As in Case II.,

$$\log a = \log c + 10 - \log \sin C;$$

$\log c$ . . .	(15) . .	1.176091	
(a. c.) $\log \sin C$	(29° 58′ 54″)	0.301271	
$\log a$		1.477362	$\therefore a = 30.017.$

Ans. $C = 29° 58' 54''$, $B = 60° 01' 06''$, and $a = 30.017$.

2. Given $b = 1052$ yds., and $c = 347.21$ yds., to find B, C, and a.

$B = 71° 44' 05''$, $C = 18° 15' 55''$, and $a = 1107.82$ yds.

3. Given $b = 122.416$, and $c = 118.297$, to find B, C, and a.

$B = 45° 58' 50''$, $C = 44° 1' 10''$, and $a = 170.235$.

4. Given $b = 103$, and $c = 101$, to find B, C, and a.

$B = 45° 33' 42''$, $C = 44° 26' 18''$, and $a = 144.256$.

CASE IV.

Given the hypothenuse and either side about the right angle, to find the remaining parts.

41. The angle at the base may be found by one of formulas (10) and 11), and the remaining side may then be found by one of formulas (7) and (8).

Examples.

1. Given $a = 2391.76$, and $b = 385.7$, to find C, B, and c.

Operation.

Applying logarithms to formula (11), we have

$$\log \cos C = \log b + 10 - \log a;$$

$\log b$	(385.7) . . .	2.586250	
(a. c.) $\log a$	(2391.76) . .	6.621282	
$\log \cos C$	. . .	9.207532	$\therefore C = 80° 43' 11'';$

$$B = 90° - 80° 43' 11'' = 9° 16' 49''.$$

From formula (7), we have

$$\log c = \log a + \log \sin C - 10;$$

$\log a$	(2391.76) .	3.378718	
$\log \sin C$	(80° 43′ 11″)	9.994278	
$\log c$	. . . . .	3.372996	$\therefore c = 2360.45.$

Ans. $B = 9° 16' 49''$, $C = 80° 43' 11''$, and $c = 2360.45$.

2. Given $a = 127.174$ yds., and $c = 125.7$ yds., to find C, B, and b.

Operation.

From formula (10), we have

$$\log \sin C = \log c + 10 - \log a;$$

$\log c$	(125.7) . . .	2.099335	
(a. c.) $\log a$	(127.174) . .	7.895602	
$\log \sin C$	. . .	9.994937	$\therefore C = 81° 16' 6'';$

$$B = 90° - 81° 16' 6'' = 8° 43' 54''.$$

From formula (8), we have

$$\log b = \log a + \log \cos C - 10;$$

$\log a$	(127.174) . .	2.104398	
$\log \cos C$	(81° 16′ 6″) .	9.181292	
$\log b$	. . . . . .	1.285690	$\therefore b = 19.3.$

Ans. $B = 8° 43' 54''$, $C = 81° 16' 6''$, and $b = 19.3$ yds.

3. Given $a = 100$, and $b = 60$, to find B, C, and c.
Ans. $B = 36° 52' 11''$, $C = 53° 7' 49''$, and $c = 80$.

4. Given $a = 19.209$, and $c = 15$, to find B, C, and b.
Ans. $B = 38° 39' 30''$, $C = 51° 20' 30''$, $b = 12$.

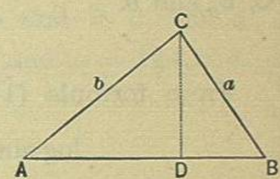
SOLUTION OF OBLIQUE-ANGLED TRIANGLES.

42. In the solution of oblique-angled triangles, *four* cases may arise. We shall discuss these cases in order.

CASE I.

Given one side and two angles, to determine the remaining parts.

43. Let ABC represent any oblique-angled triangle. From the vertex C, draw CD perpendicular to the base, forming two right-angled triangles ACD and BCD. Assume the notation of the figure.



From formula (1), we have

$$CD = b \sin A,$$

$$CD = a \sin B.$$

Equating these two values, we have,

$$b \sin A = a \sin B;$$

whence (B. II., P. II.),

$$a : b :: \sin A : \sin B. \quad \cdot \cdot \cdot \quad (13.)$$

Since a and b are any two sides, and A and B the angles lying opposite to them, we have the following principle:

The sides of a plane triangle are proportional to the sines of their opposite angles.

It is to be observed that formula (13) is true for any value of the radius. Hence, to solve a triangle, when a side and two angles are given:

First find the third angle, by subtracting the sum of the given angles from 180°; then find each of the required sides by means of the principle just demonstrated.

Examples.

