 and at 180° E. it is 6 P.M. Feb. 1. In this case it is Jan. 31 in all longitudes from 90° W. westward to the date line, and Feb. 1 in all longitudes from 90° W. eastward to the date line.

When it is midnight of Jan. 31 at Greenwich, at 90° W. it is 6 P.M. Jan. 31 and at 90 E. it is 6 A.M. Feb. 1; at 180° W. it is noon Jan. 31 and at 180° E. it is noon Feb. 1. In this case it is Jan. 31 in all longitudes from Greenwich westward to the date line, and Feb. 1 in all longitudes from Greenwich eastward to the date line.

When it is 6 P.M. Jan 31 at Greenwich, at 90° W. it is noon Jan. 31, and at 90° E. it is midnight Jan. 31; at 180° W. it is 6 A.M. Jan. 31, and at 180° E. it is 6 A.M. Feb. 1. In this case it is Jan. 31 in all longitudes from 90° E. westward to the date line, and Feb. 1 in all longitudes from 90° E. eastward to the date line.

1. When it is noon February 1 at Greenwich, what date is it at Paris? at New York City? at San Francisco?

2. Imagine the midnight line of Jan. 31 as a dark line moving westward parallel to the meridian. Everywhere on this line it is midnight. Behind this line it is Feb. 1; in front, Jan. 31. On what part of the earth's surface is it Feb. 1, and on what part Jan. 31 when this imaginary midnight line has reached the prime meridian? 90° E.? 140° E.?

3. What date will be in front of this line when it reaches 180°? What date will be behind it?

4. After crossing the 180th meridian and passing on to 175° E., what date is before the line and what date behind it?

5. What change must be made in the calendar of a ship crossing this line going westward? Going eastward?

147. Table of longitudes for use in solving problems:

Ann Arbor, Mich.	83° 43′ 48″ W.	London	0° 5′ 38″ E.
Albany, N.Y. . .	73° 44′ 48″ W.	Lisbon	9° 11′ 10″ W.
Boston	71° 3′ 30″ W.	Melbourne . .	144° 58′ 42″ E.
Berlin	13° 23′ 43″ E.	New Orleans . .	90° 3′ 28″ W.
Brussels	4° 22′ 9″ E.	New York . . .	74° 0′ 3″ W.
Chicago	87° 36′ 42″ W.	Paris	2° 20 15″ E.
Cincinnati . . .	84° 26′ 0″ W.	Peking	116° 26′ 0″ E.
Cambridge, Eng.	0° 5′ 41″ E.	Rome	12° 27′ 14″ E.
Cape Town . . .	18° 28′ 45″ E.	San Francisco .	122° 26′ 15″ W.
Calcutta	88° 19′ 2″ E.	St. Louis . . .	90° 12′ 11″ W.
Detroit	83° 5′ 7″ W.	Sydney	151° 11′ 0″ E.
Dublin	6° 2′ 30″ W.	Tokyo	139° 42′ 30″ E.
Honolulu . . .	157° 52′ 0″ W.	Washington . .	77° 1 W.

EXERCISE 23

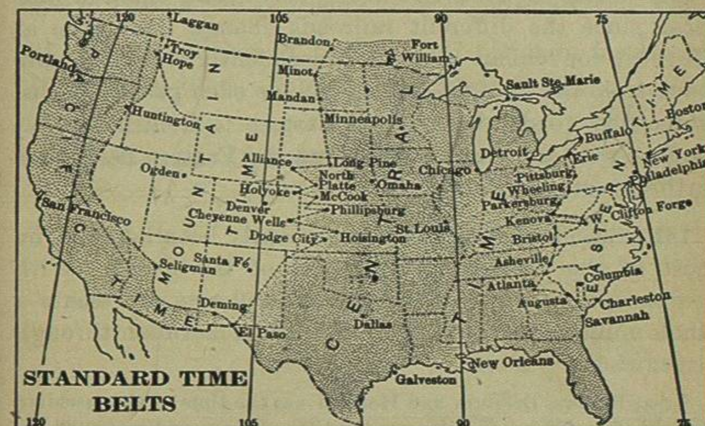
1. Determine the time and date at Ann Arbor, Berlin, Cape Town and Peking when it is midnight May 15 at Greenwich.

