GRAPHICAL REPRESENTATIONS

217. Graphical methods of representing relations between different measurements are so extensively used in many lines of work that it seems best to give a brief treatment of the subject here. Such graphical representations as are given in the following exercises show relations pictorially in a much clearer manner than can be shown by a mere statement of figures.

Ex. 1. Explain graphically the relation between an inch and a centimeter. The two lines drawn accurately to scale represent graphically the relation between the inch and the centimeter.

1 in.

1 cm.

Ex. 2. Draw a line 1.5 in. long and find the number of centimeters in it.

Ex. 3. Explain graphically the relation between the pound and the kilogram, given $1^{Kg} = 2.2$ lb.

1 lb.

$1^{Kg} = 2.2$ lb.

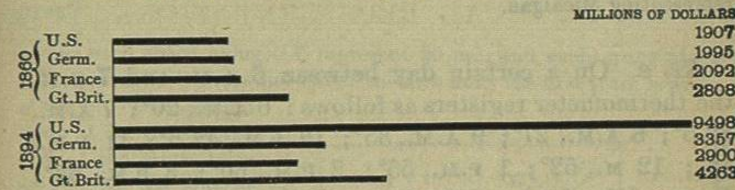
Ex. 4. Explain graphically the relation between a pint and a liter, given $1^{l} = 1.76$ pt.

Ex. 5. From a diagram find (*a*) the number of centimeters in 4 in., (*b*) the number of liters in a gallon, (*c*) the number of pounds in 5^{Kg}.

Ex. 6. The values of manufactures produced in the United States, Germany, France and Great Britain in 1860 were \$1907000000, \$1995000000, \$2092000000,

\$2808000000 respectively, and in 1894 they were \$9498000000, \$3357000000, \$2900000000, \$4263000000 respectively.

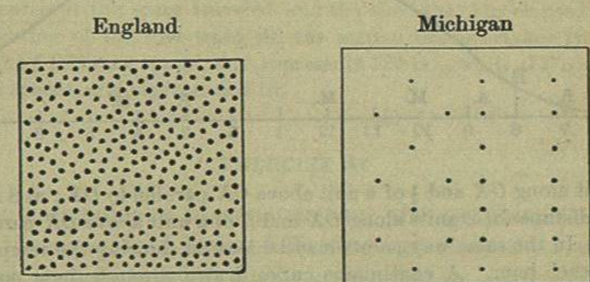
These facts may be represented graphically as follows:



These measurements, drawn accurately to a scale, show at a glance the comparative growth in manufactures produced in the different countries mentioned from 1860 to 1894.

Ex. 7. The areas of England and Michigan are 50839 and 58915 square miles respectively. The populations are approximately 31000000 and 2421000. Represent graphically the comparative sizes and the comparative density in population of the two.

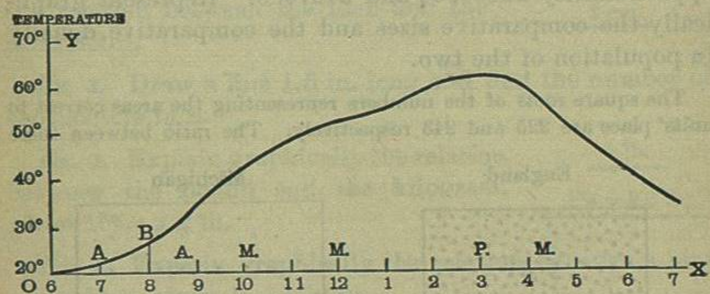
The square roots of the numbers representing the areas correct to units' place are 225 and 243 respectively. The ratio between these



two numbers reduces to 5 to 5.4. If some convenient unit of measure be taken, and squares be constructed with sides equal to 5 and 5.4 of these units, these squares will represent graphically the comparative areas. The comparative density in population will be represented by the number of dots that appear in each square, it being assumed that a dot represents 100000 in population. There will then be 310 dots in the square representing England and 24 in the square representing Michigan.