1. Given $B = 58° 07'$, $C = 22° 37'$, and $a = 408$, to find A, b, and c.

Operation.

$$\begin{array}{ll} B \cdot \cdot \cdot \cdot \cdot \cdot \cdot & 58° 07' \\ C \cdot \cdot \cdot \cdot \cdot \cdot \cdot & \underline{22° 37'} \\ A \cdot \cdot \cdot \; 180° - & 80° 44' = 99° 16'. \end{array}$$

To find b, write the proportion,

$$\sin A : \sin B :: a : b;$$

that is, *the sine of the angle opposite the given side, is to the sine of the angle opposite the required side, as the given side is to the required side.*

Applying logarithms, we have (Ex. 4, P. 15)

$$\log b = \text{(a. c.)} \log \sin A + \log \sin B + \log a - 10;$$

$$\begin{array}{lll} \text{(a. c.)} \log \sin A & (99° 16') \cdot \cdot \cdot & 0.005705 \\ \log \sin B & (58° 07') \cdot \cdot \cdot & 9.928972 \\ \log a & (408) \cdot \cdot \cdot \cdot & \underline{2.610660} \\ \log b & \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot & \underline{2.545337} \quad \therefore b = 351.024. \end{array}$$

In like manner,

$$\sin A : \sin C :: a : c;$$

and $\log c = \text{(a. c.)} \log \sin A + \log \sin C + \log a - 10$;

(a. c.) log sin A	(99° 16′)	0.005705	
log sin C	(22° 37′)	9.584968	
log a	(408)	2.610660	
log c		2.201333	∴ $c = 158.976$.

Ans. $A = 99° 16'$, $b = 351.024$, and $c = 158.976$.

2. Given $A = 38° 25'$, $B = 57° 42'$, and $c = 400$, to find C, a, and b.

Ans. $C = 83° 53'$, $a = 249.974$, $b = 340.04$.

3. Given $A = 15° 19' 51''$, $C = 72° 44' 05''$, and $c =$ 250.4 yds., to find B, a, and b.

Ans. $B = 91° 56' 04''$, $a = 69.328$ yds., $b = 262.066$ yds.

4. Given $B = 51° 15' 35''$, $C = 37° 21' 25''$, and $a =$ 305.296 ft., to find A, b, and c.

Ans. $A = 91° 23'$, $b = 238.1978$ ft., $c = 185.3$ ft.

CASE II.

Given two sides and an angle opposite one of them, to find the remaining parts.

44. The solution, in this case, is commenced by finding a second angle by means of formula (13), after which we may proceed as in Case I.; or, the solution may be completed by a continued application of formula (13).

Examples.

1. Given $A = 22° 37'$, $b = 216$, and $a = 117$, to find B, C, and c.

From formula (13), we have

$$a : b :: \sin A : \sin B;$$

that is, *the side opposite the given angle, is to the side opposite the required angle, as the sine of the given angle is to the sine of the required angle.*

Whence, by the application of logarithms,

$$\log \sin B = \text{(a. c.)} \log a + \log b + \log \sin A - 10;$$

(a. c.) log a	(117)	7.931814	
log b	(216)	2.334454	
log sin A	(22° 37′)	9.584968	
log sin B		9.851236	∴ $B = 45° 13' 55''$,
			and $B' = 134° 46' 05''$.

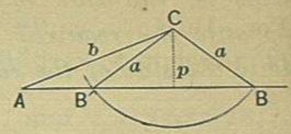
Hence, we find two values of B, which are supplements of each other, because the sine of any angle is equal to the sine of its supplement. This would seem to indicate that the problem admits of two solutions. It now remains to determine under what conditions there will be *two solutions, one solution,* or *no solution.*

There may be two cases: the given angle may be *acute,* or it may be *obtuse.*

Represent the given parts of the triangle by A, a, b. The particular letters employed are of no consequence in the discussion, and, therefore, in the results, C or B may be substituted for A, provided that, at the same time, like changes are made in the corresponding small letters.

1st Case: $A < 90°$.

Let ABC represent the triangle, in which the angle A, and the sides a and b are given. From C let fall a perpendicular upon AB, prolonged if necessary, and denote its length by p. We shall have, from formula (1), Art. 37,

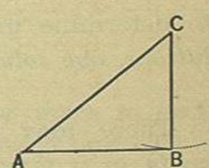


$$p = \frac{b \sin A}{R};$$

from which the value of p may be computed.

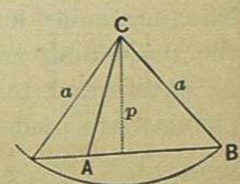
If a is greater than p and less than b, there will be *two solutions*. For, if with C as a centre, and a as a radius, an arc be described, it will cut the line AB in two points, B and B′, each of which being joined with C, will give a triangle, and we shall thus have two triangles, ABC and AB′C, which will conform to the conditions of the problem.

In this case, the angles B′ and B, of the two triangles AB′C and ABC, will be supplements of each other.



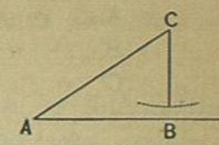
If $a = p$, there will be but *one solution*. For, in this case, the arc will be tangent to AB, the two points B and B′ will unite, and there will be but one triangle formed.

In this case, the angle ABC will be equal to 90°.



If a is greater than both p and b, there will also be but one solution. For, although the arc cuts AB in two points, and consequently gives two triangles, only one of them, ABC, conforms to the conditions of the problem.

In this case, the angle ABC will be less than A and consequently acute.

