METHOD OF ATTACK

230. In solving any arithmetical problem the **student** will find the following suggestions useful:

(1) The first essential is a thorough understanding of the proper relations between the conditions given. This requires some form of **analysis** leading to a complete **statement** of the conditions.

(2) The solution should involve no **unnecessary work**. Cancellation and other convenient short methods should be used if possible.

(3) All arithmetical work should be carefully **checked**. The student must realize that **accuracy** is of the highest importance and that to secure accuracy his work must always be checked. Any arithmetical work that has an error in it is valueless. The check also gives the student a means of knowing for himself whether he has a correct result or not. He has no need of answers to his problems.

Ex. **1.** If the time of the beat of a pendulum varies as the square root of its length, and the length of a pendulum that beats seconds is 39.2 in., find the length of a pendulum that beats 50 times a minute.

Solution. The given pendulum beats 60 times per minute, the required pendulum beats 50 times per minute.

Since the longer the pendulum the more slowly it beats, the required pendulum is longer than the given one.

Therefore, the square root of the lengths of the pendulums are in the ratio $\frac{60}{50}$, or $\frac{6}{5}$.

Let l = the length of the required pendulum.

Then,
$$\frac{\sqrt{l}}{\sqrt{39.2}} = \frac{6}{5};$$

or
$$\frac{l}{39.2} = \frac{6^2}{5^2},$$

or
$$l = \frac{6 \times 6 \times 39.2}{5 \times 5} \text{ in.} = \frac{6 \times 6 \times 39.2 \times 4}{100} \text{ in.} = 56.448 \text{ in.}$$

Check either by changing the order of the factors and performing the multiplication again, or by casting out the nines.

Ex. **2.** The greatest possible sphere is cut from a cube, one of whose edges is 3 ft. Find the portion of the cube cut away.

Solution. The volume of the cube is 3^3 cu. ft.

The volume of the sphere is $\frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3$ cu. ft.

Therefore the portion cut away is 3^3 cu. ft. $- \frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3$ cu. ft.

Without performing the operations indicated the student can by cancellation and combination of terms write the result thus,

$$3^2\left(3 - \frac{\pi}{2}\right)\text{cu. ft.} = 3^2\left(\frac{6 - 3.1416}{2}\right)\text{cu. ft.} = 9 \times 1.4292 \text{ cu. ft.} = 12.8628 \text{ cu. ft.}$$

Check as before.

Ex. **3.** Find the area of a square field whose diagonal is 50 rods.

Solution. Let x = one side of the square field.

Then
$$x^2 + x^2 = 50^2,$$

or
$$2x^2 = 50^2.$$

$\therefore x^2$, or the area of the field in square rods, $= \frac{50^2}{2}$ sq. rd. $= 7\frac{1}{18}$ acres.

Check each step in the work.

Ex. **4.** Find the area of the circle which is equal in area to two circles whose radii are 5 in. and 7 in.

Solution. Let r = the radius of the required circle.

Then its area in square inches = $\pi r^2 = \pi \times 5^2 + \pi \times 7^2 = \pi(5^2 + 7^2)$ = $\pi \times 74$, or 232.48 sq. in.

Check each step in the work.

Here, instead of multiplying π by 25 and then by 49 and adding the results, time is saved by adding 25 and 49 and multiplying π by the sum, 74.

231. The **foot pound** is used as a unit of work. This unit is defined as the amount of work required to overcome the resistance of one pound through a space of one foot. The **rate** of work is generally defined by using the term *horse power.* An engine of one horse power can do 33000 foot pounds of work in one minute, *i.e.* can overcome a resistance of 33000 pounds through a space of one foot in one minute.

Ex. **5.** What horse power is an engine exerting that draws a train with a uniform speed of 40 miles an hour against a resistance of 1000 pounds?

Solution. The amount of work done in one hour is $1000 \times 40 \times 5280$ foot pounds.

The amount of work done in one minute is $\dfrac{1000 \times 40 \times 5280}{60}$ foot pounds.

Therefore, the rate of doing work is $\dfrac{1000 \times 40 \times 5280}{60 \times 33000}$ horse power

$= \dfrac{10 \times 2 \times 16}{3}$ horse power = $106\frac{2}{3}$ horse power.