Ex. 8. On a certain day between 6 A.M. and 7 P.M. the thermometer registers as follows: 6 A.M., 20°; 7 A.M., 22.5°; 8 A.M., 27°; 9 A.M., 35°; 10 A.M., 42.5°; 11 A.M., 48°; 12 M., 52°; 1 P.M., 55°; 2 P.M., 60°; 3 P.M., 62°; 4 P.M., 60°; 5 P.M., 50°; 6 P.M., 42°; 7 P.M., 35°. Illustrate graphically this variation in temperature.

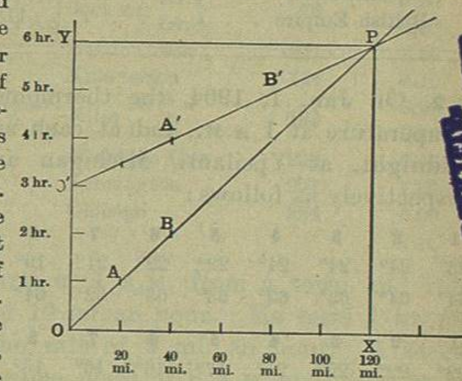
Draw two straight lines perpendicular to each other. Measure off on the horizontal line *OX* equal spaces, each representing 1 hr., and on the perpendicular line *OY* equal spaces, each one representing 10°. The temperature at 6 A.M. is shown at *O*; at 7 A.M. at *A*, a distance



of 1 unit along *OX* and ½ of a unit above *OX* parallel to *OY*; at 8 A.M. at *B*, a distance of 2 units along *OX* and .7 of a unit above *OX* parallel to *OY*. In the same way points may be located showing the temperature at each hour. A continuous curve drawn through these points is the temperature curve for the day from 6 A.M. to 7 P.M. This curve shows at a glance the variation in temperature between the hours given.

Ex. 9. Two trains leave a certain place traveling in the same direction, one at the rate of 20 mi. an hour, and the other at the rate of 40 mi. an hour. If the second train leaves 3 hr. after the first, when and where will it pass the first?

Let each space along *OX* represent 20 mi., and each space along *OY* represent 1 hr. At the end of the first hour the first train is at *A*; at the end of the second hour at *B*; and at the end of the sixth hour at *P*. At the end of the fourth hour the second train, which starts from *O′*, 3 spaces above *O*, since it starts 3 hr. later, is at *A′*; at the end of the fifth hour at *B′*; and at the end of the sixth hour at *P*. The point *P*, where the line *OP* and *O′P* cross, is the place where the second train overtakes the first. If from *P* perpendiculars *PX* and *PY* are dropped upon *OX* and *OY*, then the distances *OX* and *OY* will represent the space traveled and the time that has elapsed since the starting of the first train till the second one overtakes it. *OX* contains 6 distance spaces, and represents 120 mi., while *OY* contains 6 time spaces, and represents 6 hr.



EXERCISE 47

For convenience in constructing the graphical representations required in the following exercises, the student should provide himself with paper ruled in small squares.

1. Illustrate graphically the comparative areas and the comparative density in population in the following cases:

	Area	Population
(*a*) Alaska	590884	63592
Greenland	837837	12000
(*b*) Mexico	767258	13606000
Texas	265780	3048710
(*c*) United States (including foreign possessions)	3806279	84907156
British Empire	11391036	383165494

2. On Jan. 1, 1904, the thermometer registered the temperature at 1 A.M., and at each succeeding hour till midnight, at Ypsilanti, Michigan and Havana, Cuba, respectively as follows:

1	2	3	4	5	6	7	8	9	10	11	N.
23°	24°	24°	24°	22°	22°	21°	19°	18°	20°	21°	22°
64°	64°	63°	63°	63°	63°	62°	64°	67°	68°	70°	72°

1	2	3	4	5	6	7	8	9	10	11	Mt.
22°	22°	22°	21°	17°	16°	16°	15°	13°	12°	12°	12°
73°	73°	73°	73°	72°	70°	69°	68°	67°	66°	65°	65°

Illustrate each graphically.

