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ELEMENTS

OF

GEOMETRY AND TRIGONOMETRY

FROM THE WORKS OF

A. M. LEGENDRE

ADAPTED TO THE COURSE OF MATHEMATICAL INSTRUCTION
IN THE UNITED STATES

BY CHARLES DAVIES, LL.D.
AUTHOR OF A FULL COURSE OF MATHEMATICS

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NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

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PREFACE

Of the various treatises on Elementary Geometry which have appeared during the present century, that of **M. Legendre** stands pre-eminent. Its peculiar merits have won for it not only a European reputation, but have also caused it to be selected as the basis of many of the best works on the subject that have been published in this country.

In the original treatise of **Legendre,** the propositions are not enunciated in general terms, but by means of the diagrams employed in their demonstration. This departure from the method of **Euclid** is much to be regretted. The propositions of Geometry are general truths, and ought to be stated in general terms, without reference to particular diagrams. In the following work, each proposition is first enunciated in general terms, and afterward with reference to a particular figure, that figure being taken to represent any one of the class to which it belongs. By this arrangement, the difficulty experienced by beginners in comprehending abstract truths is lessened, without in any manner impairing the generality of the truths evolved.

The term ***solid,*** used not only by **Legendre,** but by many other authors, to denote a limited portion of space, seems calculated to introduce the foreign idea of matter into a science which deals only with the abstract properties and relations of figured space. The term ***volume*** has been introduced in its place, under the belief that it corresponds more exactly to the idea intended. Many other departures have been made from the original text, the value and utility of which have been made manifest in the practical tests to which the work has been subjected.

In the present edition, numerous changes have been made, both in the Geometry and in the Trigonometry. The definitions have been carefully revised—the demonstrations have been harmonized, and, in many instances, abbreviated—the principal object being to simplify the subject as much as possible, without departing from the general plan. These changes are due to Professor Peck, of the Department of Pure Mathematics

and Astronomy in Columbia College. For his aid, in giving to the work its present permanent form, I tender him my grateful acknowledgments.

The edition of **Legendre**, referred to in the last paragraph, will not be altered in form or substance; and yet, Geometry must be made a more practical science. To attain this object, without deranging a system so long used, and so generally approved, an Appendix has been prepared and added to **Legendre**, embracing many Problems of Geometrical construction, and many applications of Algebra to Geometry.

It would be unjust to those giving instruction, to add to their daily labors, the additional one, of finding appropriate solutions to so many difficult problems: hence, a Key has been made for the use of Teachers, in which the best methods of construction and solution are fully given.

CHARLES DAVIES.

FISHKILL-ON-HUDSON, *June*, 1875.

NOTE. — The edition of **Legendre** referred to in the foregoing preface was prepared by the late Professor Davies the year before his lamented death. The present edition is the result of a careful re-examination of the work, into which have been incorporated such emendations, in the way of greater clearness of expression or of proof, as could be made without altering it in form or substance.

Practical exercises have been placed at the end of the several books, and comprise additional theorems, problems, and numerical exercises upon the principles of the Book or Books preceding. They will, it is hoped, be found of service in accustoming students, early in and throughout their course, to make for themselves practical application of geometric principles, and constitute, in addition, a large body of review and test questions for the convenience of teachers.

The Trigonometry has been carefully revised throughout, to simplify the discussions and to make the treatment conform in every particular to the latest and best methods.

It is believed that in clearness and precision of definition, in general simplicity and rigor of demonstration, in orderly and logical development of the subject, and in compactness of form, **Davies' Legendre** is superior to any work of its grade for the general training of the logical powers of pupils, and for their instruction in the great body of elementary geometric truth.

J. H. VAN AMRINGE,
Editor of Davies' Course of Mathematics.

COLUMBIA COLLEGE, N. Y., *June*, 1885.

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ELEMENTS
OF
GEOMETRY.

INTRODUCTION.

DEFINITIONS OF TERMS.

1. QUANTITY is any thing which can be increased, diminished, and measured.

To measure a thing, is to find out how many times it contains some other thing, of the same kind, taken as a standard. The assumed standard is called the *unit of measure.*

2. In GEOMETRY, there are four species of quantity, viz.: LINES, SURFACES, VOLUMES, and ANGLES. These are called GEOMETRICAL MAGNITUDES.

