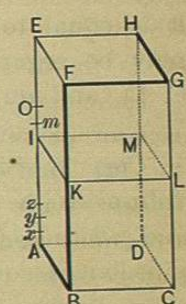
PROPOSITION XI. THEOREM.

Two rectangular parallelopipedons having a common lower base, are to each other as their altitudes.

Let the parallelopipedons AG and AL have the common lower base ABCD: then are they to each other as their altitudes AE and AI.

1°. Let the altitudes be commensurable, and suppose, for example, that AE is to AI, as 15 is to 8.

Conceive AE to be divided into 15 equal parts, of which AI contains 8; through the points of division let planes be passed parallel to ABCD. These planes divide the parallelopipedon AG into 15 parallelopipedons, which have equal bases (P. II., C.) and equal altitudes; hence, they are equal (P. X., Cor. 3).



Now, AG contains 15, and AL 8 of these equal parallelopipedons; hence, AG is to AL, as 15 is to 8, or as AE is to AI. In like manner, it may be shown that AG is to AL, as AE is to AI, when the altitudes are to each other as any other whole numbers.

2°. Let the altitudes be incommensurable.

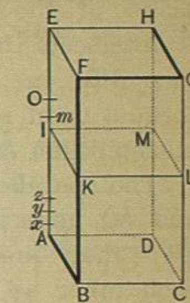
Now, if AG is not to AL, as AE is to AI, let us suppose that

$$AG : AL :: AE : AO,$$

in which AO is greater than AI.

Divide AE into equal parts, such that each is less than OI; there is at least one point of division m, between O and I. Let P denote the parallelopipedon, whose base is ABCD, and altitude Am; since the altitudes AE, Am, are to each other as two whole numbers, we have,

$$AG : P :: AE : Am.$$



But, by hypothesis, we have,

$$AG : AL :: AE : AO;$$

therefore (B. II., P. IV., C.),

$$AL : P :: AO : Am.$$

But AO is greater than Am; hence, if the proportion is true, AL must be greater than P. On the contrary, it is less; consequently, the fourth term of the proportion can not be greater than AI. In like manner, it may be shown that the fourth term can not be less than AI; it is, therefore, equal to AI. In this case, therefore, AG is to AL as AE is to AI.

Hence, in all cases, the given parallelopipedons are to each other as their altitudes; *which was to be proved.*

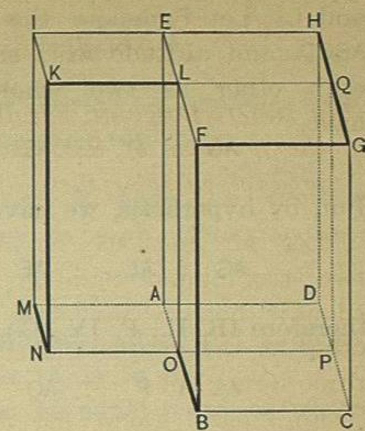
Sch. Any two rectangular parallelopipedons whose bases are equal in all respects, are to each other as their altitudes.

PROPOSITION XII. THEOREM.

Two rectangular parallelopipedons having equal altitudes, are to each other as their bases.

Let the rectangular parallelopipedons AG and AK have the same altitude AE: then are they to each other as their bases.

For, place them so that the plane angle EAO shall be common, and produce the plane of the face NL, until it intersects the plane of the face HC, in PQ; we thus form a third rectangular parallelopipedon AQ.



The parallelopipedons AG and AQ have a common base AH; they are therefore to each other as their altitudes AB and AO (P. XI.): hence, we have the proportion,

$$vol.\ AG : vol.\ AQ :: AB : AO.$$

The parallelopipedons AQ and AK have the common base AL; they are therefore to each other as their altitudes AD and AM: hence,

$$vol.\ AQ : vol.\ AK :: AD : AM.$$

Multiplying these proportions, term by term (B. II., P. XII.), and omitting the common factor, *vol.* AQ, we have,

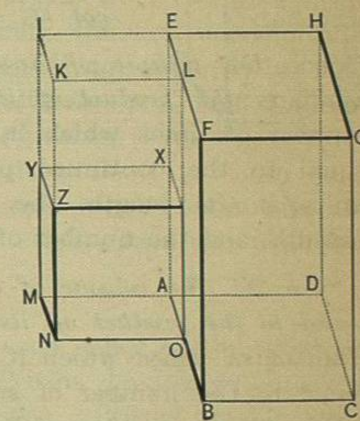
$$vol.\ AG : vol.\ AK :: AB \times AD : AO \times AM.$$

But $AB \times AD$ is equal to the area of the base ABCD, and $AO \times AM$ is equal to the area of the base AMNO: hence, two rectangular parallelopipedons having equal altitudes, are to each other as their bases; *which was to be proved.*

PROPOSITION XIII. THEOREM.

