In practice the work may be arranged as follows:

	7 1. 3
	362467.097
	343
14700	19467.
14911	14911.
15123	4556.097
15186.99	4556.097

EXERCISE 28

1. Separate each of the following numbers into periods: 2500, 2.5, 3046.2971, 0.0125, 486521.3.

2. In extracting the cube root of 208527857, does the division by $3a^2$ give the second figure of the root correctly? Why is $3a^2$ called the trial divisor? Why is $3a^2 + 3ab + b^2$ called the complete divisor?

3. In the above example explain how $3a^2$ can equal both 14700 and 15123. Explain how $3a^2 + 3ab + b^2$ can equal both 14911 and 15186.99.

4. Show how cube root may be checked by casting out the 9's.

5. Extract the cube root of each of the following, using the formula: (*a*) 472729139, (*b*) 278.445077. (*c*) 1054.977832, (*d*) 19683, (*e*) 205379, (*f*) 25153.757.

6. Extract the cube root of $\frac{1331}{3375}$, $\frac{216}{343}$, $\frac{10648}{13824}$.

7. By reducing to a decimal, extract the cube root of $\frac{2}{3}$, $\frac{4}{9}$, $\frac{51}{56}$, $16\frac{2}{3}$, $33\frac{1}{3}$.

8. By first making the denominator a perfect cube, extract the cube root of $\frac{7}{8}$, $\frac{11}{14}$, $\frac{3}{7}$, $\frac{5}{9}$.

9. Find correct to 0.01 the cube root of 12.5, 125, 1.25.

MENSURATION

176. Certain measurements have been in very common use in the development of arithmetical knowledge from the earliest times.

177. The Babylonians and Egyptians used a great variety of geometrical figures in decorating their walls and in tile floors. The sense perception of these geometrical figures led to their actual measurement and finally to abstract geometrical reasoning.

178. The Greeks credited the Egyptians with the invention of geometry and gave as its origin the measurement of plots of land. Herodotus says that the Egyptian king, Sesostris (about 1400 B.C.), divided Egypt into equal rectangular plots of ground, and that the annual overflow of the Nile either washed away portions of the plot or obliterated the boundaries, making new measurements necessary. These measurements gave rise to the study of geometry (from *ge*, earth, and *metron*, to measure).

179. Ahmes, in his arithmetical work, calculates the contents of barns and the area of squares, rectangles, isosceles triangles, isosceles trapezoids and circles. There is no clew to his method of calculating volumes. In finding the area of the isosceles triangle he multiplies a side by half of the base, giving the area of a triangle whose sides are 10 and base 4 as 20 instead of 19.6, the result obtained by multiplying the altitude by half of the base. For the area of the isosceles trapezoid he multiplies a side by half the sum of the parallel bases, instead of finding the altitude and multiplying that by half the sum of the two parallel sides. He finds the area of the circle by subtracting from the diameter $\frac{1}{9}$ of its length and squaring the remainder. This leads to the fairly correct value of 3.1604 for π.

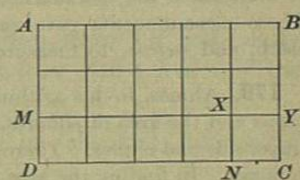
180. The Egyptians are also credited with knowing that in special cases the square on the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. They were careful to locate their temples and other public buildings on north and south, and east and west lines. The north and south line they determined by means of the stars. The east and west line was determined at right angles to the other, probably by stretching around three pegs, driven into the ground, a rope measured into three parts which bore the same relation to each other as the numbers 3, 4 and 5. Since $3^2 + 4^2 = 5^2$, this gave the three sides of a right triangle.

181. The ancient Babylonians knew something of rudimentary geometrical measurements, especially of the circle. They also obtained a fairly correct value of π.

182. It was the Greeks who made geometry a science and gave rigid demonstrations of geometrical theorems.

183. The importance of certain measurements gives mensuration a prominent place in arithmetic to-day. The rules and formulæ of the present chapter will be developed without the aid of formal demonstration.

184. **The Rectangle.** If the unit of measure $CYXN$ is 1 sq. in., then the strip $CYMD$ contains 5×1 sq. in. $= 5$ sq. in., and the whole area contained in the three strips will be 3×5 sq. in., or 15 sq. in.

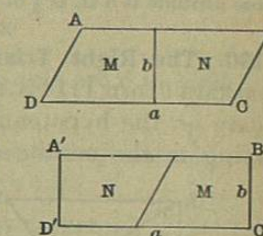


185. The dimensions of a rectangle are its **base** and **altitude**, and the area *is equal to the product of the base and altitude. That is, the number of square units in the area is equal to the product of the numbers that represent the base and altitude.* If we denote the area by A, the number of units in the base by b, and the numbers of units in the altitude by a, then $A = ab$ and any one of the three quantities, A, b, a, can be determined when the other two are given.

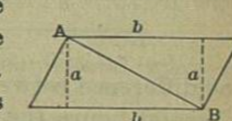
Thus, if the area of a rectangle is 54 sq. in. and the altitude is 6 in., the base can be determined from $6b = 54$, or $b = 9$ in.