3. The mean temperature for January (average for the 31 da. of the month) for the same hours and places as in Ex. 2 was as follows:

1	2	3	4	5	6	7	8	9	10	11	N.
14.2°	14.3°	14.2°	14.2°	14.1°	14.3°	14.2°	14.5°	15.3°	17.1°	18.1°	19.7°
67.2°	67.0°	66.7°	66.4°	66.0°	65.6°	65.4°	66.6°	68.9°	71.0°	73.1°	74.0°

1	2	3	4	5	6	7	8	9	10	11	Mt.
20.4°	20.7°	20.6°	19.9°	18.8°	18.1°	17.5°	16.5°	16.0°	15.4°	14.5°	16.6°
74.7°	74.9°	75.0°	74.7°	74.2°	72.9°	71.6°	70.6°	69.8°	69.0°	68.4°	67.9°

Illustrate graphically.

4. Illustrate graphically, as in Ex. 9, the point where and time at which the two trains given in the annexed time-table pass each other.

Going East A.M.	Miles		Miles	Going West A.M.
10.00	284	Detroit	0	12.35
8.54	247	Ann Arbor	37	1.25
8.00 / 7.50	214	Lv. / Ar. Jackson Ar. / Lv.	76	2.20 / 2.25
6.10	164	Battle Creek	121	3.30
4.55	141	Kalamazoo	144	4.10
3.25 / 3.15	93	Lv. / Ar. Niles Ar. / Lv.	192	5.28 / 5.33
1.55	56	Michigan City	228	6.32
12.40	13	Kensington	271	7.30
12.00 night	0	Chicago	284	8.00 A.M.

5. A cyclist starts at 7 A.M. from a town and rides 2 hr. at the rate of 10 mi. an hour. He rests 1 hr. and then returns at the rate of 9 mi. an hour. A second cyclist leaves the same place at 8 A.M. and rides at the rate of 6 mi. an hour. When and where will they meet?

6. Two cyclists start from the same place at the same time. The first rides for 2 hr. at the rate of 9 mi. an hour, rests 15 min., and then continues at 6 mi. an hour. The second one rides without stopping at the rate of 7 mi. an hour. Where will the second cyclist overtake the first?

7. The average yield of wheat per acre in the United States for the years from 1893 to 1903 in bushels was as follows: 11.4, 13.2, 13.7, 12.4, 13.4, 15.3, 12.3, 12.3, 15.0, 14.5, 12.9. The highest Chicago cash price per bushel for the same years given in cents was: 64.5, $63\frac{5}{8}$, $64\frac{3}{4}$, $93\frac{1}{8}$, 109, 70, $69\frac{1}{2}$, $75\frac{5}{8}$, $79\frac{1}{2}$, $77\frac{3}{4}$, 87. Illustrate graphically, putting the two curves in one figure.

8. The average yield of corn per acre in the United States for the years from 1893 to 1903 in bushels was as follows: 22.5, 19.4, 26.2, 28.2, 23.8, 24.8, 25.3, 25.3, 16.7, 26.8, 26.5. The highest Chicago cash price per bushel for the same years given in cents was: $36\frac{1}{2}$, $47\frac{1}{4}$, $26\frac{3}{4}$, $23\frac{3}{4}$, $27\frac{1}{2}$, 38, $31\frac{1}{4}$, $40\frac{1}{4}$, $67\frac{1}{4}$, $57\frac{1}{4}$, $43\frac{3}{4}$. Illustrate graphically, putting the two curves in one figure.

9. The average summer daily temperature in Paris at the foot and top of the Eiffel tower in 1900 was as follows:

2	4	6	8	10	N.	2	4	6	8	10	Mt.
57.2°	55.4°	58.1°	63.5°	67.8°	69.8°	70.1°	69.8°	68°	62.1°	60.7°	58.9°
57.4°	55.7°	57.2°	58.1°	60.1°	63.5°	63.9°	64°	64.4°	61.2°	60.7°	59.1°

Illustrate graphically, putting the two curves in one figure.

RATIO AND PROPORTION

218. The **ratio** of one number to another of the same kind is their quotient. The former number is called the **antecedent**, and the latter the **consequent**. The terms of the ratio therefore bear the same relation to each other as the terms of a fraction. Thus, the ratio of a to b may be written $a : b$ (read the ratio of a to b), $\frac{a}{b}$ or $a \div b$. The forms $a : b$, and $\frac{a}{b}$, are generally used. The ratio of 3 ft. to 5 ft. is 3 : 5. This may also be expressed by $\frac{3}{5}$ or 0.6.

