**24.** My broker, after selling for me 200 shares of Wabash preferred, remitted to me $9975. At what price did he sell the stock?

**25.** How much must be invested in U.S. 3's coup. to bring an annual income of $500?

**26.** A bank with a capital stock of $150000, declares a semiannual dividend of 3%. What is the amount of the dividend, and how much will a person receive who owns 25 shares?

**27.** A gas company declares a 6% dividend and distributes $120000 among its stockholders. What is its capital stock?

**28.** A cement company divides $80000 among its stockholders. What is the rate of dividend, the capital stock being $1000000? How much is paid to a person who owns 902 shares of $10 each?

**29.** A broker bought for a customer 500 shares of copper stock, par value $25, at a total cost of $18015.63. Find the market quotation and brokerage.

**30.** A man bought 200 shares of New York Central at 143. The market price declined till it reached 139 and then rallied to 141½. Believing that another decline was coming, he sold 500 shares (300 of them short) at 141½. The price continued to rally, however, and he covered by buying 300 shares at 142¼. What was the net loss on the whole transaction, making no allowance for interest, but allowing ⅛% brokerage for each sale and purchase?



## INSURANCE

**308.** There are two general classes of **insurance**: insurance on the **person** in the form of life, endowment, accident, and health insurance, and insurance on **property** in the form of fire, marine, live stock, tornado, plate glass, **boiler insurance**, insurance against bad debts, etc.

### PROPERTY INSURANCE

**309.** The principal kinds of property insurance are **fire** insurance, or insurance against loss by fire; **marine** insurance, or insurance against loss of vessels at sea, or property on board of vessels at sea; **tornado** insurance, or insurance against loss by storms, etc.

**310.** The written agreement between the company and the person insured is called the **policy**, and the sum to be paid by the company in case of loss, the **face** of the policy. The person insured is called the **insured**, and the amount paid by the insured to the company for insurance, the **premium**.

**311.** The premium is usually computed as a certain per cent of the face of the policy, or as a certain sum on each $100 of insurance. In either case it is called the **rate of insurance**.

*Ex.* A house valued at $5000 is insured for ⅘ of its value at 1.1% per annum. What is the annual premium?

How much would the owner lose if the house were burned after seven premiums had been paid? How much would the company lose?

*Solution.* Valuation of house is $\frac{4}{5}$ of \$5000 = \$4000.

Premium = 1.1% of \$4000 = \$44.

Loss of owner = \$5000 − \$4000 + 7 × \$44 = \$1308.

Loss of company = \$4000 − 7 × \$44 = \$3692.

The above rate of insurance might have been stated as \$1.10 on each \$100 insured.

### EXERCISE 72

1. A house valued at \$6000 is insured for $\frac{2}{3}$ of its value at $\frac{3}{4}\%$ per annum. What is the annual premium? How much does the owner lose if the house is burned after 10 premiums have been paid? How much does the company lose?

2. How much would the owner lose in case the house were damaged by fire to the extent of \$1500 after 3 premiums had been paid?

3. How much would the owner lose if the house were damaged by fire to the extent of \$350 after 9 premiums had been paid?

4. A residence valued at \$3500 is insured for $\frac{3}{4}$ of its full value at $\frac{4}{5}\%$ per annum. The company will insure the house for 3 yr. on the payment of $2\frac{1}{2}$ times the annual premium in advance. How much will it cost to insure the house for 3 yr.? They will insure for 5 yr. on the payment of 4 times the annual premium in advance. How much will it cost to insure the house for 5 yr.?



5. How much will it cost to insure a manufacturing plant valued at \$65000 at $\frac{3}{4}\%$ and the machinery valued at \$30000 at $\frac{9}{10}\%$?

6. The insurance in Example 5 is placed in four companies, as follows: building, \$25000, \$20000, \$15000, \$5000; machinery, \$12000, \$8000, \$6000, \$4000. What is the annual premium paid each company?

7. A farmer takes the following insurance on his property: house valued at \$2500 at $1\frac{1}{4}\%$; barn valued at \$1800 at $1\frac{1}{2}\%$; live stock valued at \$2600 at $\frac{1}{5}\%$; grain valued at \$1800 at $\frac{2}{3}\%$; he also takes tornado insurance for \$3000 and pays 40 ct. per \$100 for 5 yr. He pays 4 times the annual premium for fire insurance for 5 yr. and 3 times for live stock insurance. What is his total premium for 5 yr.?