Check each step.

232. The student will notice that in each of the above exercises, first, *the relations between the given conditions are carefully established ;* and second, *a complete statement of these conditions is written out and the work shortened as much as possible by cancellation or otherwise, before the processes*

of multiplication and division are used. Frequently students in solving such problems will perform the operations indicated at each step, thus doing a large amount of unnecessary work. By carefully studying these model solutions the student will see where the unnecessary work can be avoided.

As indicated in Art. 41, it is a good plan, whenever possible, to estimate the result mentally and to compare this rough estimate with the result found by solving the problem. This will prevent large errors and such errors as arise from misplacing the decimal point.

EXERCISE 5.

1. Find the area bounded by 6 equal coins whose centers are at the vertices of a regular hexagon, the diameter of each coin being 2.38^{cm}.

2. A crescent is bounded by a semi-circumference of a circle whose radius is 15 inches, and by the arc of another circumference whose center is on the first arc produced. Find the area and perimeter of the crescent.

3. A horse is tied with a 50 ft. rope to one corner of a barn 30 ft. by 40 ft. Find the area he can graze over.

4. A well 30 ft. deep and 4 ft. in diameter is to be dug. If a cubic foot of earth weighs 72 lb., how much work is to be done?

5. A horse drawing a wagon along a level road at the rate of 2 mi. an hour does 29216 foot pounds of work in 3 min. What pull in pounds does he exert in drawing the wagon?

6. A uniform heavy bar, 12 ft. long and weighing 80 lb., rests on 2 props in the same horizontal plane, so that 2 ft. project over one of the props; find the distance between the props so that the pressure on one may be double that on the other ; also find the pressures.

7. It is proved in geometry that similar volumes are to each other as the cubes of their like dimensions. If a cubical bin whose edge is 4 ft. holds 52 bu. of wheat, how many bushels will a bin 6 ft. on an edge hold?

8. The temperature remaining the same, the space occupied by a gas varies inversely as the pressure. At a constant temperature a mass of air occupies 25 cu. ft. under a pressure of 10 lb. to the square inch ; what space will it occupy under a pressure of 26 lb. to the square inch?

9. A cubic foot of water weighs 1000 oz., and the pressure of the air is 336 oz. per square inch ; find the pressure on a square foot at a depth of 10 ft. below the surface of a pond.

10. If the specific gravity of mercury is 13.598 and the weight of a cubic inch of water is 252.6 grains, find the pressure of air per square inch in pounds when the mercury in the barometer stands at 30.5 in.

11. An iceberg (specific gravity 0.925) floats in sea water (specific gravity 1.025). Find the ratio of the part out of water to the part immersed.

12. A piece of lead placed in a cylindrical vessel, the radius of whose base is 1.2^{dm}, causes the liquid in the vessel to rise 3^{cm}. What is the volume of the piece of lead, and how much does it weigh if lead is 11.35 times as heavy as water?

MISCELLANEOUS EXERCISE 52

Express the ratio of :

1. A cubic decimeter to a liter.
2. A cubic centimeter to a cubic millimeter.
3. A cubic decimeter to a cubic meter.
4. A kilogram to a centigram.
5. A meter to a yard.
6. A quart to a liter.
7. A kilogram to a pound.
8. A milligram to a kilogram.
9. A kilogram to 40 grams.
10. A kilometer to 200 centimeters.

Find the value of :

11. $(60 - \frac{12}{6}) \times 3$.

12. $\dfrac{120}{12 \times 50} + 1$.

13. $(\frac{143}{11} - 5) \times 6$.

14. $(\frac{369}{9} + 2) \times 4$.

15. $\left(\dfrac{522}{6} - \dfrac{2727}{22} + \dfrac{144}{130} + \dfrac{8 \times 9}{4 \times 5}\right) \times 12$.

16. $\dfrac{(26 \div 13) \times 7}{2 + 15 \div 3}$.

17. $\dfrac{5 \times 8 - 17 \times 2}{17 - 14}$.

18. $\dfrac{\frac{2}{9} \times 4\frac{1}{2}}{\frac{1}{3} + 1\frac{5}{6}}$.

19. $\dfrac{7\frac{1}{2} + 3\frac{1}{3}}{1\frac{5}{6}}$ of $\dfrac{3\frac{1}{3}}{1\frac{1}{2} \times \frac{3}{4}}$.