219. The ratio is always an abstract number, since it is the relation of one number to another of the same kind. There can be no ratio between 5 hr. and $10, nor between 7 lb. and 6 ft. But there can be a ratio between 3 ft. and 6 in., since the quantities are of the same kind. Both terms must, however, be reduced to the same unit. Thus, 3 ft. = 36 in., and 36 in. : 6 in. = $\frac{36}{6}$ = 6.

The ratio $\frac{b}{a}$ is called the **inverse** or **reciprocal** of the ratio $\frac{a}{b}$.

EXERCISE 48

1. How is the value of a ratio affected by multiplying or dividing both terms by the same number?

2. How is the value affected by multiplying or dividing the antecedent? by multiplying or dividing the consequent?

Express the ratio of:

3. 100 to 25.
4. $16\frac{2}{3}$ to 100.
5. $33\frac{1}{3}$ to 100.
6. $2^{m}\ 4^{cm}$ to 50^{cm}.
7. \$15 to 50 cents.
8. $7\frac{1}{2}$ to $37\frac{1}{2}$.
9. $\frac{2}{3}$ to $16\frac{2}{3}$.
10. $12\frac{1}{2}$ to 100.
11. 14 hr. 30 min. 3 sec. to a day.
12. 2 mo. 10 da. to a year.
13. What number has to 10 the ratio 2? to 5 the ratio 0.3?
14. If $x : 3 = 5$, find x.
15. If $x : \frac{1}{2} = 2$, find x.
16. Which ratio is the greater, $\frac{5}{13}$ or $\frac{6}{17}$? $\frac{12}{13}$ or $\frac{16}{17}$?
17. The ratio of the circumference of a circle to its diameter being 3.1416, find the diameter of a circle whose circumference is 125 ft. correct to inches.
18. A map is drawn on the scale of 1 in. to 75 mi. In what ratio are the lengths diminished? In what ratio is the area diminished?
19. Two rooms are 14 ft. long, 12 ft. wide, and 12 ft. long, 10 ft. wide respectively. What is the ratio of the cost of carpeting them?
20. What is the ratio of a square field 20 rd. on a side to one 25 rd. on a side?
21. What is the ratio of the circumferences of two circles whose diameters are 2 in. and 4 in.? of two circumferences whose diameters are 5 in. and 7 in.? of two circumferences whose diameters are d and d'? Hence in general the ratio of two circumferences is equal to what?

22. What is the ratio of the areas of two circles whose radii are 3 in. and 5 in.? of the areas of two circles whose radii are 4 in. and 6 in.? of the areas of two circles whose radii are r and r'? Hence in general the ratio of the areas of two circles is equal to what?

23. What is the ratio of the volumes of two spheres whose radii are 2 in. and 3 in.? of the volumes of two spheres whose radii are 5 in. and 6 in.? of the volumes of two spheres whose radii are r and r'? Hence in general the ratio of the volumes of two spheres is equal to what?

220. Specific Gravity. The specific gravity of a substance is the ratio of its weight to the weight of an equal volume of some other substance taken as a standard.

221. Distilled water at its maximum density, 4° C., is the standard of specific gravity for solids and liquids.

222. Since 1^{cm^3} of water weighs 1 gram, the same number that expresses the weight of any substance in grams will also express its specific gravity. Thus, 1^{cm^3} of water weighs 1^{g}; hence, 1 is the specific gravity of water. 1^{cm^3} of lead weighs 11.35^{g}; hence, this being 11.35 times as heavy as an equal volume of water, the specific gravity of lead is 11.35.

SPECIFIC GRAVITIES OF SUBSTANCES

Copper	8.92	Tin	7.29	Sea Water	1.026
Iron (cast)	7.21	Anthracite Coal	1.30	Sulphuric Acid	1.841
Gold	19.26	Cork	0.24	Milk	1.032
Lead	11.35	Pine	0.65	Alcohol	0.84
Platinum	21.50	Oak	0.845	Ice	0.92
Mercury	13.598	Beech	0.852	Rock Salt	2.257

1 cu. ft. of water weighs about 1000 oz., or 62.5 lb.