186. If the dimensions of a rectangle are equal, the figure is a square and the area is equal to the second power of a number denoting the length of its side, or $A = a^2$. For this reason the **second power** of a number is called its **square**.

187. **The Parallelogram.** Any parallelogram has the same area as a rectangle with the same base and altitude, as can be shown by dividing the parallelogram $ABCD$ into two parts, M and N, and placing them as in $A'B'C'D'$, thus forming a rectangle with the same base and altitude as the given parallelogram. Therefore, *the area of a parallelogram is equal to the product of its base and altitude*, or $A = ab$.

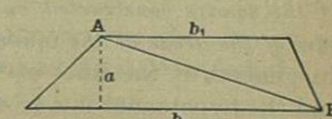


188. **The Triangle.** Since the line AB divides the parallelogram into two equal triangles with the same base and altitude as the parallelogram, the area of the triangle is equal to one half of the area of the parallelogram. But the area of the parallelogram is equal to the product of its base and altitude. Therefore, *the area of the triangle is one half the product of its base and altitude*, or $A = \frac{1}{2}ab$.



Thus, the area of a triangle with base 6 in. and altitude 5 in. is $\frac{1}{2}$ of 6×5 sq. in. $= 15$ sq. in.

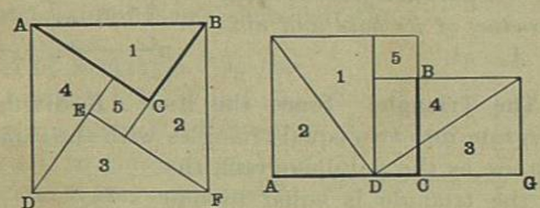
189. **The Trapezoid.** The line AB divides the trapezoid into two triangles whose bases are the upper and lower bases of the trapezoid and whose common altitude is the



altitude of the trapezoid. The areas of the triangles are respectively $\frac{1}{2}ab_1$ and $\frac{1}{2}ab_2$, and since the area of the trapezoid equals the sum of the areas of the triangles, therefore the area of the trapezoid is $\frac{1}{2}ab_1 + \frac{1}{2}ab_2 = \frac{1}{2}a(b_1 + b_2)$, or *the area of a trapezoid is equal to one half of the product of its altitude and the sum of the upper and lower bases*, or $A = \frac{1}{2}a(b_1 + b_2)$.

Thus, the area of a trapezoid whose bases are 20 ft. and 17 ft. and whose altitude is 6 ft. is $\frac{1}{2}$ of $6 \times (20 + 17)$ sq. ft. $= 111$ sq. ft.

190. The Right Triangle. The Hindu mathematician Bhaskara (born 1114 A.D.) arranged the figure so that the square on the hypotenuse contained four right triangles, leaving in the middle a small square whose side equals



the difference between the sides of the right triangle. In a second figure the small square and the right triangles were arranged in a different way so as to make up the squares on the two sides. Bhaskara's proof consisted simply in drawing the figure and writing the one word "Behold." From these figures it is evident that *the area of the square constructed on the hypotenuse will equal the sum of the areas of the squares constructed on the two sides.* In general, if the sides of the right triangle are a and b and the hypotenuse is c, $a^2 + b^2 = c^2$.

Thus, the hypotenuse of the right triangle whose sides are 5 and 12 is $\sqrt{25 + 144} = 13$.

This theorem is known by the name of the Pythagorean theorem, because it is supposed to have been first proved by the Greek mathematician Pythagoras, about 500 B.C.

191. If either side and the hypotenuse of a right triangle are known, the other side can be found from the equation $a^2 + b^2 = c^2$.

Thus, if one side is 3 and the hypotenuse is 5, the other side is $\sqrt{25 - 9} = 4$.

EXERCISE 29

1. The two sides of a right triangle are 6 in. and 8 in. Find the length of the hypotenuse.

   $x = \sqrt{6^2 + 8^2}$ $x^2 = 6^2 + 8^2$ $x^2 = 36 + 64$ $x^2 = 100$ $x = 10$

2. Find the area of an isosceles triangle, if the equal sides are each 10 ft. and the base is 4 ft.

3. Find the area of an isosceles trapezoid, if the bases are 10 ft. and 18 ft. and the equal sides are 8 ft.

4. What is the area in hectares, etc., of a field in the form of a trapezoid of which the bases are 475^m and 580^m and the altitude is 1270^m?

5. Show that the altitude of an equilateral triangle, each of whose sides is a, is $\frac{a}{2}\sqrt{3}$. $h^2 = a^2 - (\frac{1}{2}a)^2$

6. The hypotenuse of a right triangle with equal sides is 10 ft. Find the length of the two equal sides.

7. The diagonal of a square field is 80 rd. How many acres does the field contain?

8. Find correct to square centimeters the area of an equilateral triangle each side of which is 1^m in length.

192. The Circle. If the circumference (c) and the diameter (d) of a number of circles are carefully measured, and if the quotient $\frac{c}{d}$ is taken in each case, the quotients will be found to have nearly the same value. If absolutely correct measurements could be made, the quotient in each case would be the same and equal to 3.14159+, *i.e.* the ratio of the circumference of a circle to its diameter is the same for all circles. The ratio is denoted by the Greek letter π (pī). The value of π found in geometry is 3.14159+. In common practice π is taken as 3.1416. The value of π cannot be exactly expressed by any number, but can be found correct to any desired number of decimal places.