20. What is a decimal fraction ?

21. How is the units' place distinguished ?

22. What is the place value of a digit one place to the right of units ? three places to the right ?

23. What is the importance of the symbol 0 in the decimal scale of notation?

24. If a decimal fraction is multiplied by a digit in units' place, do the place values of the digits in the product differ from the place value of the digits in the multiplicand? If the decimal fraction is multiplied by the same digit two orders lower, is there a difference in the place value of the digits in the product?

25. If a decimal fraction is divided by a digit in units' place, do the place values of the digits in the quotient differ from the place values of the digits in the dividend? If the decimal fraction is divided by the same digit three orders higher, what is the difference in the place values of the digits in the quotient?

26. What is a divisor of a number? a common divisor of two or more numbers? the greatest common divisor of two or more numbers?

27. What is a multiple of a number? a common multiple of two or more numbers? the least common multiple of two or more numbers?

28. What is a prime number? What is a prime factor of a number? When are two numbers prime to each other?

29. What is the shortest piece of rope that can be cut exactly into pieces 12, 15 or 20 ft. long?

30. Find the l. c. m. of the first five odd numbers, also of the first six even numbers.

31. Find the g. c. d. of 125, 340 and 735.

32. Evaluate $3\frac{1}{2} + 5\frac{1}{3} + 7\frac{1}{4} + 9\frac{1}{6}$.

33. Evaluate $\frac{7}{8} + \frac{7}{12} + \frac{7}{16} + \frac{7}{20} - \frac{5}{6} - \frac{5}{10} - \frac{5}{14}$.

34. Evaluate $4\frac{5}{18} + 2 \times 5\frac{5}{9} - 3 \times \frac{3}{4} \div \frac{1}{2}$.

35. A cubic foot of water weighs 1000 oz. How many tons, etc., of water are there in a canal 30 ft. wide, 8 ft. deep and 10 mi. long?

36. How many feet per second are equal to 40 mi. an hour?

37. Find the square root of 0.4; the cube root of 0.27.

38. If I walk 7.2^{Km} in 1 hr., how far shall I go in 6 hr. and 20 min. at the same rate?

39. How many cubic centimeters of air are there in a room $9\frac{1}{4}^{m}$ long, $6\frac{1}{2}^{m}$ wide and 3.15^{m} high?

40. What is the area of a cube that has the same volume as a box 2 ft. 6 in. by 2 ft. 3 in. by 2 ft.?

41. How many cubic meters of water pass under a bridge in one minute when the river is 20^{m} wide, 4^{m} deep and is running 3^{Km} per hour?

42. Write three numbers of four figures each that are divisible by both 8 and 3.

43. Write three numbers of six figures each that are divisible by both 9 and 11.

44. Replace the zeros in 205006 so that the number may be divisible by both 9 and 11.

45. What is the cost per hour of lighting a room with 40 burners, each consuming $2\frac{1}{2}$ cu. in. of gas per second, the price of gas being $1.25 per thousand cubic feet?

46. A roller used in rolling a lawn is $6\frac{1}{2}$ ft. in circumference and $2\frac{3}{4}$ ft. wide. If the roller makes 10 revolutions in crossing the lawn once and must pass back and forth 12 times to cover the whole lawn, find the area of the lawn.

47. Find the sum of $\frac{1}{8} + \frac{1}{6} + \frac{3}{20} + \frac{4}{30}$ correct to four decimal places.

48. Find each of the following products correct to five significant figures :

(a) 20.361 × 40.482.   (b) 1.5674 × 75.429.
(c) 824.763 × 45.   (d) 103.64 × 0.033.
(e) 0.423 × 0.00765.

49. Find each of the following quotients correct to 0.01 :

(a) 22 ÷ 3.1416; (b) 42.567 ÷ 21.268; (c) 0.4 ÷ 0.75;
(d) 237.64 ÷ 2.1473; (e) 2 ÷ 9.97.