Any two rectangular parallelopipedons are to each other as the products of their bases and altitudes; that is, as the products of their three dimensions.

Let AZ and AG be any two rectangular parallelopipedons: then are they to each other as the products of their three dimensions.



For, place them so that the plane angle EAO shall be common, and produce the faces necessary to complete the rectangular parallelopipedon AK. The parallelopipedons AZ and AK have a common base AN; hence (P. XI.),

$$vol.\ AZ : vol.\ AK :: AX : AE.$$

The parallelopipedons AK and AG have a common altitude AE; hence (P. XII.),

$$vol.\ AK : vol.\ AG :: AMNO : ABCD.$$

Multiplying these proportions, term by term, and omitting the common factor, *vol.* AK, we have,

$$vol.\ AZ : vol.\ AG :: AMNO \times AX : ABCD \times AE;$$

or, since AMNO is equal to $AM \times AO$, and ABCD to $AB \times AD$,

$$vol.\ AZ : vol.\ AG :: AM \times AO \times AX : AB \times AD \times AE;$$

which was to be proved.

Cor. 1. If we make the three edges AM, AO, and AX, each equal to the linear unit, the parallelopipedon AZ becomes a cube constructed on that unit, as an edge; and consequently, it is the unit of volume. Under this supposition, the last proportion becomes,

$$1 \;:\; vol.\ AG \;::\; 1 \;:\; AB \times AD \times AE;$$

whence, $$vol.\ AG = AB \times AD \times AE.$$

Hence, *the volume of any rectangular parallelopipedon is equal to the product of its three dimensions;* that is, the number of times which it contains the unit of volume, is equal to the continued product of the number of linear units in its length, the number of linear units in its breadth, and the number of linear units in its height.

Cor. 2. *The volume of a rectangular parallelopipedon is equal to the product of its base and altitude;* that is, the number of times which it contains the unit of volume, is equal to the number of superficial units in its base, multiplied by the number of linear units in its altitude.

Cor. 3. The volume of any parallelopipedon is equal to the product of its base and altitude (P. X., C. 1).

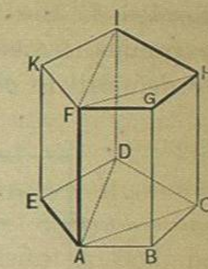
PROPOSITION XIV. THEOREM.

The volume of any prism is equal to the product of its base and altitude.

Let ABCDE–K be any prism: then is its volume equal to the product of its base and altitude.

For, through any lateral edge, as AF, and the other lateral edges not in the same faces, pass the planes AH, AI, dividing the prism into triangular prisms. These prisms all have a common altitude equal to that of the given prism.

Now, the volume of any one of the triangular prisms, as ABC–H, is equal to half that of a parallelopipedon constructed on the edges BA, BC, BG (P. VII., C.); but the volume of this parallelopipedon is equal to the product of its base and altitude (P. XIII., C. 3); and because the base of the prism is half that of the parallelopipedon, the volume of the prism is also equal to the product of its base and altitude: hence, the sum of the triangular prisms, which make up the given prism, is equal to the sum of their bases, which make up the base of the given prism, into their common altitude; *which was to be proved.*



Cor. Any two prisms are to each other as the products of their bases and altitudes. Prisms having equal bases are to each other as their altitudes. Prisms having equal altitudes are to each other as their bases.

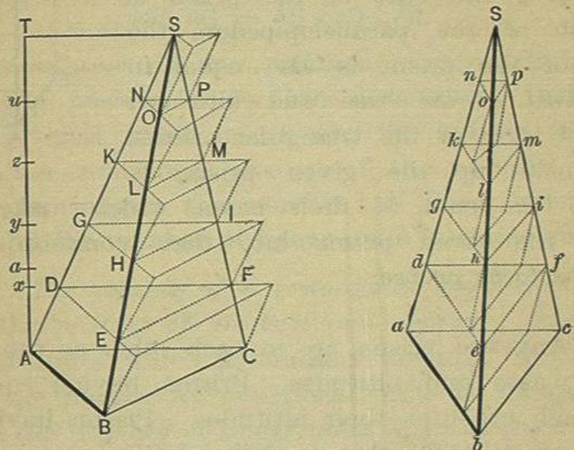
PROPOSITION XV. THEOREM.