8. A dealer in Buffalo ordered his Chicago agent to buy 4000 bu. of wheat at 72 ct., 2500 bu. of oats at 26 ct., 7200 bu. of corn at $37\frac{1}{2}$ ct., paying 2% commission for buying. The grain was shipped by boat, and a policy at $1\frac{1}{8}\%$ taken to cover the cost of grain and all charges. What was the amount of the policy and what was the premium?

9. In a town where the regular police force consists of 20 or more patrolmen a company will insure a bank against burglary for 1 yr. for 50 ct. per \$100 up to \$3000, and 25 ct. per \$100 above that amount. How much will it cost to insure a bank for \$50000 against burglary in such a town?

10. The annual premium for insuring a plate glass window 6 ft. by 10 ft. is \$3.30. How much will it cost a merchant to insure two such windows for 5 yr.?

## LIFE AND ACCIDENT INSURANCE

**312. Life** insurance is an agreement to pay to the heirs of a person a specified sum upon his death.

**313. Endowment** insurance is an agreement to pay a specified sum to the person insured if living at the end of a definite period of years, or to his heirs in case of death within that period.

**314. Accident** insurance is an agreement to pay the person insured a weekly indemnity for loss of time while incapacitated from accident, or a fixed amount in case of permanent injury, such as the loss of both hands, both feet, the entire sight of the eyes, etc., or a fixed amount to his heirs in case of death by accident.

**315. Health** insurance is an agreement to pay a weekly sum in case of sickness from specified diseases. In addition to the weekly indemnity, health insurance sometimes guarantees the payment of all doctor's fees and special amounts to cover cost of surgical operations.

**316.** The following tables show the annual rates per $1000 charged by one of the leading life insurance companies doing business in the United States. These rates are for life and endowment policies. Insurance companies also issue rates payable semiannually or quarterly. Such rates are slightly in advance of the annual rate, due to the fact that interest is charged on the amounts not paid at the time when the whole premium is due.