50. Find the cost of carpeting a room 12 ft. 3 in. long and 10 ft. 9 in. wide with carpet 27 in. wide at $1.12 a yard.

51. Find the cost of 8 T. 1450 lb. of coal at $7.25 a ton.

52. Multiply 7644 by $33\frac{1}{3}$ and divide the result by $16\frac{2}{3}$.

53. Divide 8350 by 25 and multiply the result by $12\frac{1}{2}$.

Find the value of :

54. 0.0001 × 0.0001; 6.74 × 21.023.

55. 1.1 × 0.011; 7.6 × 0.76.

56. 2.5 × 25 × 250 , 2.5 × 0.25 × 0.025.

57. 0.002 × 3.01; 0.0005 × 0.01 × 5000000.

58. 15.625 ÷ 25; 0.15625 ÷ 2.5.

59. 8 ÷ 0.002; 50 ÷ 0.25.

60. 9.065 ÷ 0.049; 0.005 ÷ 0.01.

61. 0.00128 ÷ 8.192; 1708.4592 ÷ 0.00024.

Find correct to 4 decimal places :

62. 0.138138 + 0.1425876 + 2.060606 + 0.008964.

63. 7.427525 − 2.347596.   **65.** $0.33\frac{1}{3} ÷ 0.37\frac{1}{2}$.

64. $0.33\frac{1}{3} × 0.37\frac{1}{2}$.   **66.** 0.0404 ÷ 7692.

67. If the length of Jupiter's day is 9 hr. 56 min., how many more days has Jupiter than the earth in one year?

68. If $500 can be counted in one minute, how long will it take to count $1000000?

69. What is the difference between the daily income of a man whose salary is $1200 a year and of one whose salary is $1600?

70. Counting 12 hr. a day, how long would it take to count a billion at the rate of 750 a minute?

71. How many days old was a person Oct. 5, 1904, who was born July 27, 1861?

72. The ancient Roman mile is 0.917 of the English mile. Express the diameter of the earth (7926 English miles) in Roman miles.

73. The diameter of a fly wheel is found by measurement to be 20.12 in. Find its circumference.

74. The specific gravity of copper is 8.92; of gold, 19.26; of lead, 11.35. Find the weight of a lump of each equal in bulk to a liter of water.

75. The diameter of the earth is 7926 mi. The sun's diameter is 111.454 times the earth's diameter. Find the sun's diameter correct to miles.

76. A lump of iron containing 12 cu. ft. is drawn out into a rod 50 ft. long. What is the diameter of the rod?

77. The true length of the year is 365.2426 da. What error is made by calculating the year as 365 da., and adding a day every leap year, omitting three leap years in four centuries?

78. The edge of a cube is 12 in. What is the edge of a cube three times its volume?

79. How many miles an hour does a person walk who takes two steps a second and 1900 steps to the mile?

80. Express in words 0.12071 and 12000.00071.

81. How many steps 0.8 of a meter long will a person take in walking 10^{Km}?

82. A clock which gains one minute in 10 hr. is correct on Monday noon. What is the correct time when it indicates Monday noon of the next week?

In scientific work, when numbers depend upon measurements and therefore cannot be expressed with absolute accuracy the index notation is frequently used. Thus, the wave length of blue light, determined by the physicist to be 0.000431^{mm} would usually be written $4.31 \times 10^{-4\,mm}$. The distance from the sun to the earth is determined by the astronomer to be approximately 93000000 mi. In index notation it would be written 9.3×10^7 mi.

83. Express the following in the index notation:

0.0000025; 36500000000; $\frac{1}{200000000}$; 41100000.

84. Express in the common notation 1.1×10^{-5}; 3.6×10^6; 4.321×10^{-8}; 5×10^{-4}; 5×10^6.

85. From 3542_6 subtract 2131_6.

86. Find the sum of 34.6_{12}, 121.51_{12}, and 25.11_{12} and express it in the decimal notation.

87. If brass weighs 525 lb. per cubic foot, find the weight of a circular brass plate 21 in. in diameter and $\frac{1}{2}$ in. thick.

88. If a cubic foot of gold may be made to cover uniformly 432000000 sq. in., find the thickness of the gold.

89. If a gallon of water contains 231 cu. in., and a cubic foot of water weighs 1000 oz., how much does a pint of water weigh? How many gallons will weigh a ton?

90. Four circles each 1 ft. in diameter are so placed that two of them touch two of the others, and the remaining two both touch three of the others; find the area of the figure whose angles are at the four centers.