2. When it is noon March 1 at Rome, what time and date is it at San Francisco? at Sydney? at Detroit?

3. If a man were to travel westward around the earth in 121 da., in how many days would he actually make the trip by the local time of the places he passes through? In how many days would he make the trip traveling eastward?

4. When it is noon Sunday, Jan. 31, on the 90th meridian west, what part of the world has Sunday? What is the day and date on the other part?

5. When it is 3 P.M. Feb. 5 on the 45th meridian east, what part of the world has Feb. 5, and what is the date on the other part?



148. Standard Time. In order to secure uniform time over considerable territory, in 1883 the railroad companies of the United States decided to adopt standard time. They divided the country into four time belts, each of approximately 15° of longitude in width. The time in the various belts will therefore differ by hours, while the minute and second hand of all timepieces will remain the same. The correct time is distributed by telegraph throughout the United States from the Naval Observatory at Washington each day.

149. The **Eastern** time belt lies approximately $7\frac{1}{2}°$ each side of the 75th meridian, and has throughout the local time of the 75th meridian. Similarly, the **Central, Mountain** and **Pacific** time belts lie approximately $7\frac{1}{2}°$ each side of the 90th, 105th and 120th meridians, and the time throughout in each belt is determined by the local time of the 90th, 105th and 120th meridians.

150. These divisions are not by any means equal or uniform, since the different railroads change their time at the most convenient places. Consequently, variations are made from the straight line to include such places. Thus, while most roads change from Eastern to Central time at Buffalo, the Canadian roads extend the Eastern belt much farther west.

151. Standard time has more recently been adopted by most of the leading governments of the world. With few exceptions the standard meridian chosen represents a whole number of hours from the prime meridian through Greenwich.

Great Britain, Belgium and Holland use the time of the meridian through Greenwich. France uses the time of the meridian (2° 20′ E.) through Paris. Cape Colony uses the time of the meridian 22° 30′ E. Germany, Italy, Austria, Denmark, Norway and Sweden use the time of the 15th meridian east. Roumania, Bulgaria and Natal use the time of the 30th meridian east. Western Australia uses the time of the 120th meridian east. Southern Australia and Japan use the time of the 135th meridian east. New South Wales, Victoria, Queensland, and Tasmania use the time of the 150th meridian east. New Zealand uses the time of 170° 30′ E.

EXERCISES 24

1. When it is 7 P.M. at Philadelphia, what time is it at London? at Paris? at Berlin?

2. What is the difference in time between Boston and Rome? Melbourne and Tokyo?

3. It is 7 A.M. March 1 at St. Louis. What is the time and date at Tokyo?

4. When it is 1.30 P.M. at Buffalo, what time is it at Cleveland?

5. What is the difference between the local and standard time of Boston? of Chicago? of St. Louis? of New York?

6. The local time of Detroit is 27.658 min. faster than the standard time. Find the longitude of Detroit.

7. When it is noon local time at Boston, what is the standard time at Ogden, Utah?

THE EQUATION

152. An **equation** is a statement of equality of two numbers or expressions. Thus, $5 = 3 + 2$ and $3 \times 5 = 15$ are equations.

153. The equation is one of the most powerful instruments used in mathematics and can be used to decided advantage in the solution of many arithmetical problems.

154. The book written by the Egyptian priest Ahmes, and referred to elsewhere, is one of the very oldest records of the extent of mathematical knowledge among the ancients. In this book we find a very close relation between arithmetic and algebra. A number of problems are given leading to the simple equation. Here the unknown quantity is called *hau* or "heap," and the equation is given in the following form: *heap* its $\frac{2}{3}$, its $\frac{1}{2}$, its $\frac{1}{7}$, its whole, gives 33, or $\frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x + x = 33$. (Cajori, "History of Elementary Mathematics," p. 23.)