### WHOLE LIFE POLICIES

PARTICIPATING

| Age | Payments for Life | 20 Payments | 15 Payments | 10 Payments | 5 Payments | Single Payment |
|---|---|---|---|---|---|---|
| 18 | | $28 05 | $33 75 | $45 37 | $80 70 | $364 89 |
| 19 | | 28 47 | 34 24 | 46 03 | 81 84 | 369 96 |
| 20 | | 28 90 | 34 76 | 46 71 | 83 02 | 375 19 |
| 21 | $19 50 | 29 35 | 35 29 | 47 41 | 84 24 | 380 58 |
| 22 | 19 93 | 29 82 | 35 84 | 48 13 | 85 50 | 386 15 |
| 23 | 20 38 | 30 30 | 36 41 | 48 88 | 86 80 | 391 89 |
| 24 | 20 86 | 30 81 | 37 00 | 49 65 | 88 14 | 397 82 |
| 25 | 21 35 | 31 33 | 37 61 | 50 45 | 89 52 | 403 93 |
| 26 | 21 87 | 31 87 | 38 24 | 51 28 | 90 95 | 410 24 |
| 27 | 22 42 | 32 43 | 38 90 | 52 14 | 92 43 | 416 74 |
| 28 | 22 99 | 33 01 | 39 57 | 53 02 | 93 96 | 423 45 |
| 29 | 23 59 | 33 61 | 40 28 | 53 94 | 95 53 | 430 37 |
| 30 | 24 22 | 34 24 | 41 01 | 54 89 | 97 16 | 437 50 |
| 31 | 24 89 | 34 89 | 41 77 | 55 87 | 98 84 | 444 86 |
| 32 | 25 59 | 35 58 | 42 55 | 56 89 | 100 58 | 452 44 |
| 33 | 26 33 | 36 29 | 43 37 | 57 94 | 102 38 | 460 25 |
| 34 | 27 11 | 37 03 | 44 21 | 59 03 | 104 23 | 468 30 |
| 35 | 27 93 | 37 80 | 45 10 | 60 16 | 106 14 | 476 58 |
| 36 | 28 80 | 38 61 | 46 01 | 61 33 | 108 11 | 485 12 |
| 37 | 29 72 | 39 45 | 46 97 | 62 54 | 110 15 | 493 91 |
| 38 | 30 69 | 40 34 | 47 96 | 63 80 | 112 26 | 502 95 |
| 39 | 31 71 | 41 26 | 48 99 | 65 10 | 114 43 | 512 24 |
| 40 | 32 80 | 42 24 | 50 07 | 66 45 | 116 67 | 521 80 |
| 41 | 33 95 | 43 26 | 51 20 | 67 85 | 118 98 | 531 62 |
| 42 | 35 17 | 44 34 | 52 38 | 69 30 | 121 37 | 541 71 |
| 43 | 36 47 | 45 48 | 53 62 | 70 82 | 123 83 | 552 07 |
| 44 | 37 84 | 46 68 | 54 92 | 72 40 | 126 38 | 562 70 |
| 45 | 39 31 | 47 95 | 56 28 | 74 04 | 129 01 | 573 59 |
| 46 | 40 86 | 49 30 | 57 72 | 75 75 | 131 72 | 584 76 |
| 47 | 42 52 | 50 73 | 59 23 | 77 54 | 134 52 | 596 18 |
| 48 | 44 29 | 52 25 | 60 82 | 79 40 | 137 42 | 607 85 |
| 49 | 46 17 | 53 87 | 62 49 | 81 35 | 140 41 | 619 76 |
| 50 | 48 17 | 55 59 | 64 26 | 83 38 | 143 48 | 631 89 |
| 51 | 50 31 | 57 43 | 66 13 | 85 50 | 146 65 | 644 22 |
| 52 | 52 58 | 59 38 | 68 10 | 87 72 | 149 92 | 656 74 |
| 53 | 55 00 | 61 47 | 70 19 | 90 03 | 153 28 | 669 43 |
| 54 | 57 59 | 63 71 | 72 40 | 92 46 | 156 74 | 682 28 |
| 55 | 60 34 | 66 10 | 74 75 | 94 99 | 160 30 | 695 27 |
| 56 | 63 28 | 68 66 | 77 24 | 97 66 | 163 97 | 708 38 |
| 57 | 66 42 | 71 41 | 79 90 | 100 45 | 167 75 | 721 60 |
| 58 | 69 78 | 74 37 | 82 74 | 103 39 | 171 65 | 734 91 |
| 59 | 73 37 | 77 55 | 85 77 | 106 50 | 175 68 | 748 28 |
| 60 | 77 20 | 80 97 | 89 02 | 109 77 | 179 84 | 761 71 |