Two triangular pyramids having equal bases and equal altitudes are equal in volume.

Let S–ABC, and S–*abc*, be two pyramids having their equal bases ABC and *abc* in the same plane, and let AT be their common altitude: then are they equal in volume.

For, if they are not equal in volume, suppose one of them, as S–ABC, to be the greater, and let their difference be equal to a prism whose base is ABC, and whose altitude is A*a*.

Divide the altitude AT into equal parts, Ax, xy, &c., each of which is less than Aa, and let k denote one of these parts; through the points of division pass planes parallel to the plane of the bases; the sections of the two pyramids, by each of these planes, are equal, namely, DEF to def, GHI to ghi, &c. (P. III., C. 2).



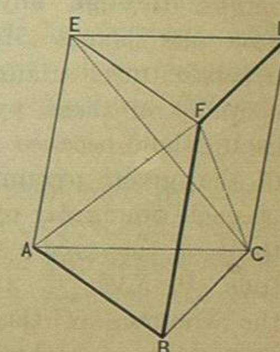
On the triangles ABC, DEF, &c., as lower bases, construct exterior prisms whose lateral edges are parallel to AS, and whose altitudes are equal to k: and on the triangles def, ghi, &c., taken as upper bases, construct interior prisms, whose lateral edges are parallel to aS, and whose altitudes are equal to k. It is evident that the sum of the exterior prisms is greater than the pyramid S–ABC, and also that the sum of the interior prisms is less than the pyramid S–abc: hence, the difference between the sum of the exterior and the sum of the interior prisms, is greater than the difference between the two pyramids.

Now, beginning at the bases, the second exterior prism EFD–G, is equal to the first interior prism efd–a, because

they have the same altitude k, and their bases EFD, efd, are equal: for a like reason, the third exterior prism HIG–K, and the second interior prism hig–d, are equal, and so on to the last in each set: hence, each of the exterior prisms, excepting the first BCA–D, has an equal corresponding interior prism; the prism BCA–D, is, therefore, the difference between the sum of all the exterior prisms, and the sum of all the interior prisms. But the difference between these two sets of prisms is greater than that between the two pyramids, which latter difference was supposed to be equal to a prism whose base is BCA, and whose altitude is equal to Aa, greater than k; consequently, the prism BCA–D is greater than a prism having the same base and a greater altitude, which is impossible: hence, the supposed inequality between the two pyramids can not exist; they are, therefore, equal in volume; *which was to be proved.*

PROPOSITION XVI. THEOREM.

Any triangular prism may be divided into three triangular pyramids, equal to each other in volume.



Let ABC–D be a triangular prism: then can it be divided into three equal triangular pyramids.

For, through the edge AC, pass the plane ACF, and through the edge EF pass the plane EFC. The pyramids ACE–F and ECD–F, have their bases ACE and ECD equal, because they are halves of the same parallelogram ACDE; and they have a common altitude, because their bases are in the same plane AD, and their vertices at the same point F; hence, they are equal in volume (P. XV.). The pyramids ABC–F and DEF–C, have their bases ABC and DEF, equal, because they are the bases of the given prism, and their altitudes are equal because each is equal to the altitude of the prism; they are, therefore, equal in volume: hence, the three pyramids into which the prism is divided, are all equal in volume; *which was to be proved.*

Cor. 1. A triangular pyramid is one third of a prism having an equal base and an equal altitude.

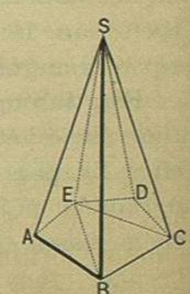
Cor. 2. The volume of a triangular pyramid is equal to one third of the product of its base and altitude.

PROPOSITION XVII. THEOREM.

The volume of any pyramid is equal to one third of the product of its base and altitude.

Let S–ABCDE, be any pyramid: then is its volume equal to one third of the product of its base and altitude.