193. Since $\frac{c}{d} = \pi$ and $d = 2r$, where r stands for the radius of the circle, then $c = \pi d = 2\pi r$, and $d = \frac{c}{\pi}$ and $r = \frac{c}{2\pi}$. Hence, if the radius, diameter or circumference of a circle is known, the other parts can be found.

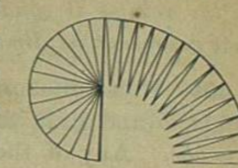
Thus, the circumference of a circle whose radius is 10 in. is $2 \times 3.1416 \times 10$ in. = 62.832 in.

EXERCISE 30

1. Find the circumference of a circle whose diameter is 20 in.
2. Find the radius of a circle whose circumference is 250 ft.
3. If the length of a degree of the earth's meridian is 69.1 mi., what is the diameter of the earth?
4. If the radius of a circle is 8 in., what is the length of an arc of 15° 20′?
5. The diameter of a circle is 10 ft. How many degrees are there in an arc 16 ft. long?

3.1416×10 = 31.416 360 ÷ by 31.416 = 11.4

194. The Area of a Circle. The circle may be divided into a number of equal figures that are essentially triangles. The sum of the bases of these triangles is the circumference of the circle, and the altitudes are radii of the circle. Treating these figures as triangles, their areas will be $\frac{1}{2} c \times r$. Therefore, since $c = 2\pi r$, $A = \frac{1}{2}$ of $2\pi r \times r = \pi r^2$. It is proved in geometry that this result is exactly correct.

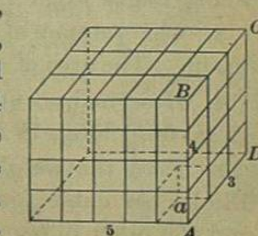


The area of a circle whose radius is 5 ft. is $3.1416 \times 5 \times 5$ sq. ft. = 78.54 sq. ft.

EXERCISE 31

1. Find the area of a circle whose radius is 10 in.
2. Find the area of a circle whose circumference is 25 ft.
3. Find the radius of a circle whose area is 100 sq. ft.
4. The areas of two circles are 60 sq. ft. and 100 sq. ft. Find the number of degrees in an arc of the first that is equal in length to an arc of 45° in the second.
5. Find the side of a square that is equal to a circle whose circumference is 50 in. longer than its diameter.

195. The Volume of a Rectangular Parallelopiped. If the unit of measure a is 1 cu. in., then the column AB is 4 cu. in., and the whole section $ABCD$ will contain 3 of these columns, or 3×4 cu. in. Since there are five of these sections in the parallelopiped, the entire volume (V) is $5 \times 3 \times 4$ cu. in., or 60 cu. in. Therefore, *the vol-*



ume of a rectangular parallelopiped is equal to the products of its three dimensions. That is, the number of cubic units in the volume is equal to the product of the three numbers that represent its dimensions.

196. If the dimensions of the rectangular parallelopiped are a, b and c, it can be shown in the same way that $V = abc$. Any of these four quantities, V, a, b, c, can be determined when the other three are known.

Ex. If the volume of a rectangular parallelopiped is 36 cu. in. and two of the dimensions are 6 in. and 2 in., the third dimension is $\frac{36}{6 \times 2} = 3$. $\therefore$ 3 in. is the other dimension.

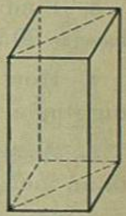
197. If the dimensions of a rectangular parallelopiped are equal, the figure is a cube and the volume is equal to the third power of the number denoting the length of its edge (a), or $V = a^3$. For this reason the **third power** of a number is called its **cube**.

198. It is proved in geometry that any parallelopiped has the same volume as a rectangular parallelopiped with the same base and altitude.

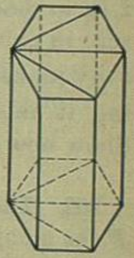
EXERCISE 32

1. Find the volume of a cube 3 in. on an edge.
2. Find the volume of a rectangular parallelopiped whose edges are 3^{cm}, 5^{cm} and 11^{cm}.
3. The volume of a rectangular parallelopiped is 100 cu. in. The area of one end is 20 sq. in. Find the length.
4. How many cubic feet of air are there in a room 12 ft. 6 in. long, 10 ft. 8 in. wide and 9 ft. high?
5. Find the weight of a rectangular block of stone at 135 lb. per cubic foot, if the length of the block is $9\frac{1}{2}$ ft. and the other dimensions are 2 ft. and 5 ft.