## ENDOWMENT POLICIES

PAYMENTS FOR FULL TERM. PARTICIPATING

| Age | Due in 10 Years | Due in 15 Years | Due in 20 Years | Due in 25 Years | Due in 30 Years | Due in 35 Years | Due in 40 Years |
|---|---|---|---|---|---|---|---|
| 18 | $102 37 | $66 25 | $48 55 | $38 22 | $31 58 | $27 08 | $23 93 |
| 19 | 102 45 | 66 34 | 48 65 | 38 32 | 31 70 | 27 21 | 24 09 |
| 20 | 102 54 | 66 44 | 48 75 | 38 43 | 31 82 | 27 35 | 24 26 |
| 21 | 102 64 | 66 54 | 48 86 | 38 55 | 31 96 | 27 51 | 24 44 |
| 22 | 102 73 | 66 64 | 48 97 | 38 68 | 32 10 | 27 68 | 24 65 |
| 23 | 102 83 | 66 75 | 49 09 | 38 81 | 32 26 | 27 86 | 24 86 |
| 24 | 102 94 | 66 87 | 49 22 | 38 96 | 32 42 | 28 06 | 25 10 |
| 25 | 103 06 | 66 99 | 49 36 | 39 11 | 32 60 | 28 27 | 25 36 |
| 26 | 103 18 | 67 13 | 49 50 | 39 28 | 32 79 | 28 50 | 25 64 |
| 27 | 103 30 | 67 27 | 49 66 | 39 46 | 33 00 | 28 75 | 25 95 |
| 28 | 103 44 | 67 42 | 49 83 | 39 65 | 33 23 | 29 03 | 26 28 |
| 29 | 103 58 | 67 58 | 50 00 | 39 85 | 33 48 | 29 33 | 26 65 |
| 30 | 103 74 | 67 75 | 50 20 | 40 08 | 33 75 | 29 66 | 27 05 |
| 31 | 103 90 | 67 93 | 50 41 | 40 32 | 34 04 | 30 02 | 27 48 |
| 32 | 104 08 | 68 12 | 50 63 | 40 59 | 34 37 | 30 41 | 27 96 |
| 33 | 104 26 | 68 34 | 50 87 | 40 88 | 34 72 | 30 84 | 28 48 |
| 34 | 104 46 | 68 56 | 51 14 | 41 20 | 35 10 | 31 31 | 29 04 |
| 35 | 104 68 | 68 81 | 51 43 | 41 55 | 35 53 | 31 83 | 29 66 |
| 36 | 104 91 | 69 07 | 51 74 | 41 93 | 35 99 | 32 39 | 30 33 |
| 37 | 105 16 | 69 36 | 52 09 | 42 35 | 36 50 | 33 01 | 31 06 |
| 38 | 105 43 | 69 68 | 52 47 | 42 81 | 37 07 | 33 69 | 31 85 |
| 39 | 105 72 | 70 02 | 52 88 | 43 31 | 37 68 | 34 43 | 32 71 |
| 40 | 106 04 | 70 40 | 53 34 | 43 87 | 38 36 | 35 24 | 33 64 |
| 41 | 106 38 | 70 82 | 53 84 | 44 49 | 39 10 | 36 12 | |
| 42 | 106 76 | 71 27 | 54 40 | 45 16 | 39 92 | 37 09 | |
| 43 | 107 18 | 71 78 | 55 01 | 45 91 | 40 82 | 38 14 | |
| 44 | 107 64 | 72 33 | 55 69 | 46 73 | 41 81 | 39 29 | |
| 45 | 108 14 | 72 95 | 56 44 | 47 64 | 42 90 | 40 54 | |
| 46 | 108 70 | 73 63 | 57 27 | 48 64 | 44 09 | | |
| 47 | 109 32 | 74 39 | 58 18 | 49 75 | 45 39 | | |
| 48 | 110 00 | 75 22 | 59 20 | 50 97 | 46 82 | | |
| 49 | 110 76 | 76 14 | 60 31 | 52 31 | 48 38 | | |
| 50 | 111 59 | 77 15 | 61 54 | 53 78 | 50 07 | | |
| 51 | 112 51 | 78 27 | 62 89 | 55 38 | | | |
| 52 | 113 51 | 79 49 | 64 38 | 57 15 | | | |
| 53 | 114 61 | 80 84 | 66 01 | 59 07 | | | |
| 54 | 115 82 | 82 33 | 67 81 | 61 18 | | | |
| 55 | 117 16 | 83 96 | 69 78 | 63 47 | | | |
| 56 | 118 62 | 85 76 | 71 94 | | | | |
| 57 | 120 22 | 87 73 | 74 31 | | | | |
| 58 | 121 98 | 89 91 | 76 91 | | | | |
| 59 | 123 92 | 92 30 | 79 75 | | | | |
| 60 | 126 05 | 94 93 | 82 85 | | | | |



The premiums on life policies are to be paid during the entire life of the insured, or during the period indicated in the preceding table. The face of the policy is to be paid at the death of the insured. The endowment policy provides for the payment of the face of the policy in 10, 15, 20, 25, 30, 35, or 40 yr. from the date of issue, or at the death of the insured if it occurs before the close of the stated period.

**317.** The premiums charged by the life insurance companies are determined by three considerations: (1) the probability that the insured will live as long as a healthy person of his age may be expected to live; (2) the rate of interest the company can earn on the premiums paid in; (3) the necessary expense of managing the company.

In order to secure safety of the policy contract, premiums are made higher than the above considerations render necessary. The portion of the premium remaining unused at the end of any year may be returned to the policy holder in the form of an **annual dividend**, or it may be allowed to accumulate for a term of years, called the **accumulation period**. The period is usually 10, 15, or 20 years. In the latter case, no dividend is paid unless the policy is kept in force to the end of the accumulation period.

The excess of assets over liabilities due to accumulated dividends, interest earned, etc., forms the **surplus** of the company. The **reserve** of a company is the amount held to meet the payment of policies when due.

Great care is taken by life insurance companies to protect the insured against forfeiture through nonpayment of premiums.