For, through any lateral edge, as SE, pass the planes SEB, SEC, dividing the pyramid into triangular pyramids. The altitudes of these pyramids are equal to each other, because each is equal to that of the given pyramid. Now, the volume of each triangular pyramid is equal to one third of the product of its base and altitude (P. XVI., C. 2); hence, the sum of the volumes of the triangular pyramids, is equal to one third of the product of the sum of their



bases by their common altitude. But the sum of the triangular pyramids is equal to the given pyramid, and the sum of their bases is equal to the base of the given pyramid: hence, the volume of the given pyramid is equal to one third of the product of its base and altitude; *which was to be proved.*

Cor. 1. The volume of a pyramid is equal to one third of the volume of a prism having an equal base and an equal altitude.

Cor. 2. Any two pyramids are to each other as the products of their bases and altitudes. Pyramids having equal bases are to each other as their altitudes. Pyramids having equal altitudes are to each other as their bases.

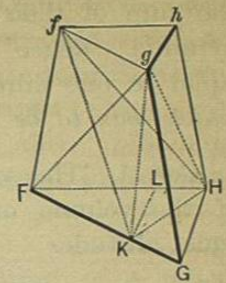
Scholium. The volume of a polyedron may be found by dividing it into triangular pyramids, and computing their volumes separately. The sum of these volumes is equal to the volume of the polyedron.

PROPOSITION XVIII. THEOREM.

The volume of a frustum of any triangular pyramid is equal to the sum of the volumes of three pyramids whose common altitude is that of the frustum, and whose bases are the lower base of the frustum, the upper base of the frustum, and a mean proportional between the two bases.

Let FGH–*h* be a frustum of any triangular pyramid: then is its volume equal to that of three pyramids whose common altitude is that of the frustum, and whose bases are the lower base FGH, the upper base *fgh*, and a mean proportional between these bases.

For, through the edge FH, pass the plane FH*g*, and through the edge fg, pass the plane fgH, dividing the frustum into three pyramids. The pyramid g–FGH, has for its base the lower base FGH of the frustum, and its altitude is equal to that of the frustum, because its vertex g is in the plane of the upper base. The pyramid H–fgh, has for its base the upper base fgh of the frustum, and its altitude is equal to that of the frustum, because its vertex lies in the plane of the lower base.

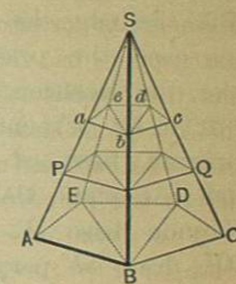


The remaining pyramid may be regarded as having the triangle FfH for its base, and the point g for its vertex. From g, draw gK parallel to fF, and draw also KH and Kf. Then the pyramids K–FfH and g–FfH, are equal; for they have a common base, and their altitudes are equal, because their vertices K and g are in a line parallel to the base (B. VI., P. XII., C. 2).

Now, the pyramid K–FfH may be regarded as having FKH for its base and f for its vertex. From K, draw KL parallel to GH; it is parallel to gh: then the triangle FKL is equal to fgh, for the side FK is equal to fg, the angle F to the angle f, and the angle K to the angle g. But, FKH is a mean proportional between FKL and FGH (B. IV., P. XXIV., C.), or between fgh and FGH. The pyramid f–FKH, has, therefore, for its base a mean proportional between the upper and lower bases of the frustum, and its altitude is equal to that of the frustum; but the pyramid f–FKH is equal in volume to the pyramid g–FfH: hence, the volume of the given frustum is equal to that of three pyramids whose common altitude is equal to that of the frustum, and whose bases are the upper base, the lower base, and a mean proportional between them; *which was to be proved.*

Cor. The volume of the frustum of any pyramid is equal to the sum of the volumes of three pyramids whose common altitude is that of the frustum, and whose bases are the lower base of the frustum, the upper base of the frustum, and a mean proportional between them.

For, let ABCDE–e be a frustum of a pyramid whose vertex is S, and let PQ be a section parallel to the bases, such that distance from S is a mean proportional between the distances from S to the two bases of the frustum. Let planes be passed through SB, and SE, SD, dividing the frustum into triangular frustums; the section of each of the triangular frustums is a mean proportional between its bases (P. III., C. 4). Now the sum of the triangular frustums is equal to the sum of three sets of pyramids, whose altitude is that of the given frustum. The sum of the bases of the first set is the lower base of the frustum, the sum of the bases of the second set is the upper base of the frustum, and the sum of the bases of the third set is a mean proportional between these bases. Hence, the sum of the partial frustums, that is, the given frustum, is equal to the sum of three pyramids having the same altitude as the given frustum, and whose bases are the two bases of the frustum and a mean proportional between them.