91. What (standard) time is it in Boston when it is 4.30 P.M. in San Francisco?

92. A ship's clock is corrected at 1 o'clock each day. If the ship passes over 10° 30′ each day, what change must be made in the clock (a) if the ship is sailing from W. to E.; (b) from E. to W.?

93. Find the remainders (without dividing) after 471321 has been divided by all of the numbers (except 7) from 2 to 12 inclusive.

94. Show without dividing that 133056 is divisible by 792.

95. A ship's clock is corrected every day at 1 P.M.; how much must it be put back or forward at 1, if the ship has passed over 11° of longitude from east to west?

96. When it is noon (standard time) Wednesday, Dec. 7, at Chicago, what time and date is it at Rome? at Tokyo?

97. A meter is defined as 1×10^{-7} of the distance from the pole to the equator. Find the circumference of the earth in kilometers.

98. Find the circumference of the earth in miles if the meter is equal to 39.37 in.

99. If 1 cu. ft. of water weighs 1000 oz., and platinum is 20.337 times as heavy as water, how many feet of platinum wire $\frac{1}{30000}$ of an inch in diameter would weigh a grain?

PERCENTAGE

233. $\frac{1}{2} = \frac{50}{100} = 0.50 = 50\% = 50$ per cent,

and $\frac{1}{3} = \frac{33\frac{1}{3}}{100} = 0.33\frac{1}{3} = 33\frac{1}{3}\% = 33\frac{1}{3}$ per cent.

These are different ways of denoting the same **fractional part.** In business operations it is customary to express fractions in hundredths, but in stating problems the denominator 100 is omitted and the per cent symbol, %, or the expression **per cent** is used. Percentage is therefore only an application of the decimal fraction and not a separate department of arithmetic.

234. The word **percentage** is derived from the Latin *per centum,* meaning *by the hundredths.*

235. The number denoting how many hundredths are taken is called the **rate per cent.** Thus, if 5% of a number is to be taken, 5 is called the **rate per cent,** and 5% the rate.

236. The following examples illustrate several closely related operations frequently used in business transactions.

Ex. **1.** What is 8% of $750?

Solution. 8% of $750 = 0.08 of $750 = $60.

Ex. **2.** 12 is what per cent of 240?

Solution. Let $x\% =$ the rate.

Then $x\%$ of 240 = 12,

$x\% = \frac{12}{240} = \frac{1}{20} = 0.05 = 5\%$.

Ex. **3.** 20 is 6% of what number?

Solution. Let $x =$ the number.

Then 6% of $x = 20$,

$x = \frac{20}{0.06} = 333\frac{1}{3}$.

EXERCISE 53

1. Express the following fractions in **per cent,** also as decimals: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{9}{10}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{2}{5}$, $\frac{1}{7}$, $\frac{13}{10}$, $\frac{12}{5}$.

2. 3 is what per cent of 4? 8 is what per cent of 4? 18 is what per cent of 27? 25 is what per cent of 200? 7 is what per cent of 2?

3. The population of a town is 7200. What is the population five years later if it has increased 7% in that time?

4. A town of 11750 inhabitants decreases 12% in ten years. What is its population after this loss?

5. Express the following as decimals: $\frac{1}{8}\%$, $33\frac{1}{3}\%$, 0.5%, 125%.

6. What is $\frac{1}{2}\%$ of 75? $\frac{1}{3}\%$ of 100? 0.4% of $\frac{5}{6}$? $\frac{2}{3}\%$ of $4\frac{5}{8}$?

7. Write as per cent $1\frac{1}{2}$, $2\frac{3}{4}$, $\frac{1}{8}$, $0.00\frac{1}{3}$, 10, 2, 0.25, 2.5, $0.16\frac{2}{3}$.

8. The attendance in a certain school increased in one year from 318 to 425; find the rate per cent of increase.

9. In a certain school there are 291 boys and 315 girls. What percentage of the attendance is boys and what percentage is girls?

10. In a certain town the total school enrollment is 962; of this 156 are in the high school. What percentage of the whole enrollment is in the high school?

11. If 0.8% of those living at the age of 24 die within a year, how many out of 6625 persons of this age die during that period?