**318.** The following tables illustrate the **loan value**, or the amount the company agrees to loan the insured if the policy is assigned to the company as security; the **cash value**, or the amount the company agrees to pay the insured on surrender of the policy; the **paid-up policy**, or the face of a paid-up policy the company agrees to exchange for the original policy if surrendered; the **extended** insurance, or the time the company will continue the full amount of insurance without further payment. These privileges are granted in consideration of the premiums already paid.

20-PAYMENT LIFE

| YEAR | Age 35. | | PREMIUM: A. $37.80 S. A. 19.60 Q. 9.98 | | |
|---|---|---|---|---|---|
| | Loan | Cash Value | Paid up Policy | Extended Insurance | |
| | | | | Years | Days |
| 3 | $47 | $53 | $131 | 6 | 194 |
| 4 | 70 | 78 | 183 | 8 | 276 |
| 5 | 94 | 105 | 235 | 10 | 317 |
| 6 | 118 | 132 | 287 | 12 | 284 |
| 7 | 144 | 160 | 339 | 14 | 167 |
| 8 | 170 | 189 | 391 | 15 | 332 |
| 9 | 196 | 218 | 442 | 17 | 55 |
| 10 | 224 | 249 | 493 | 18 | 75 |
| 11 | 252 | 281 | 544 | 19 | 41 |
| 12 | 281 | 313 | 595 | 19 | 324 |
| 13 | 312 | 347 | 646 | 20 | 205 |
| 14 | 343 | 382 | 696 | 21 | 56 |
| 15 | 376 | 418 | 746 | 21 | 251 |
| 16 | 408 | 454 | 797 | 22 | 69 |
| 17 | 441 | 491 | 847 | 22 | 250 |
| 18 | 476 | 529 | 898 | 23 | 76 |
| 19 | 511 | 568 | 948 | 23 | 287 |
| 20 | 548 | 609 | | | |
| 25 | 599 | 666 | | | |
| 30 | 650 | 723 | Policy full paid. | | |

20-YEAR ENDOWMENT

| YEAR | Age 35. | | PREMIUM: A. $51.43 S. A. 26.67 Q. 13.57 | | | |
|---|---|---|---|---|---|---|
| | Loan | Cash Value | Paid up Policy | Extended Insurance | | Endowment |
| | | | | Years | Days | |
| 3 | $82 | $92 | $147 | 10 | 196 | |
| 4 | 118 | 132 | 208 | 13 | 348 | |
| 5 | 155 | 173 | 259 | 15 | 0 | $41 |
| 6 | 193 | 215 | 314 | 14 | 0 | 121 |
| 7 | 233 | 259 | 368 | 13 | 0 | 198 |
| 8 | 274 | 305 | 421 | 12 | 0 | 272 |
| 9 | 316 | 352 | 474 | 11 | 0 | 343 |
| 10 | 360 | 401 | 525 | 10 | 0 | 411 |
| 11 | 405 | 451 | 576 | 9 | 0 | 477 |
| 12 | 453 | 504 | 626 | 8 | 0 | 541 |
| 13 | 502 | 558 | 675 | 7 | 0 | 602 |
| 14 | 553 | 615 | 724 | 6 | 0 | 660 |
| 15 | 606 | 674 | 771 | 5 | 0 | 716 |
| 16 | 659 | 733 | 818 | 4 | 0 | 781 |
| 17 | 716 | 796 | 865 | 3 | 0 | 842 |
| 18 | 774 | 861 | 910 | 2 | 0 | 898 |
| 19 | 835 | 928 | 955 | 1 | 0 | 951 |
| 20 | | 1000 | | | | |

### EXERCISE 73

From the tables find the annual premium required for:

1. A life policy for $2500, age 24.

2. A ten-payment life policy for $4000, age 29.

3. A ten-year endowment policy for $5000, age 40.

4. A twenty-payment life policy for $3000, age 37.

5. A twenty-year endowment policy for $6000, age 37.

6. At age 24 Mr. Robbins takes out a life policy for $5000; if he dies at the age of 41, how much does the face of the policy exceed the premiums paid?

7. If money is worth 6% per annum, what do the premiums paid in Ex. 6 amount to? How much does the face of the policy exceed the amount?