Ex. **1.** Find a number such that, if 30 is subtracted, $\frac{2}{3}$ of the original number will remain.

The given relation may be expressed as follows:

The number diminished by 30 equals $\frac{2}{3}$ of the number.
From this relation we are to find the number.
Let x = the number.
Then $x - 30$ = the number diminished by 30, and $\frac{2}{3}x = \frac{2}{3}$ of the number.
$\therefore x - 30 = \frac{2}{3}x$ is the equation expressing the relation between the number given in the conditions of the problem and the letter representing the number to be found.

Solution. Since we wish to obtain an equation with x alone on the left side and only numerical quantities on the right side, we proceed as follows:

Subtracting $\frac{2}{3}x$ from both sides of the equation, we have

$$x - 30 - \frac{2}{3}x = \frac{2}{3}x - \frac{2}{3}x, \qquad \text{Axiom 3.}$$

or

$$\frac{1}{3}x - 30 = 0.$$

Adding 30 to both sides of the equation, we have

$$\frac{1}{3}x - 30 + 30 = 30, \qquad \text{Axiom 2.}$$

or

$$\frac{1}{3}x = 30.$$

Multiplying both terms by 3, we have

$$x = 90.$$

$\therefore$ 90 is the required number.

To *check* the result, substitute 90 for the unknown number in the problem.

This gives us

$$90 - 30 = \frac{2}{3} \text{ of } 90,$$

or

$$60 = 60.$$

$\therefore x = 90$ is the correct result.

Ex. **2.** The sum of two numbers is 20 and their difference is 4. What are the numbers?

Solution. Let x = the larger number.

Then $x - 4$ = the smaller number.

$$\therefore x + x - 4 = 20.$$

$$2x = 24, \qquad \text{Axiom 2.}$$

or

$$x = 12 = \text{the larger number.} \qquad \text{Axiom 5.}$$

Check. $12 - 4 = 8$ = the smaller number.

$12 + 8 = 20$. $12 - 8 = 4$.

EXERCISE 25

1. The sum of two numbers is 31 and their difference is 11. What are the numbers?

2. The difference of two numbers is 60, and if both numbers are increased by 5 the greater becomes four times as large as the smaller. What are the numbers?

3. Find two numbers such that their difference is 95 and the smaller divided by the greater is $\frac{4}{9}$.

4. After spending $\frac{2}{3}$ of his money a man pays bills of \$25, \$40 and \$14, and finds that he has \$132 left. How much money had he at first?

5. Find a number such that its half, third and fourth parts shall exceed its fifth part by 106.

6. A father wishes to divide \$28,000 among his two sons and a daughter so that the elder son shall receive twice as much as the younger, and the younger son twice as much as the daughter. Find the share of each.

POWERS AND ROOTS

155. Archimedes (287–212 B.C.) in his measurements of the circle computed the approximate value of a number of square roots, but nothing is known of his method. A little later, Heron of Alexandria also used the approximation $\sqrt{a^2 + b} = a + \frac{b}{2a}$, *e.g.* $\sqrt{85} = \sqrt{9^2 + 4} = 9 + \frac{4}{18}$.

156. The Hindus included powers and roots among the fundamental processes of arithmetic. As early as 476 A.D. they used the formulæ $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ in the extraction of square and cube root, and they separated numbers into periods of two and three figures each.

157. The Arabs also extracted roots by the formula for $(a + b)^n$. They introduced a radical sign by placing the initial letter of the word *jird* (root) over the number.

158. The **power** of a number is the product that arises by multiplying the number by itself any number of times. The second power is called the **square** of the number and the number itself is called the **square root** of its second power. The third power is called the **cube** of the number, and the number itself is called the **cube root** of its third power. Thus, 4 is the square of 2, and 2 is the square root of 4. 125 is the cube of 5, and 5 is the cube root of 125.

159. Since the square root of a number is one of the two equal factors of a perfect second power, numbers that are not exact squares have no square roots. However, they are treated as having approximate square roots. These approximate square roots can be found to any required