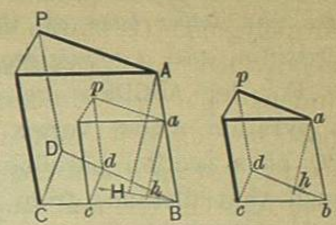


PROPOSITION XIX. THEOREM.

Similar triangular prisms are to each other as the cubes of their homologous edges.

Let CBD–P, cbd–p, be two similar triangular prisms, and let BC, bc, be any two homologous edges: then is the prism CBD–P to the prism cbd–p, as $\overline{BC}^3$ to $\overline{bc}^3$.

For, the homologous angles B and b are equal, and the faces which bound them are similar (D. 16): hence, these triedral angles may be applied, one to the other, so that the angle cbd will coincide with CBD, the edge ba with BA. In this case, the prism cbd–p will take the position Bcd–p. From A draw AH perpendicular to the common base of the prisms:



then the plane BAH is perpendicular to the plane of the common base (B. VI., P. XVI.). From a, in the plane BAH, draw ah perpendicular to BH: then ah is also perpendicular to the base BDC (B. VI., P. XVII.); and AH, ah, are the altitudes of the two prisms.

Since the bases CBD, cbd, are similar, we have (B. IV., P. XXV.),

$$\textit{base}\ \mathrm{CBD} : \textit{base}\ cbd :: \overline{\mathrm{CB}}^2 : \overline{cb}^2.$$

Now, because of the similar triangles ABH, aBh, and of the similar parallelograms AC, ac, we have,

$$\mathrm{AH} : ah :: \mathrm{CB} : cb;$$

hence, multiplying these proportions term by term, we have,

$$\textit{base}\ \mathrm{CBD} \times \mathrm{AH} : \textit{base}\ cbd \times ah : \overline{\mathrm{CB}}^3 : \overline{cb}^3.$$

But, *base* CBD × AH is equal to the volume of the prism CDB–A, and *base* $cbd \times ah$ is equal to the volume of the prism cbd–p: hence,

$$\textit{prism}\ \mathrm{CDB–P} : \textit{prism}\ cbd\text{–}p :: \overline{\mathrm{CB}}^3 : \overline{cb}^3;$$

which was to be proved.

Cor. 1. *Any two similar prisms are to each other as the cubes of their homologous edges.*

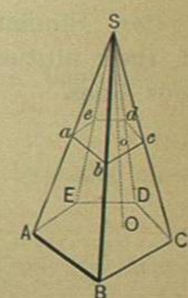
For, since the prisms are similar, their bases are similar polygons (D. 16); and these similar polygons may each be divided into the same number of similar triangles, similarly placed (B. IV., P. XXVI.); therefore, each prism may be divided into the same number of triangular prisms, having their faces similar and like placed; consequently, the triangular prisms are similar (D. 16). But these triangular prisms are to each other as the cubes of their homologous edges, and being like parts of the polygonal prisms, the polygonal prisms themselves are to each other as the cubes of their homologous edges.

Cor. 2. Similar prisms are to each other as the cubes of their altitudes, or as the cubes of any other homologous lines.

PROPOSITION XX. THEOREM.

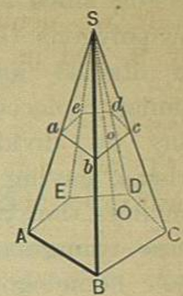
Similar pyramids are to each other as the cubes of their homologous edges.

Let S–ABCDE, and S–$abcde$, be two similar pyramids, so placed that their homologous angles at the vertex shall coincide, and let AB and ab be any two homologous edges: then are the pyramids to each other as the cubes of AB and ab.



For, the face SAB, being similar to Sab, the edge AB is parallel to the edge ab, and the face SBC being similar to Sbc, the edge BC is parallel to bc; hence, the planes of the bases are parallel (B. VI., P. XIII.).