12. At the age of 15, 735 out of 96285 die within a year. What is the rate per cent of deaths?

13. At the age of 25, 718 out of 89032 die within one year. Is the death rate higher or lower than at the age of 15?

14. A man owns a farm worth $7500. His annual taxes are $68.50. How much must he make in order to clear 6% from his farm each year?

15. A house depreciates in value each year at the rate of 5% of its value at the beginning of the year, and its value at the end of three years is $4225; find the original value.

16. A man sold two horses for $200 each; on the purchase price of one he made 20%, and on the other he lost 25%. Did he gain or lose and how much?

17. The wholesale grocer buys coffee at 25 ct. per pound and sells it at 30 ct. The retail grocer buys it at 30 ct. and sells it at $37\frac{1}{2}$ ct. What per cent does each make?

18. If a person spends 60% of his income and saves $1000, what is his income?

19. Which investment returns the larger per cent, flour costing $1.98 per hundred pounds and sold for $2.10, or sugar costing $3\frac{1}{2}$ ct. a pound and sold for $4\frac{1}{2}$ ct.?

20. A man owning a $\frac{2}{3}$ interest in a store sold $\frac{1}{2}$ of his interest. What per cent of his share did he sell, and what per cent of the store did he still own?

21. A merchant sold out his stock of goods at a discount of 10% of the cost and realized $14756.34. How much did his goods cost him?

22. A house rents for $300 a year, which represents 6% of its value. How much is it worth?

23. In 1880 the population of the United States was 50152866, in 1890 it was 63069756, and in 1900 it was 75994575. During which decade was the per cent of increase greater and how much?

24. What is the difference, in square yards, between $\frac{3}{4}$ of an acre and $\frac{3}{4}$% of an acre?

25. The population of a city is 14553, and is 35% more than it was 10 yr. ago. What was the population then?

26. On Nov. 1, 1897, the amount of money in circulation in the United States was: gold (including gold certificates), $576000000; silver (including silver certificates), $496000000; paper, $634000000. Nov. 1, 1902, the corresponding amounts were $967000000, $623000000 and $736000000. What was the per cent of increase in each case during the 5 yr., and what was the total per cent of increase? .

27. The following tables show the total receipts and disbursements of three of the largest life insurance companies in the United States for the year 1902:

Total Income	Expenses and Taxes	Death Claims	Other Disbursements
1073636984	183485217	252617938	316541543
782424835	156329328	163663466	185702274
330651136	54403289	60459793	69056722

Find the per cent of the total income remaining in the hands of each company at the end of the year.   Find the per cent of expense to income and of death claims to income in each case.

28. In 1890 the total foreign population in the United States was 9249547, of whom 2784894 were born in Germany and 1871509 in Ireland.   The population of the United States in 1890 being 63069756, what per cent of the population was born in Germany, and what per cent in Ireland ?

29. In 1890 the total number of negroes in the United States was 7470000, which was 11.8% of the total population at that time.   Determine the population correct to thousands.

30. In 1898 the total value of the exports from the United States was $1231482330, the total value of imports was $616049654.   By what per cent did the value of the exports exceed the value of the imports ?

COMMERCIAL DISCOUNTS

237. Manufacturers, publishers and wholesale dealers have a fixed price list for their products.   Their customers are allowed certain discounts from their list price, determined by the current market value.   Thus, a book may be published at $1.50 with a discount of 20% to dealers. The $1.50 is the **list price** and 20% is the **discount**.   The list price less the discount ($1.50 − 20% of $1.50 = $1.20) is the **net price**, or **cost.**

238. To avoid the inconvenience and expense of issuing a new catalogue whenever the market values change, business houses generally print a new trade price list giving new discounts, without issuing a new catalogue.   The discount is changed either by increasing or diminishing the single rate of discount already allowed, according as the cost of production is diminished or increased.   If the discount is to be increased, the change is generally made by quoting a further discount.   Thus, in a catalogue of electrical goods a 32 candle power lamp is quoted at $1.20.   In trade price list A, accompanying the catalogue, a discount of 50 % is allowed on small orders. In trade price list B, issued later on account of a change in the cost of production, a discount of 50 % and 15 % is allowed.   A dealer buying 10 lamps according to trade price list A would pay 10 × $1.20 − 50 % of 10 × $1.20 = $6, while according to trade price list B he would pay $6 − 15 % of $6 = $5.10.