8. At age 35 Mr. Andrews takes out a $5000 twenty-payment life policy; what is the face of the paid-up life policy that will be given to him if he stops paying premiums and surrenders his policy at age 46? What is the guaranteed cash value of the policy at age 45?

9. At age 31 a man took out a $2500 life policy and at age 36 a $1500 twenty-five-year endowment policy and a $1000 twenty-year endowment policy. How much does the insurance exceed the premiums paid if he dies at the age of 43?

10. If the annual dividends on a twenty-payment life policy, age 35, average 21% of the premiums, how much has a $1000 policy cost at the end of 20 years, money being worth 5%?

11. If dividends are not paid annually, but are allowed to accumulate for a period of twenty years on the above twenty-payment life policy, the insured would be privileged to withdraw the accumulated surplus in cash and still retain a full-paid policy for $1000 payable at death. Should the accumulated surplus amount to $391.78 at the end of twenty years, how much does the policy cost, money being worth 5%?

12. Mr. Young takes out a $5000 fifteen-payment life policy Nov. 19, 1887, at age 40. In 1902, instead of continuing the insurance, he surrenders for a cash value of $4036.75, which includes the accumulated dividends. Allowing $15 per annum per $1000 for protection afforded, what rate of interest has his money earned in the 15 years?

# TAXES AND DUTIES

**319.** The expenses of the **United States government** for pensions, army and navy, salaries of the President, congressmen, and other officials, etc., amount to something over $1000000 a day. The **state** must have money for the care of the insane, blind, deaf and dumb, criminals; for educational purposes, salaries of state officials, etc. The **county** needs money for public buildings, bridges, salaries, educational purposes, etc. The **city** and **village** must have public improvements, fire protection, police, schools, etc. These expenses are met by **taxes.**

## TAXES

**320.** The expenses for the support of the state, county, city, etc., are paid by taxes on real estate and personal property. In addition to the property tax most states collect a **poll** tax of from $1 to $3 from each male citizen over 21 years of age and under 50.

**321. The rate of taxation** is usually expressed as a certain number of mills on each dollar, or as a certain number of cents on each $100 of valuation.

### EXERCISE 74

1. The valuation of the property of a certain county is $7500000. If the general state tax and the general county tax are each 60 ct. on each $100 and in addition the





bridge tax is 40 ct. and the school tax 30 ct., what is the total tax of the county and what is the amount set aside for each of the above purposes?

2. What are the taxes of a man who owns 160 acres of land in the above county worth $60 an acre and assessed at $\frac{2}{5}$ of its value, and personal property amounting to $1850?

3. The total assessed value of property in Michigan in 1901 was $1578100000. What amount did the State University receive in 1903 from a $\frac{1}{4}$ of a mill tax?

4. How much of this tax did a farmer have to pay who owns 200 acres of land valued at $75 an acre and assessed at $\frac{2}{5}$ of its value?

5. A certain city is bonded for $6000; its taxable property is valued at $7500000. How much of the above bonded indebtedness does a man worth $10000 pay?

6. Suppose the above city wishes to build a high school building valued at $50000, what will be the tax on each $100?

7. The taxable property of a certain county is $125000000. What will be the tax on each $100 to build a courthouse worth $90000?

8. The Michigan State Normal College received from the state, in 1903, $103200. How much of this did a man pay who owns $7500 worth of taxable property, the state having property listed at $1578100000?

9. The assessed value of a town is $250000 and the amount of tax to be raised is $3500; what is the rate of taxation?

### DUTIES

**322.** The income for the support of the national government is derived largely from **custom revenue** (tariff or duty on imports, collected at **customhouses** established by the government at ports of entry), and **internal revenue** (taxes on spirits, tobacco, etc.).

**323.** Merchandise brought into the country is subject to **ad valorem duty** (a certain per cent of the cost of the goods), **specific duty** (a certain amount of weight, number, or measure, without regard to value), or both **ad valorem** and **specific duty**. Some goods are admitted duty free.

*Illustrations.* Cut glass and laces pay an ad valorem duty of 60%. Machinery pays 45%. Tin plates pay a specific duty of 1½ ct. per pound, horses valued at $150 or less pay $30, and wheat pays 25 ct. per bushel. Cigars pay a duty of $4.50 per pound and 25%, and lead pencils pay 45 ct. per gross and 25%. Books published in foreign languages are admitted duty free.