Draw SO perpendicular to the base ABCDE; it will also be perpendicular to the base *abcde*. Let it pierce that plane at the point *o*; then SO is to S*o*, as SA is to S*a* (P. III.), or as AB is to *ab*; hence,



$$\tfrac{1}{3}\mathrm{SO} \ : \ \tfrac{1}{3}\mathrm{S}o \ :: \ \mathrm{AB} \ : \ ab.$$

But the bases being similar polygons, we have (B. IV., P. XXVII.),

$$base\ \mathrm{ABCDE} \ : \ base\ abcde \ :: \ \overline{\mathrm{AB}}^2 \ : \ \overline{ab}^2.$$

Multiplying these proportions, term, P. term, we have,

$$base\ \mathrm{ABCDE}\times\tfrac{1}{3}\mathrm{SO} \ : \ base\ abcde\times\tfrac{1}{3}\mathrm{S}o \ :: \ \overline{\mathrm{AB}}^3 \ : \ \overline{ab}^3.$$

But, *base* ABCDE $\times \frac{1}{3}$SO is equal to the volume of the pyramid S–ABCDE, and *base abcde* $\times \frac{1}{3}$S*o* is equal to the volume of the pyramid S–*abcde*; hence,

$$pyramid\ \mathrm{S{-}ABCDE} \ : \ pyramid\ \mathrm{S{-}}abcde \ :: \ \overline{\mathrm{AB}}^3 \ : \ \overline{ab}^3;$$

which was to be proved.

Cor. Similar pyramids are to each other as the cubes of their altitudes, or as the cubes of any other homologous lines.

GENERAL FORMULAS.

If we denote the volume of any prism by V, its base by B, and its altitude by H, we shall have (P. XIV.),

$$\mathrm{V} = \mathrm{B}\times\mathrm{H} \quad \cdots\cdots \quad (1.)$$

If we denote the volume of any pyramid by V, its base by B, and its altitude by H, we have (P. XVII.),

$$\mathrm{V} = \mathrm{B}\times\tfrac{1}{3}\mathrm{H} \quad \cdots\cdots \quad (2.)$$

If we denote the volume of the frustum of any pyramid by V, its lower base by B, its upper base by *b*, and its altitude by H, we shall have (P. XVIII., C.),

$$\mathrm{V} = (\mathrm{B} + b + \sqrt{\mathrm{B}\times b})\times\tfrac{1}{3}\mathrm{H} \quad \cdots \quad (3.)$$

REGULAR POLYEDRONS.

A REGULAR POLYEDRON is one whose faces are all equal regular polygons, and whose polyedral angles are equal, each to each.

There are five regular polyedrons, namely:

1. The TETRAEDRON, or *regular pyramid*—a polyedron bounded by four equal equilateral triangles.

2. The HEXAEDRON, or *cube*—a polyedron bounded by six equal squares.

3. The OCTAEDRON—a polyedron bounded by eight equal equilateral triangles.

4. The DODECAEDRON—a polyedron bounded by twelve equal and regular pentagons.

5. The Icosaedron—a polyedron bounded by twenty equal equilateral triangles.

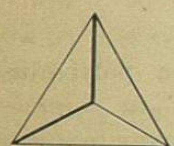
In the Tetraedron, the triangles are grouped about the polyedral angles in sets of three, in the Octaedron they are grouped in sets of four, and in the Icosaedron they are grouped in sets of five. Now, a greater number of equilateral triangles can not be grouped so as to form a salient polyedral angle; for, if they could, the sum of the plane angles formed by the edges would be equal to, or greater than, four right angles, which is impossible (B. VI., P. XX.).

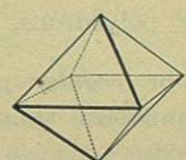
In the Hexaedron, the squares are grouped about the polyedral angles in sets of three. Now, a greater number of squares can not be grouped so as to form a salient polyedral angle; for the same reason as before.

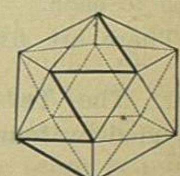
In the Dodecaedron, the regular pentagons are grouped about the polyedral angles in sets of three, and for the same reason as before, they can not be grouped in any greater number so as to form a salient polyedral angle.

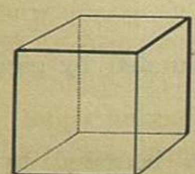
Furthermore, no other regular polygons can be grouped so as to form a salient polyedral angle; therefore,

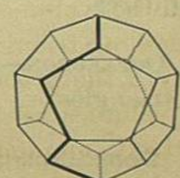
Only five regular polyedrons can be formed.