Since the unit of measure of a quantity is of the same kind as the quantity measured, there are four kinds of units of measure, viz.: *Units of Length, Units of Surface, Units of Volume,* and *Units of Angular Measure.*

3. GEOMETRY is that branch of Mathematics which treats of the properties, relations, and measurement of the Geometrical Magnitudes.

4. In Geometry, the quantities considered are generally represented by means of the straight line and curve. The operations to be performed upon the quantities, and the relations between them, are indicated by signs, as in Analysis.

The following are the principal signs employed:

The *Sign of Addition*, $+$, called *plus:*

Thus, $A + B$, indicates that B is to be added to A.

The *Sign of Subtraction*, $-$, called *minus:*

Thus, $A - B$, indicates that B is to be subtracted from A.

The *Sign of Multiplication*, $\times$:

Thus, $A \times B$, indicates that A is to be multiplied by B.

The *Sign of Division*, $\div$:

Thus, $A \div B$, or, $\frac{A}{B}$, indicates that A is to be divided by B.

The *Exponential Sign:*

Thus, A^3, indicates that A is to be taken three times as a factor, or raised to the third power.

The *Radical Sign*, $\sqrt{\ }$:

Thus, $\sqrt{A}$, $\sqrt[3]{B}$, indicate that the square root of A, and the cube root of B, are to be taken.

When a compound quantity is to be operated upon as a single quantity, its parts are connected by a vinculum or by a parenthesis:

Thus, $\overline{A + B} \times C$, indicates that the sum of A and B is to be multiplied by C; and $(A + B) \div C$, indicates that the sum of A and B is to be divided by C.

A number written before a quantity, shows how many times it is to be taken.

Thus, $3(A + B)$, indicates that the sum of A and B is to be taken three times.

The *Sign of Equality*, $=$:

Thus, $A = B + C$, indicates that A is equal to the sum of B and C.

The expression, $A = B + C$, is called an equation. The part on the left of the sign of equality is called the *first member;* that on the right, the *second member.*

The *Sign of Inequality*, $<$:

Thus, $\sqrt{A} < \sqrt[3]{B}$, indicates that the square root of A is less than the cube root of B. The opening of the sign is towards the greater quantity.

The sign, $\therefore$ is used as an abbreviation of the word *hence,* or *consequently.*

The symbols, 1°, 2°, etc., mean 1st, 2d, etc.

5. The general truths of Geometry are deduced by a course of logical reasoning, the premises being definitions and principles previously established. The course of reasoning employed in establishing any truth or principle is called *a demonstration.*

6. A THEOREM is a truth requiring demonstration.

7. An AXIOM is a self-evident truth.

8. A PROBLEM is a question requiring solution.

9. A POSTULATE is a self-evident Problem.

Theorems, Axioms, Problems, and Postulates, are all called *Propositions.*

10. A LEMMA is an auxiliary proposition.

11. A COROLLARY is an obvious consequence of one or more propositions.

12. A SCHOLIUM is a remark made upon one or more propositions, with reference to their connection, their use, their extent, or their limitation.

13. An HYPOTHESIS is a supposition made, either in the statement of a proposition, or in the course of a demonstration.

14. Magnitudes are equal to each other, when each contains the same unit an equal number of times.

15. Magnitudes are equal *in all respects*, when they may be so placed as to coincide throughout their whole extent; they are equal *in all their parts* when each part of one is equal to the corresponding part of the other, when taken either in the same or in the reverse order.

ELEMENTS OF GEOMETRY.

BOOK I.

ELEMENTARY PRINCIPLES.

DEFINITIONS.

1. GEOMETRY is that branch of Mathematics which treats of the properties, relations, and measurements of the Geometrical Magnitudes.

2. A POINT is that which has position, but not magnitude.

3. A LINE is that which has length, but neither breadth nor thickness.

Lines are divided into two classes, *straight* and *curved.*

4. A STRAIGHT LINE is one which does not change its direction at any point.

5. A CURVED LINE is one which changes its direction at every point.

When the sense is obvious, to avoid repetition, the word *line*, alone, is commonly used for *straight line;* and the word *curve*, alone, for *curved line.*

6. A line made up of straight lines, not lying in the same direction, is called a *broken line.*

7. A SURFACE is that which has length and breadth without thickness.