#### EXERCISE 75

1. What will be the duty on 1 T. 4 cwt. of tin plate?
2. What will be the duty on 20 gross of lead pencils?
3. What is the cost per gross of lead pencils on which the two rates of duty are equal?
4. The duty on ready-made clothing is 50%. What is the duty on $6000 worth?
5. If the duty on linen collars and cuffs is 40 ct. per dozen and 20%, what is the duty on 10 doz. collars at 75 ct. a dozen and 10 pairs of cuffs at 25 ct. a pair?
6. What is the duty at 50% on 500 doz. kid gloves at 75 francs a dozen?
7. Find the duty on an importation of £750 8*s*. 4*d*. worth of English crockery at 40%.

## THE PROGRESSIONS

**324.** By a **series** is meant a succession of terms formed according to some common law.

**325.** An **arithmetical progression** (A. P.) is a series in which each term differs from the preceding by a constant quantity called the **common difference.**

Thus, 2, 5, 8, 11, ···, and 15, 10, 5, 0, − 5, − 10, ···, are arithmetical progressions. In the first, 3 is the common difference and is added to each term to form the next; in the second, − 5 is the common difference and is added to each term to form the next.

**326.** A **geometrical progression** (G. P.) is a series in which each term after the first is derived by multiplying the preceding term by a constant multiplier called the **ratio.**

Thus, 2, 4, 8, 16, ···, and 18, − 6, 2, $-\frac{2}{3}$, $\frac{2}{9}$, ···, are geometrical progressions, the ratios being respectively 2 and $-\frac{1}{3}$.

**327. Last Term.** If $a$ is the first term, $l$ the last term, $d$ the common difference, $r$ the ratio, and $n$ the number of terms, we have from the definitions, —

| | 1st | 2d | 3d | | nth |
|---|---|---|---|---|---|
| A. P. | $a$ | $(a+d)$ | $(a+2d)$ | ··· | $a+(n-1)d$ |
| G. P. | $a$ | $ar$ | $ar^2$ | ··· | $ar^{n-1}$ |

∴ the formulas for the last of $n$ terms are:

A. P. $l = a + (n-1)d$.

G. P. $l = ar^{n-1}$.

*Ex.* Find the last term in an A. P. in which the first term is 10, the common difference 4, and the number of terms 12.

*Solution.* $l = a + (n-1)d = 10 + (12-1) \times 4 = 54$.

*Ex.* Find the last term in a G. P. in which the first term is 2, the common ratio 3, and the number of terms 5.

*Solution.* $l = ar^{n-1} = 2 \times 3^4 = 162$.

### 328. Sum of Series.

A. P. Take the series 3, 5, 7, 9, 11, in which $a=3$, $d=2$, $l=11$, and the sum $(S)=35$.

Then $S = 3 + (3+2) + (3+4) + (3+6) + (3+8)$,

and in reverse order

$$S = 11 + (11-2) + (11-4) + (11-6) + (11-8).$$

Adding and canceling the 2, 4, 6, and 8,

$$2S = (3+11) + (3+11) + (3+11) + (3+11) + (3+11)$$

$$= 5(3+11).$$

$$\therefore S = \tfrac{5}{2}(3+11) = 35,$$

or *the sum of the series equals one half of the number of terms times the sum of the first and last terms.*

Take the general series $a$, $a+d$, $a+2d$, $\cdots$, $a+(n-1)d$. In this series it will be noticed that each term is formed by adding to the first term the common difference multiplied by the number of the term less one.

Then $S = a + (a+d) + (a+2d) + \cdots (l-d) + l$,

and in reverse order

$$S = l + (l-d) + (l-2d) + \cdots (a+d) + a.$$



Adding and canceling the $d$'s.

$$2S = a + l + a + l + a + l + \cdots a + l + a + l = n(a+l).$$

$$\therefore S = \frac{n}{2}(a+l).$$

G. P. Take the series 2, 6, 18, 54, 162, in which $a=2$, $r=3$, $l=162$, $n=5$, and $S=242$.

Then $S = 2 + 2\times3 + 2\times3^2 + 2\times3^3 + 2\times3^4$.

Multiplying by 3, $3S = 2\times3 + 2\times3^2 + 2\times3^3 + 2\times3^4 + 2\times3^5$.