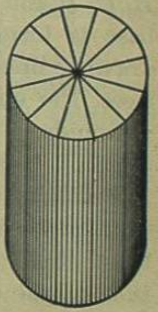
6. If a cubic foot of water weighs 1000 oz., find the edge of a cubical tank that will hold 2 T.
7. Show why the statement that the volume of a rectangular parallelopiped is equal to the product of its three dimensions is the same as the statement that its volume is equal to the product of its altitude and the area of its base.



199. The Volume of a Prism. A rectangular parallelopiped can be divided into two equal triangular prisms with the same altitude and half the base. Hence, the volume of the prism is half the volume of the parallelopiped. But the base of the parallelopiped is twice the base of the prism, therefore, *the volume of a triangular prism is equal to the product of its altitude and the area of its base.*



200. Since any prism can be divided into triangular prisms, as in the figure, it follows that *the volume of any prism is equal to the product of its altitude and the area of its base.*



201. The Volume of a Cylinder. The cylinder may be divided into a number of solids that are essentially prisms, as indicated in the figure. The sum of the bases of these prisms is the base of the cylinder and the altitude of the prisms is the same as the altitude of the cylinder. Therefore, *the volume of a cylinder is the product of its altitude and the area of its base.* $V = a \times \pi r^2$.

$1\frac{2}{3} \times 1\frac{2}{3} \times 12$ = cu. ft in prism
= 33.33

EXERCISE 33

1. Find the volume of a prism with square ends, each side measuring 1 ft. 8 in., and the height being 12 ft.

2. Find the volume of a prism whose ends are equilateral triangles, each side measuring 11 in. and the height being 20 in.

3. Find the volume of a cylinder if the diameter of its base is 20 in. and the altitude is 30 in.

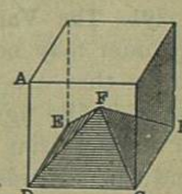
4. How many cubic yards of earth must be removed in digging a well 45 ft. deep and 3 ft. in diameter?

5. A cubic foot of copper is to be drawn into a wire $\frac{1}{10}$ of an inch in diameter. Find the length of the wire.

6. How many revolutions of a roller $3\frac{1}{2}$ ft. in length and 2 ft. in diameter will be required in rolling a lawn $\frac{3}{4}$ of an acre in extent.

7. Show how to find the surface of a cylinder by dividing it into figures that are essentially parallelograms. Show how to find the surface of a prism.

202. The Volume of a Pyramid. Let AB be a cube and F the middle point of the cube, then by connecting F with B, C, D and E a pyramid with a square base is formed. It is evident that by drawing lines from F to each of the vertices, the cube will consist of six such pyramids. Hence, the volume of the pyramid is $\frac{1}{6}$ of the volume of the cube. The volume of the cube is the product of its altitude and the area of its base $BCDE$. Therefore, the volume of the pyramid is $\frac{1}{6}$ of the product of the altitude of the cube and the area of its base. But the base



of the pyramid is the base of the cube and its altitude is $\frac{1}{2}$ of the altitude of the cube, *hence, the volume of the pyramid is one third of the product of its altitude and the area of its base.* In geometry this is proved true of *any* pyramid.

Ex. If the altitude of a pyramid is 45^m, and a side of its square base is 60^m, its volume is $\frac{1}{3}$ of $45 \times (60^2)^{m^2} = 54000^{m^3}$.

203. The Volume of a Cone. The cone may be divided into a number of equal figures that are essentially pyramids as indicated in the figure. The sum of the bases of these pyramids is the base of the cone, and their altitudes are the same as the altitude of the cone. Therefore, *the volume of a cone is equal to one third of the product of its altitude and the area of its base.*

Ex. If the altitude of a cone is 10 ft. and the radius of its base 4 ft., its volume is $\frac{1}{3}$ of $10 \times 3.1416 \times 4^2$ cu. ft. = 167.55 cu. ft.

EXERCISE 34

1. Show that the pyramid with a square base can be divided into two equal pyramids with triangular bases and the same altitude as the original pyramid, and hence show how any pyramid may be similarly divided.

2. Find the volume of a cone if the diameter of the base is 16 in. and the altitude is 12 in. Find the volume if the diameter of the base is 16 in. and the slant height is 12 in.

3. Show how to find the surface of a cone by dividing it into figures that are essentially triangles. Show how to find the surface of a pyramid.

4. Find the volume of a pyramid if the area of its base is 4 sq. ft. and its altitude is 2 ft. Find the volume if the base is 2 feet square and the slant height is 2 ft.

5. How much canvas is necessary for a conical tent 8 ft. high, if the diameter of the base is 8 ft.?

6. The radius of a cylinder is 8 ft. and its altitude is 10 ft. Find the altitude of a cone with the same base and volume.

204. The Surface of a Sphere. The surface of a sphere is proved in geometry to be equal to the area of 4 great circles or $4\pi r^2$, r being the radius of the sphere. This can be shown by winding a firm cord to cover a hemisphere and a great circle as indicated in the figure. It will be found that twice as much cord is used to cover the hemisphere as the great circle, therefore, to cover the whole sphere 4 times as much would be required.