The discount is frequently increased in case of large orders.   Thus, in the above trade price list, a discount of 50 % is allowed on all orders for less than 25 lamps, a discount of 50 % and 20 % is allowed on all orders for 25 to 100 lamps, and a discount of 50 %, 20 % and 10 % on orders for 100 or over.

239. Bills are generally made out payable in 30, 60 or 90 days, subject to a certain discount for cash, or if paid before due. Business houses usually print on their bill heads their terms of discount for cash, *e.g.* "Terms: 60 days, or 2% discount for cash." "Terms: net 90 days, or 3% in 10 days."

Ex. **1.** On March 12, 1903, E. C. Horner & Co. bought of James Bros., Chicago, 50 plows, listed at $6.50, less 25% and 10%. Terms: 90 days, 3% in 10 days.

Bill Rendered

Chicago, Ill., *March 12, 1903.*

E. C. Horner & Co.

Bought of James Bros.

Terms: 90 days; 3% 10 days.

50 Plows	@ $6.50	$325	00		
	Discount, 25%	81	25		
		243	75		
	Discount, 10%	24	38	$219	37

If Horner & Co. avail themselves of cash payment, they will deduct 3% of $219.37 = $6.58, and send the remainder, $212.79, to James Bros. If the bill is not paid till the 90 days expire, they will send $219.37.

Ex. **2.** Find the cost of a bill of goods amounting to $75 less 20%, 5% and 2% for cash.

Solution. Let x = the cost.

Then $x = 0.98 \times 0.95 \times 0.80$ of $75 = $55.86.

Analysis. $75 is the list price.

Then $75 − 20% of $75 = 0.80 of $75 is the amount left after the first discount. And 0.80 of $75 − 5% of 0.80 of $75 = 0.95 × 0.80 of $75 is the amount left after the second discount. And 0.95 × 0.80 of

$75 − 2% of $0.95 × 0.80 of $75 = 0.98 × 0.95 × 0.80$ of $75 is the amount left after the third discount.

∴ $0.98 × 0.95 × 0.80$ of $75 = $55.86 is the net price or cost.

Second Solution.　5)$75 = list price.

　　　$15 = 20% discount.

　20)$60

　　　$3 = 5% discount

　50)$57

　　　$1.14 = 2% discount for cash.

　　　$55.86 = cost of the goods.

Ex. **3.** What must be the list price of goods in order to realize $243 after deducting discounts of 25%, 10% and 10%?

Solution. Let x = the list price.

Then $0.90 × 0.90 × 0.75$ of $x = $243.

$$\therefore x = \frac{\$243}{0.90 \times 0.90 \times 0.75} = \frac{\$243 \times 10000}{9 \times 9 \times 75} = \$400.$$

EXERCISE 54

1. Find the net amount of the bill to render in each of the following cases:

　(*a*) $750 less $33\frac{1}{3}$%.

　(*b*) $1250 less 25% and 15%.

　(*c*) $525 less 20%, 10% and 5%.

　(*d*) $525 less 5%, 10% and 20%.

　(*e*) $5050.75 less 50% and 10%.

2. March 1, 1903, the Manhattan Electrical Supply Co. sold George J. Fiske & Co. the following bill of goods, 60 da., 2% 10 da.: 2 electrical gongs at $17.22 each, less 40% and 10%; 2 hotel annunciators at $15 each, less 60%; 2 spools of wire at 75 ct. each, less 50% and 10%. Find the amount to be remitted if paid March 11, and write the bill rendered.

3. A piano listed at $750 was sold at a discount of 40% and 10%. If the freight was $4.87 and drayage $3, what was the net cost of the piano?

4. Find the net cost of a piece of Rogers's statuary listed at $65 and discounted at 35%, 20%, 10% and 5%.

5. A merchant buys $1750 worth of goods at a discount of $33\frac{1}{3}$% and 10%. If he sells the goods at the list prices, what is the rate of gain on the cost?

6. A car load of flour weighing 195 hundredweight cost a grocer $1.85 a hundredweight. If he is allowed a discount of 1% for cash and sells the flour for $2.10 a hundredweight, how much does he make?