Ex. 1. A mass of cast iron weighs 3500 lb. How many cubic feet does it contain?

Since 1 cu. ft. of water weighs 62.5 lb., 1 cu. ft. of iron weighs 7.21 × 62.5 lb.

$\therefore \frac{3500}{7.21 \times 62.5} = 7.77$, the number of cubic feet.

Ex. 2. In France wood is sold by weight. How much does 1 stere of beech wood weigh, allowing $\frac{1}{3}$ for space not filled?

Since 1 $^{m^3}$ of water weighs 1000 ^{Kg}, 1 stere of beech wood weighs 0.852 × 1000 ^{Kg} − $\frac{1}{3}$ of 0.852 × 1000 ^{Kg} = 568 ^{Kg}.

EXERCISE 49

1. What is the ratio of the weight of 1 stere to 1 cord of oak wood, allowing $\frac{1}{3}$ for waste space?

2. Allowing $\frac{1}{3}$ for waste space, how many tons of coal will a bin 9 ft. long, 8 ft. wide and 8 ft. deep hold?

3. What is the weight of a cubic decimeter of each of the substances in the above table? of a cubic foot?

4. A flask will hold 6 oz. of water. How much alcohol will it hold? how much mercury?

5. To what depth will a cubic foot of cork sink in sea water? in alcohol?

6. How much does a piece of copper 20^{cm} long, 15^{cm} wide and 5^{mm} thick weigh?

7. If 1 lb. of rock salt is dissolved in 1 cu. ft. of water without increasing its volume, what will be the specific gravity of the solution?

8. How much does a boat weigh that displaces 7000 cu. ft. of water?

9. If a boat is capable of displacing 3000 cu. ft., what weight will be required to sink it?

223. Proportion. A **proportion** is an equality of ratios and is expressed in the following way:

$$\frac{a}{b} = \frac{c}{d},$$

$$a : b = c : d,$$

$$a : b :: c : d.$$

224. The method of solving problems by proportion is often called the **Rule of Three**, since problems which give three quantities so related that two of them sustain the same ratio to each other as the third to the quantity required, can readily be solved by proportion.

225. Thus, if any three of the four terms of a proportion are known, the other one can be found.

If $\frac{x}{3} = \frac{5}{7}$, then, $x = 3 \times \frac{5}{7} = 2\frac{1}{7}$.

Check by putting $2\frac{1}{7}$ for x, then $\frac{2\frac{1}{7}}{3} = \frac{5}{7}$, or $\frac{15}{21} = \frac{5}{7}$.

226. The first and last terms of a proportion are called the **extremes**, and the second and third terms the **means**.

227. *In any proportion the product of the means is equal to the product of the extremes.*

If $\frac{a}{b} = \frac{c}{d}$, then by clearing of fractions $ad = bc$. This proves the proposition, since a and d are the extremes, and b and c the means.

228. If 1 lb. of sugar costs 4 ct., 2 lb. will cost 8 ct. and 4 ct. : 8 ct. = 1 lb : 2 lb. At the same rate 3 lb. would cost 12 ct., etc. The ratio of costs in each case is equal to the ratio of the weights. The cost of sugar is said to be **directly proportional** to its weight.

Ex. The Washington monument is 555 ft. high. What is the height of a post that casts a shadow 1 ft. 9 in. when the monument casts a shadow 192 ft. 6 in.?

Solution by proportion.

Let $x =$ the height of the post.

Then $$\frac{x}{555} = \frac{1.75}{192.5}$$

$$\therefore x = \frac{\overset{111}{\cancel{555}} \times \overset{0.05}{\cancel{1.75}}}{\underset{1.1}{\underset{\cancel{38.5}}{\cancel{192.5}}}} = 5.04 \text{ ft.}$$

Solution by unitary analysis.

A shadow 192 ft. 6 in. long is cast by a monument 555 ft. high.

$\therefore$ a shadow 1 ft. long will be cast by a post $\frac{555}{192.5}$ ft. high.

$\therefore$ a shadow 1 ft. 9 in. long will be cast by a post $\frac{1.75 \times 555}{192.5}$ ft. high, or 5.04 ft. high.