Ex. A sphere with a radius of 6 in. has a surface of $4 \times 3.1416 \times 6^2$ sq. in. $= 452.39+$ sq. in.

205. The Volume of a Sphere. The sphere may be divided into a number of figures that are essentially pyramids, as indicated in the figure. The sum of the bases of these pyramids is the surface of the sphere and the altitude of each pyramid is its radius. Therefore, the volume of all these pyramids is equal to $\frac{1}{3}r \times 4\pi r^2 = \frac{4}{3}\pi r^3$.

Ex. The volume of a sphere whose radius is 3 in. is $\frac{4}{3} \times 3.1416 \times 3^3$ cu. in. $= 113.1$ cu. in.

206. Board Measure. In measuring lumber the **board foot** is used. It is a board 1 ft. long, 1 ft. wide and 1 in. or less thick. Lumber more than 1 in. thick is measured by the number of square feet of boards 1 in. thick to which it is equal.

Thus, a board 10 ft. long, 1 ft. wide and $1\frac{1}{2}$ in. thick, contains 15 board feet.

Lumber is usually sold by the 1000 board feet. A quotation of $17 per M, means $17 per 1000 board feet.

EXERCISE 35

1. Find the cost of 12 boards 16 ft. long, 6 in. wide, and 1 inch thick at $18 per M.

2. How many board feet are there in a stick of timber 16 ft. by 16 in. by 10 in.?

3. How much is a stick of timber 15 ft. by 2 ft. by 1 ft. 4 in. worth at $22 per M?

4. How many board feet are used in laying the flooring of two rooms, each 32 ft. by 20 ft., allowing $\frac{1}{3}$ for waste in sawing and in tongue and groove.

5. What is the cost of 25 $2\frac{1}{2}$-in. planks 16 ft. long by 1 ft. wide at $22.50 per M?

6. What is the cost of 15 joists 12 ft. by 10 in. by 4 in. at $23 per M?

207. Wood Measure. The unit of wood measure is the **cord**. The cord is a pile of wood 8 ft. by 4 ft. by 4 ft.

A pile of wood 1 ft. by 4 ft. by 4 ft. is called a **cord foot**.

A cord of stove wood is 8 ft. long by 4 ft. high. The length of stove wood is usually 16 in.

EXERCISE 36

1. Find the number of cords of wood in a pile 32 ft. by 4 ft. by 4 ft.

2. At $5.75 per cord, how much will a pile of wood 52 ft. by 4 ft. by 4 ft. cost?

3. How much will a pile of stove wood 94 ft. long 4 ft. high be worth at $2.75 per cord?

208. Carpeting. A yard of carpet refers to the running measurement, regardless of the width. The cheaper grades of carpet are usually 1 yd. wide, and the more expensive, such as Brussels, Wilton, etc., are $\frac{3}{4}$ of a yard wide.

In carpeting, it is usually necessary to allow for some waste in matching the figures in patterns. Dealers count this waste in their charges. In computing the cost of carpets, dealers charge the same for a fractional width as for a whole one.

Carpets may often be laid with less waste one way of the room than the other; hence, it is sometimes best to compute the cost with the strips running both ways, and by comparison determine which involves the smaller waste.

EXERCISE 37

1. How many yards of Brussels carpet $\frac{3}{4}$ of a yard wide will be required to cover the floor of a room 15 ft. by 13 ft. 6 in., the waste in matching being 4 in. to each strip except the first? Which will be the more economical way to lay the carpet?

2. How much will it cost to cover the same room with Brussels carpet if a border $\frac{5}{8}$ of a yard wide is used, the carpet and border being $1.25 per yard, and the waste being 4 in. to each strip of carpet except the first, and $\frac{5}{8}$ of a yard of border at each corner?

3. How much will it cost to cover the same room with ingrain carpet 1 yd. wide, at $67\frac{1}{2}$ ct. per yard, the waste being 6 in. to each strip except the first?

4. At $\$1.12\frac{1}{2}$ per yard, how much will it cost to carpet a flight of stairs of 14 steps, each step being 8 in. high and 11 in. wide?

5. A room is 17 ft. by 14 ft. 9 in. Will it be cheaper to run the strips lengthwise or across the room? If the room is covered with carpet $\frac{3}{4}$ of a yard wide at $1.35 per yard, how much will it cost? Allow 1 yd. for waste in matching.

209. Papering. Wall paper is sold in single rolls 8 yd. long, or in double rolls 16 yd. long. It is usually 18 in. wide.

There is considerable waste in cutting and matching paper. Whole rolls may be returned to the dealer, but part of a roll will not usually be taken back. Paper for border is usually sold by the yard.

EXERCISE 38

1. How many rolls of paper and how many yd. of border are used in papering the walls and ceiling of a room 14 ft. by 13 ft. and 8 ft. high above the baseboard, deducting $\frac{1}{2}$ of a roll for each of 2 windows and 2 doors, the width of the border being 18 in., and 1 roll being allowed for waste in matching?

2. How much will it cost to paper the room mentioned in Ex. 1 if the paper is 12 ct. a roll and the border is 5 ct. a yd.? The paper hanger works 8 hr. at 30 ct. an hr.