7. Which is the greater, a discount of 10%, 10% and 10%, or a discount of 20%, 5% and 5%?

8. A merchant buys goods at a discount of 40% and 10% and sells at a discount of 30% and 5%. What is his gain per cent?

9. A certain publishing house allows a discount of $16\frac{2}{3}$% on all orders under $100, $16\frac{2}{3}$% and 10% on all orders between $100 and $500, and $16\frac{2}{3}$%, 10% and 5% on all orders above $500. If three dealers wish to send in orders amounting to $60, $175 and $350 respectively, how much will each one gain if they combine their orders?

10. Which is the better discount for a buyer to take:
(a) $33\frac{1}{3}$%, 10% and 5%, or 40%?
(b) 10%, 10% and 5%, or 25%?
(c) 40% and 15%, or 40%, 10% and 5%?
(d) 50% and 15%, or 60%?

11. How much above the cost must a book marked $2 be sold, if 10% is taken from the marked price and a profit of 10% on the cost is still made?

12. One firm offers to sell $500 worth of galvanized pipe at a discount of 40%, 10% and 5%, and another firm offers a discount of $33\frac{1}{3}$%, 20% and 10%. Which is the better rate of discount and what is the difference in dollars?

13. Office furniture amounting to $750 was inventoried at the end of the first year at 25% below cost and at the end of the second year at 15% below inventory. What was the loss in value?

14. If a grocer buys sugar at 3.42 ct. per pound and sells it at 4 ct., what is his gain per cent?

15. A dealer marked his goods at $33\frac{1}{3}$% above cost, but sold at a certain per cent discount and still made 15% on the cost. What was the rate per cent of discount?

16. What three equal rates of discount are equivalent to a single rate of 27.1%?

MARKING GOODS

240. Most merchants use a private mark to indicate the cost and selling price of goods. They usually select some word or phrase containing 10 different letters and use it as a **key.** These letters are used to represent the 9 digits and 0. In this way the cost and selling price will be understood only by those who know the key.

Two different keys are generally selected, one to mark the cost and the other to mark the selling price. One or more extra letters, called repeaters, are used to avoid the repetition of a figure and to prevent giving any clew to the private mark used. The cost is usually written above and the selling price below a line.

241. The words *equinoctial* (omitting the last *i*) and *importance* are adapted for use as keys, since they both contain 10 different letters. These words give the following keys:

```
1 2 3 4 5 6 7 8 9 0
e q u i n o c t a l
i m p o r t a n c e    Repeaters x and y.
```

Thus, if a merchant pays $29.98 per dozen for hats, and sells them for $3.50 each, he would mark them $\frac{\$qa.xt}{\$p.re}$.

EXERCISE 55

1. Explain why, if the cost of a dozen articles is divided by 10, the result will give the retail price of one article with a profit of 20% added.

2. Explain why, to make a profit of $33\frac{1}{3}\%$, the cost of a dozen articles may be divided by 10 and $\frac{1}{8}$ of the result added.

3. Determine short methods of finding the retail price of one article when the cost per dozen is given and the dealer wishes to make a profit of 35%; $37\frac{1}{2}\%$; 40%; 50%; 60%.

4. A merchant buys shirts for $12.50 per dozen. For what price must he sell them to make 50%? 40%?

5. A merchant retails neckties at 50 ct. and makes 50%. How much did they cost him per dozen?

Using *equinoctal* and *importance* as keys, mark the cost and selling price of the following articles:

6. Gloves costing $5 per dozen and selling for $6.50.

7. Hats costing $22.50 per dozen and selling at 20% gain.

8. Caps costing $7.50 per dozen and selling at $33\frac{1}{3}\%$ gain.

9. Shoes costing $1.98 and selling at 25% gain.

10. Rubber boots costing $2.68 and selling at $3.75.

11. Make a key of the letters contained in the words *Cumberland* and *Charleston* spelled backward, and mark the articles given in Ex. 6 to 10.

12. A merchant sold a bill of goods that cost $125; the asking price was 30% in advance of the cost, from which a wholesale discount of 15% was allowed. What was the per cent gain?

13. An invoice of hats costing $112 is marked so as to sell at 40% profit. Does the merchant gain or lose if the hats are sold at 30% discount from the marked price?