229. If 1 man can do a certain piece of work in 6 days, 2 men working at the same rate will do the work in 3 days, 3 men will do it in 2 days, etc. 2 men do the work in $\frac{1}{2}$ the time that 1 man will do it; 3 men in $\frac{1}{3}$ the time, etc. Hence, as the number of men increases, the time diminishes in the same ratio. If 2 men do the work in 3 days, 3 men will do it in $\frac{2}{3}$ of 3 days, or 2 days. Therefore the ratio of the number of men, $\frac{3}{2}$, is equal to the corresponding ratio of time inverted. Hence, the number of men is said to be **inversely proportional** to the time.

Ex. The crew and passengers of a steamship consisted of 1500 persons. The ship had sufficient provisions to last 12 weeks when the survivors of a wreck were taken on board. The provisions were then consumed in 10 weeks; how many were taken on board?

Solution by proportion.

Let x equal the total number on board.

Then $$\frac{x}{1500} = \frac{12}{10},$$

or $$x = \frac{150\cancel{0} \times 12}{1\cancel{0}} = 1800,$$

and $1800 - 1500 = 300$, the number taken on board.

Solution by unitary analysis.

There are provisions for 12 weeks for 1500 persons.

$\therefore$ there are provisions for 1 week for 12×1500 persons.

$\therefore$ there are provisions for 10 weeks for $\frac{150\cancel{0} \times 12}{1\cancel{0}}$ persons or 1800 persons.

$\therefore$ $1800 - 1500 = 300$, the number taken on board.

EXERCISE 50

State which of the following are directly proportional and which are inversely proportional:

1. The price of bread, the price of flour.
2. The number of workmen, the amount of work done in a given time.
3. The number of workmen, the time required to do a given amount of work.
4. The height of the thermometer, the temperature.
5. The velocity of a train, the time required to go a given distance.
6. The number of horses bought for a given sum, the price per horse.
7. The price of freight, the distance carried.
8. The area of a circle, the length of its diameter.

9. In how many ways can the terms of the proportion 2:3=8:12 be arranged without destroying the proportion?

10. The assessed value of a certain town is $7500000, and bonds for $6000 are issued. What part of this does a person worth $10000 pay?

11. A shadow cast by a post 6 ft. high is 9 ft. 3 in. How long is the shadow cast by a church steeple 150 ft. high?

12. A merchant fails for $12,300 and his property is worth $5720. How much will he pay a creditor whom he owes $2500?

13. A clock is set at noon on Monday; at 6 P.M. on Wednesday it is 2 minutes and 20 seconds too slow. Supposing the loss of time to be constant, what is the correct time when the clock strikes 12 on Sunday noon?

14. There are two kinds of thermometers used in this country, Fahrenheit, used to register temperature, and Centigrade, used largely in scientific work. The freezing point of water is 32° and 0° respectively, while the boiling point is 212° and 100° respectively. 68° Fahrenheit corresponds to what temperature Centigrade and 54° Centigrade to what temperature Fahrenheit?

15. There is another kind of thermometer known as Réaumur, the freezing and boiling points being 0° and 80° respectively. Express in Réaumur scale 70° on each of the other two.

16. The boiling point of alcohol is 78° Centigrade; what is the boiling point of alcohol on each of the other two?

17. A grain of gold can be beaten into a leaf of 56 sq. in. How many of these leaves will make an inch in height if 1 cu. ft. of gold weighs 1215 lb.?

18. Divide 60 into two parts proportional to 2 and 3.

19. Divide 90 into parts proportional to 2, 3 and 4.

20. Two men start in business with a capital of $7500. One of them furnishes $4000 and the other $3500. At the end of a year the profits are $3250. How much is each man's share?

21. A man starts in business with a capital of $5000 and in 3 months admits a partner with a capital of $4500. At the end of the year the profits amount to $3750. How much is each man's share?

22. A piece of work was to have been done by 10 men in 20 days, but at the end of two days 3 men left. How long did it take the remaining 7 men to complete the work?

23. If the interest on $325 is $72.50 in a given time, how much is the interest on $850 for the same time?

24. Two cog wheels work together; one has 36 cogs and the other 14. How many revolutions does the smaller one make while the larger one makes 28 revolutions?