3. At 25 ct. per roll, how much will it cost to paper the walls and ceiling of a room 18 ft. square and 9 ft.

high above the baseboard, allowing $\frac{1}{2}$ of a roll for each of 2 doors and 3 windows, the border being 18 in. wide and costing 12 ct. a yd.? The paper hanger works 11 hr. at 30 ct. an hr., and $1\frac{1}{2}$ rolls are allowed for waste in matching.

210. Painting and Plastering. The **square yard** is the unit of painting and plastering.

There is no uniform practice as to allowances to be made for openings made by windows, doors, etc., and the baseboard. To avoid complications, a definite written contract should always be drawn up.

EXERCISE 39

1. How much will it cost to plaster the walls and ceiling of a room 15 ft. by 13 ft. 6 in., and 9 ft. high, at $27\frac{1}{2}$ ct. per square yard, deducting half of the area of 2 doors, each 7 ft. by $3\frac{1}{2}$ ft., and 2 windows, each 6 ft. by $3\frac{1}{4}$ ft.?

2. How much will it cost to paint the walls and ceiling of the same room at $12\frac{1}{2}$ ct. per square yard, the same allowance being made for openings?

3. At 20 ct. per square yard, how much will it cost to paint a floor 18 ft. by 16 ft. 6 in.?

4. Allowing $\frac{1}{5}$ of the surface of the sides for doors, windows and baseboard, how much will it cost to plaster the sides and ceiling of a room 22 ft. by 18 ft. and $9\frac{1}{2}$ ft. high, at $22\frac{1}{2}$ ct. per square yard?

211. Roofing and Flooring. A **square** 10 ft. on a side, or 100 sq. ft., is the unit of roofing and flooring.

The average shingle is taken to be 16 in. long and 4 in. wide. Shingles are usually laid about 4 in. to the weather.

Allowing for waste, about 1000 shingles are estimated as needed for each square, but if the shingles are good, 850 to 900 are sufficient. There are 250 shingles in a **bundle.**

EXERCISE 40

1. At $8.60 per square, how much will it cost to shingle a roof 50 ft. by $22\frac{1}{2}$ ft. on each side?

2. How much will it cost to lay a hard-wood floor in a room 30 ft. by 28 ft., if the labor, nails, etc. cost $22.50, lumber being $28 per M, and allowing 57 sq. ft. for waste?

3. Allowing 900 shingles to the square, how many bundles will be required to shingle a roof 70 ft. by 28 ft. on each side? How much will the shingles cost at $3.75 per M?

4. At $12.50 per square, how much will the slate for a roof 40 ft. by 24 ft. on each side cost?

212. Stonework and Masonry. The **cubic yard** or the **perch** is the unit of stonework.

A **perch of stone** is a rectangular solid $16\frac{1}{2}$ ft. by $1\frac{1}{2}$ ft. by 1 ft., and therefore contains $24\frac{3}{4}$ cu. ft.

A common brick is 8 in. by 4 in. by 2 in. Bricks are usually estimated by the thousand, sometimes by the cubic foot, 22 bricks laid in mortar being taken as a cubic foot.

There is no uniformity of practice in making allowances for windows and other openings. There should be a definite written contract with the builder covering this point. The corners, however, are counted twice on account of the extra work involved in building them. It is also generally considered that the work around openings is more difficult, so that allowance is frequently made here.

EXERCISE 41

1. If 60 ct. per cubic yard was paid for excavating a cellar 30 ft. by 20 ft. by 7 ft., and \$4.75 a perch was paid for building the four stone walls, 18 in. thick and extending 2 ft. above the level of the ground, what was the total cost?

2. How many bricks will be used in building the walls of a flat-roofed building 90 ft. by 60 ft. and 20 ft. high, if the walls are 18 in. thick and 500 cu. ft. are allowed for openings?

3. How much will it cost to build the walls described in Ex. 2, if the bricks are \$8.50 per M, and the mortar and brick-laying cost \$3.50 per M?

4. How many perch of stone will be needed for the walls of a cellar 30 ft. by $22\frac{1}{2}$ ft. and 9 ft. deep from the top of the wall, the wall being 18 in. thick? How many perch will be needed for a cross wall of the same thickness, allowing for half of a door 7 ft. by 4 ft.? How much will the stone cost at \$4.50 a perch?

213. Contents of Cisterns, Tanks, etc. The **gallon** or the **barrel** is the unit of measure for cisterns, tanks, etc.

The liquid gallon contains 231 cu. in. and the barrel $31\frac{1}{2}$ gal.

EXERCISE 42

1. How many gallons of water will a tank 10 ft. long, 3 ft. wide and 3 ft. deep contain? How many barrels?

2. How many gallons of water will a cistern 10 ft. deep and 10 ft. in diameter contain? How many barrels?

3. How many barrels will a cylindrical tank 5 ft. high and 3 ft. in diameter contain?

4. How many barrels of oil will a tank 40 ft. long and 6 ft. in diameter contain?

5. Show that to find the approximate number of gallons in a cistern it is necessary only to multiply the number of cubic feet by $7\frac{1}{2}$ and subtract from the product $\frac{1}{400}$ of the product. Apply this method to each of the above exercises.