Subtracting and canceling common terms,

$$S - 3S = 2 - 2\times3^5.$$

$$\therefore S = \frac{2 - 2\times3^5}{1-3} = 242,$$

or *the sum of the series equals the first term minus the first term times the ratio raised to a power equal to the number of terms divided by one minus the ratio.*

Take the general series $a$, $ar$, $ar^2$, $\cdots$, $ar^{n-1}$. In this series it will be noticed that each term is formed by multiplying the first term by the ratio raised to a power one less than the number of the term.

$$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}.$$

Multiplying by $r$, $rS = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n$.

Subtracting, $S - rS = a - ar^n$.

$$\therefore S = \frac{a - ar^n}{1-r}.$$

*Ex.* Find the sum of the first 8 terms in an A. P. when the first term is 5 and the common difference is 3.

*Solution.* Since $S = \frac{n(a+l)}{2}$ and $l = a + (n-1)d$,

$$\therefore S = \frac{n}{2}[2a + (n-1)d] = 4(10 + 7\times3) = 124.$$

*Ex.* Sum to 6 terms the series $2 + 6 + 18 + \cdots$.

*Solution.* $S = \frac{a - ar^n}{1-r} = \frac{2 - 2\times3^6}{1-3} = 728$.

**329. Infinite Series.** Writing the formula

$$S = \frac{a - ar^n}{1 - r} = \frac{a}{1 - r} - \frac{ar^n}{1 - r},$$

we see that when $r$ is a proper fraction and $n$ becomes large, $ar^n$ becomes small. If we make $n$ sufficiently large, $ar^n$ and hence $\frac{ar^n}{1 - r}$ will approach as near to zero as we please, and hence, when $n$ becomes infinite, $S = \frac{a}{1 - r}$.

*Ex.* Sum to infinity the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$.

*Solution.* $S = \frac{1}{1 - \frac{1}{2}} = 2$.

**330. Circulating Decimals.** $\frac{1}{3} = 0.3333\cdots$ and $\frac{7}{37} = 0.189189\cdots$. In the first case 3 is repeated indefinitely, and in the second case the digits 189 are repeated indefinitely in the same order. Such decimals are called **circulating decimals**. The repeating figures are called the **repetend**. A circulating decimal is expressed by writing the repetend once and placing a dot over the first and the last figure of the part repeated.

Thus, $0.333\cdots = 0.\dot{3}$ and $0.189189\cdots = 0.\dot{1}8\dot{9}$.

*Ex.* Reduce $0.\dot{3}$ to an equivalent common fraction.

*Solution.* $0.3 = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots =$ a G. P. with the first term $= \frac{3}{10}$, and the ratio $= \frac{1}{10}$. $\therefore S = \frac{\frac{3}{10}}{1 - \frac{1}{10}} = \frac{1}{3}$.

### EXERCISE 76

1. Find the 12th term of the series 5, 7, 9, ....
2. Find the 7th term of the series $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \cdots$.
3. Find the sum of 9 terms of $\frac{1}{6} + \frac{1}{3} + \frac{1}{2} + \cdots$.



4. If a body falls $16\frac{1}{12}$ ft. the first second, 3 times as far the next second, 5 times as far the third second, and so on, how far will it fall in the twelfth second? How far will it fall in 12 sec.?
5. Find the 8th term in the series $1, \frac{1}{3}, \frac{1}{9}, \cdots$.
6. Find the sum of $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ to infinity.
7. Find the 7th term in the series $4, -2, 1, \cdots$.
8. Find the value of $0.4\dot{2}\dot{3}$.
9. Find the 5th and 9th terms of the series 3, 6, 12, ....
10. Find the 9th term of the series $\frac{1}{64}, \frac{1}{32}, \frac{1}{16}, \cdots$.
11. Sum to 5 terms the series $9, -6, 4, \cdots$.
12. Find the value of $0.\dot{2}$; $0.\dot{2}\dot{3}$; $0.2\dot{4}$; $1.71\dot{4}\dot{5}$.
13. Find the sum of $3 + 0.3 + 0.03 + \cdots$ to infinity.
14. Find the sum of the first 25 odd numbers; the first 25 even numbers.
15. What is the distance passed through by a ball before it comes to rest, if it falls from a height of 40 ft. and rebounds half the distance at each fall?