Tetraedron


Octaedron


Icosaedron


Hexaedron


Dodecaedron

EXERCISES.

1. What is the convex surface of a right prism whose altitude is 20 feet and whose base is a pentagon each side of which is 15 feet?

2. The altitude of a pyramid is 10 feet and the area of its base 25 square feet; find the area of a section made by a plane 6 feet from the vertex and parallel to the base.

3. Find the convex surface of a right triangular pyramid, each side of the base being 4 feet and the slant height 12 feet.

4. A right pyramid whose altitude is 8 feet and whose base is a square each side of which is 4 feet, is cut by a plane parallel to the base and 2 feet from the vertex; required the convex surface of the frustum included between the base and the cutting plane.

5. The three concurrent edges of a rectangular parallelopipedon are 4, 6, and 8 feet; find the length of the diagonal.

6. Of two rectangular parallelopipedons having equal bases, the altitude of the first is 12 feet and its volume is 275 cubic feet; the altitude of the second is 8 feet—find its volume.

7. Two rectangular parallelopipedons having equal altitudes are respectively 80 and 45 cubic feet in volume, and the area of the base of the first is 12 square feet; find the base of the second and the altitude of both.

8. Find the volume of a triangular prism whose base is an equilateral triangle of which the altitude is 3 feet, the altitude of the prism being 8 feet.

9. The volumes of two pyramids having equal altitudes are respectively 60 and 115 cubic yards and the base of the smaller is 8 square yards; find the base of the larger.

10. Given a pyramid whose volume is 512 cubic feet and altitude 8 feet; find the volume of a similar pyramid whose altitude is 12 feet, and find also the area of the base of each.

11. Find the volume of the frustum of a right triangular pyramid with each side of the lower base 6 feet and each side of the upper base 4 feet, the altitude being 5 feet.

12. Find the volume of the pyramid of which the frustum given in the last example is a frustum.

[Find the radii of the inscribed circles of the upper and lower bases (B. IV., P. VI., C. 2); then the altitude of the pyramid, slant height, and the two radii form two similar triangles from which the altitude may be found.]

13. Given two similar prisms; the base of the first contains 30 square yards and its altitude is 8 yards; the altitude of the second prism is 6 yards—find its volume and the area of its base.

14. A pyramid, whose base is a regular pentagon of which the apothem is 3.5 feet, contains 129 cubic feet; find the volume of a similar pyramid, the apothem of whose base is 4 feet.

15. Show that the four diagonals of a parallelopipedon bisect each other in a common point.

16. Show that the two lines joining the points of the opposite faces of a parallelopipedon, in which the diagonals of those faces intersect, bisect each other at the point in which the diagonals of the parallelopipedon intersect.

17. Show that two regular polyedrons of the same kind are similar.

18. Show that the surfaces of any two similar polyedrons are to each other as the squares of any two homologous edges

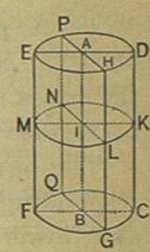
BOOK VIII.

THE CYLINDER, THE CONE, AND THE SPHERE.

DEFINITIONS.

1. A CYLINDER is a volume which may be generated by a rectangle revolving about one of its sides as an *axis*.

Thus, if the rectangle ABCD be turned about the side AB, as an axis, it will generate the cylinder FGCQ–P.



The fixed line AB is called *the axis of the cylinder;* the curved surface generated by the side CD, opposite the axis, is called *the convex surface of the cylinder;* the equal circles FGCQ, and EHDP, generated by the remaining sides BC and AD, are called *bases of the cylinder;* and the perpendicular distance between the planes of the bases is called *the altitude of the cylinder.*

The line DC, which generates the convex surface, is, in any position, called an *element of the surface;* the elements are all perpendicular to the planes of the bases, and any one of them is equal to the altitude of the cylinder.

Any line of the generating rectangle ABCD, as IK, which is perpendicular to the axis, will generate a circle whose plane is perpendicular to the axis, and which is equal to either base: hence, any section of a cylinder by a plane perpendicular to the axis, is a circle equal to either base. Any section, FCDE, made by a plane through the axis, is a rectangle double the generating rectangle.