6. How many gallons will a cask contain, the bung diameter being 24 in., the head diameter 20 in. and the length 34 in.?

Suggestion. The average or mean diameter is $\frac{24\text{ in.} + 20\text{ in.}}{2} = 22$ in.

214. Measuring Grain in the Bin, Corn in the Crib, etc. There are 2150.42 cu. in. in every bushel, stricken measure, and 2747.71 cu. in. in every bushel, heaped measure.

EXERCISE 43

1. How many bushels of wheat does a bin 8 ft. by 7 ft. by 6 ft. contain?

2. Show that multiplying by 0.8 will give the approximate number of stricken bushels in any number of cubic feet, and dividing by 0.8 will give the approximate number of cubic feet in any number of stricken bushels.

3. Show that multiplying by 0.63 will give the approximate number of heaped bushels in any number of cubic feet, and dividing by 0.63 will give the approximate number of cubic feet in any number of heaped bushels.

4. How deep must a bin 10 ft. by 8 ft. be to hold 500 bushels of wheat?

5. A farmer builds a corncrib 20 ft. long, 10 ft. high, 8 ft. wide at the bottom and 12 ft. wide at the top. How many heaped bushels of corn in the ear will the crib hold when level full? If the ridge of the roof is 3 ft. above the top level, how many bushels will the crib hold when filled to the ridge?

Suggestion. The average width of the crib is $\frac{12 \text{ ft.} + 8 \text{ ft.}}{2} = 10$ ft.

6. How many stricken bushels of shelled corn are there in the above crib if 3 half bushels of ears make one bushel of shelled corn?

215. Measuring Hay in the Mow or Stack. The only correct way to measure hay is to weigh it. However, it is sometimes convenient to be able to estimate the number of tons in a mow or stack. The results of such estimations can be only approximately correct, as different kinds of hay vary in weight. In well-settled mows or stacks, as nearly as can be estimated, 15 cu. yd. make one ton. When hay is baled, 10 cu. yd. make a ton.

EXERCISE 44

1. Approximately how many tons of hay are there in a mow 40 ft. by 22 ft. and 15 ft. deep? 40×22×15×

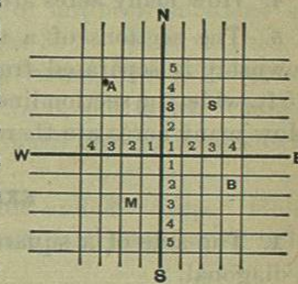
2. Approximately how many tons of hay are there in a circular stack 21 ft. high and averaging 80 ft. in circumference?

3. Approximately how many tons of hay are there in a rick averaging 35 ft. long, 15 ft. wide and 20 ft. high?

216. Land Measure. The unit of land measure is the **acre.** In the Eastern states, the land was divided, as convenient, when settled, and the description of tracts of land refer to

such natural objects as near-by bowlders, ponds, established roads, etc. But all states whose lands have been surveyed since 1802 are divided by a system of meridians and parallels into **townships** 6 miles square. Each township contains 36 square miles or **sections.** Each section contains 2 **half sections** or 4 **quarter sections.**

Public lands are located with reference to a north and south line called the **principal meridian** and an east and west line called the **base line.** The north and south rows of townships are called **ranges** and these rows are numbered from the principal meridian. The townships are numbered from the base line. A township is therefore designated by its number and the number of its range.

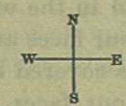


Thus, *A* is township 4 N., Range 3, W. What is *B*? *M*? *S*?

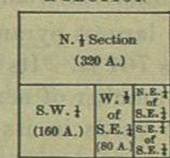
The 36 sections of a township are numbered as in the following diagram. The corners of all sections are permanently marked by stones, or otherwise.

A TOWNSHIP

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36



A SECTION



The divisions of sections into half sections, quarter sections, etc., are shown in the diagram.

Thus, the N. E. ¼ of N. E. ¼, section 6, means the northeast quarter of the northeast quarter of section 6.

EXERCISE 45

1. How many acres are there in a section? In the S. W. $\frac{1}{4}$ of S. W. $\frac{1}{4}$, section 16? In S. $\frac{1}{2}$ of N. E. $\frac{1}{4}$, section 36? Locate these sections.

2. What will be the cost of a quarter section of land at $55 an acre?

3. How many rods of fence are necessary to inclose a quarter section?

4. How many acres are there in a township?

5. The sections of a township are separated and the township is separated from adjacent townships by a road 45 ft. wide, the section lines being in the middle of the road. How many acres are there in the roads of the township?

EXERCISE 46

1. The side of a square is 100 ft. Find the length of a diagonal.

2. One side of a right-angled triangle is 16 yd. and the other side is $\frac{3}{5}$ of the hypotenuse; what is the length of the hypotenuse?

3. Find the volume of a pyramid whose base and faces are all equilateral triangles with sides 10 in. long.

4. The largest pyramid in the world has a square base with sides 764 ft. Its four faces are equilateral triangles. Find the number of acres covered by its base, the number of square yards in its four faces, and the height of the pyramid.

5. A cistern 22 ft. long, 10 ft. wide and 8 ft. deep is to be filled with water from a well 8 ft. in diameter and 40 ft. deep. If no water flows into the well while filling the cistern, find how far the water in the well is lowered.

6. Two persons start from the same place at the same time. One walks due east at the rate of 3 mi. an hour, and the other due south at the rate of $3\frac{1}{2}$ mi. an hour. In how many hours will they be 30 mi. apart?

7. What is the circumference of the earth if its diameter is 7916 mi.? $\times \pi$

8. Air being 0.00129206 as heavy as water, find in kilograms the weight of the air in a room 23^{m} long, 16^{m} wide and 10^{m} high.

9. A rectangular sheet of tin of uniform thickness is 85^{cm} wide and 2.7^{m} long, and weighs 356^{g}. Find its thickness if tin is 7.3 times as heavy as water.

10. A plate of iron weighs 277.54^{Kg}, and is 137^{cm} long, 643^{mm} wide, 43.1^{mm} thick. How much heavier than water is iron?

11. A tank is 2^{m} long, 5^{dm} wide and 8^{cm} deep. How many liters of water will it contain, and how much will the water weigh?

12. Sulphuric acid is 1.84 times as heavy as water. How many kilograms will a tank hold that is 2^{m} long, 75^{cm} wide and 50^{cm} deep?

13. A block of marble is 2 ft. long, 10 in. wide and 8 in. thick. What is the edge of a cubical block of equal volume?

14. If 1 T. of hard coal occupies a space of 36 cu. ft., how many tons will a bin 10 ft. long, $7\frac{1}{2}$ ft. wide and 9 ft. deep hold?

15. How much space will a car load of hard coal consisting of 38 T. 14 cwt. 75 lb. occupy, if one ton occupies 36 cu. ft.?

16. How long must a bin 20 ft. wide and 20 ft. deep be to hold the above car load of coal?

17. Find correct to 0.001 the diagonal of a square whose side is 10 in., and the diagonal of a cube whose edge is 10 in.

18. What will be the expense of painting the walls and ceiling of a room whose height is 10 ft. 4 in., length 16 ft. 6 in. and width 12 ft. 3 in., at 15 ct. per square yard?

19. At 11 ct. per square foot, how much will it cost to make a cement walk 5 ft. wide around a school yard in the shape of a rectangle, 18 rd. by 26 rd.?

20. Two corridors of a public building intersect at right angles near the center of the building. If the corridors are 160 ft. and 140 ft. long respectively, and 20 ft. wide, how much will it cost to cover them with a hard-wood floor at $24 per thousand feet?

21. At $18 per M, how much will it cost to cover the floor of a barn 30 ft. long and 20 ft. wide with 2-inch planks?

22. How much will it cost to fence the school yard mentioned in Ex. 19, with 1-inch boards, 6 in. wide, at $17.50 per M; the fence to be 4 boards high and built 2 ft. inside the walk?

23. How many board feet are there in 150 rafters, 14 ft long, 4 in. wide and 2 in. thick?

24. How many bunches of shingles will be required to shingle a barn with a roof 60 ft. long and rafters 18 ft. long, the shingles being laid 4 in. to the weather with a double row at the bottom?

60x18

25. What is the value of a log that will cut 36 1-inch boards, each 16 ft. long and 12 in. wide at $1\frac{3}{4}$ ct. per square foot?

26. How many board feet are there in a stick of timber $18\frac{1}{2}$ ft. long, 16 in. wide and 12 in. thick?

27. How many bricks will be used in building the walls of a building 120 ft. long, 60 ft. wide and 45 ft. high, outside measurement, if the walls are 18 in. thick and no allowance is made for doors and windows?

28. How many cubic centimeters of lead are there in a piece of lead pipe 1^{m} long, the outer diameter being 5^{cm}, and the thickness of the lead being 10^{mm}?

29. A race track 30 ft. wide with semicircular ends is constructed in a field 1050 ft. by 400 ft. Find the inside and outside lengths of the track. Also find the area of the track and the area of the field inside the track.

30. Find the volume and convex surface of a right cone, the diameter of the base being 16 in. and the altitude 18 in.

31. Find the volume and surface of a sphere whose diameter is 6 in.

32. Find the least possible loss of material in cutting a cube out of a sphere of wood 9 in. in diameter.

33. Find the least possible loss of material in cutting a sphere out of a cubical block of wood with edges 9 in. long.

34. Find the cost of making a road 200 yd. in length and 24 ft. wide; the soil being first excavated to the depth of 14 in., at a cost of 20 ct. per cubic yard; crushed stone being then put in 8 in. deep at a cost of 40 ct. per cubic yard, and gravel placed on top 6 in. thick at a cost of 45 ct. per cubic yard.

35. A map of Kansas is made on a scale of 1 in. to 100 mi. The map measures 4 in. by 2 in. Find the area of the state.