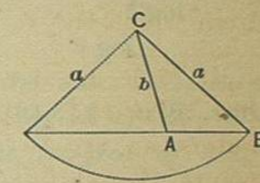


If $a < p$, there will be *no solution*. For, the arc can neither cut AB nor be tangent to it.

2d Case: $A > 90°$.

When the given angle A is obtuse, the angle ABC will be acute; the side a will be greater than b, and there will be but *one solution*.

(See B. III., Prob. XI., S.)



In the example under consideration, there are two solutions, the first corresponding to $B = 45°\ 13'\ 55''$, and the second to $B' = 134°\ 46'\ 05''$.

In the first case, we have

A	$22°\ 37'$	
B	$\underline{45°\ 13'\ 55''}$	
C . . . . 180° −	$\underline{67°\ 50'\ 55''}$	$= 112°\ 09'\ 05''$.

To find c, we have

$$\sin B \ : \ \sin C \ :: \ b \ : \ c;$$

and $\log c = (\text{a. c.}) \log \sin B + \log \sin C + \log b - 10$;

(a. c.) log sin B (45° 13′ 55″) .	0.148764	
log sin C (112° 09′ 05″) .	9.966700	
log b . (216)	$\underline{2.334454}$	
log c	$\underline{2.449918}$	$\therefore c = 281.785$.

Ans. $B = 45°\ 13'\ 55''$, $C = 112°\ 09'\ 05''$, and $c = 281.785$.

In the second case, we have,

A		$22° 37'$	
B′		$\underline{134° 46' 05''}$	
C′ . . .	$180° -$	$\underline{157° 23' 05''}$	$= 22° 36' 55''$;

and as before,

(a. c.) log sin B′	$(134° 46' 05'')$.	0.148764
log sin C′	$(22° 36' 55'')$.	9.584943
log b . . .	(216) . .	$\underline{2.334454}$
log c'		$\underline{2.068161}$ $\therefore c' = 116.993.$

Ans. $B' = 134° 46' 05''$, $C' = 22° 36' 55''$, and $c' = 116.993$.

2. Given $A = 32°$, $a = 40$, and $b = 50$, to find B, C, and c.

$$Ans. \begin{cases} B = 41° 28' 59'', & C = 106° 31' 01'', & c = 72.368. \\ B' = 138° 31' 01'', & C' = 9° 28' 59'', & c' = 12.436. \end{cases}$$

3. Given $B = 18° 52' 13''$, $b = 27.465$ yds., and $a = 13.189$ yds., to find A, C, and c.

Ans. $A = 8° 56' 05''$, $C = 152° 11' 42''$, $c = 39.611$ yds.

4. Given $C = 32° 15' 26''$, $b = 176.21$ ft., and $c = 94.047$ ft., to find B, A, and a.

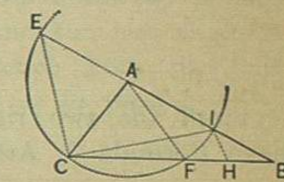
Ans. $B = 90°$, $A = 57° 44' 34''$, $a = 149.014$ ft.

CASE III.

Given two sides and their included angle, to find the remaining parts.

45. The solution, in this case, is begun by finding the half sum and the half difference of the two required angles. The half sum of these angles may be found by subtracting the given angle from 180°, and dividing the remainder by 2; the half difference may be found by means of the following principle, now to be demonstrated, viz.:

In any plane triangle, the sum of the sides including any angle, is to their difference, as the tangent of half the sum of the two other angles, is to the tangent of half their difference.



Let ABC represent any plane triangle, c and b any two sides, and A their included angle. Then we are to show that

$$c + b \ : \ c - b \ :: \ \tan \tfrac{1}{2}(C + B) \ : \ \tan \tfrac{1}{2}(C - B).$$

With A as a centre, and b, the shorter of the two sides, as a radius, describe a semicircle meeting AB in I, and the prolongation of AB in E. Draw EC and CI, and through I draw IH parallel to EC. Since the angle ECI is inscribed in a semicircle, it is a right angle (B. III., P. XVIII., C. 2); hence, EC is perpendicular to CI, at the point C; and since IH is parallel to EC, it is also perpendicular to CI.

The inscribed angle CIE is half the angle at the centre, CAE, intercepting the same arc CE. Since the angle CAE is exterior to the triangle ABC, we have (B. I., P. XXV., C. 6),

$$\text{CAE} = \text{C} + \text{B};$$

hence, $\text{CIE} = \frac{1}{2}(\text{C} + \text{B}).$

AC and AF, being radii of the same circle, are equal to each other, and therefore (B. I., P. XI.), the angle AFC is equal to the angle C; but the angle AFC is exterior to the triangle FBA, and hence we have

$$\text{AFC} \text{ or } \text{C} = \text{FAB} + \text{B};$$

hence, $\text{FAB} = \text{C} - \text{B}.$

But the inscribed angle, ICH, is half the angle at the centre, FAB, intercepting the same arc FI; hence,

$$\text{ICH} = \tfrac{1}{2}(\text{C} - \text{B}).$$

From the two right-angled triangles ICE and ICH, we have (formula 3, Art. 37),

$$\begin{aligned} \text{EC} &= \text{IC} \tan \text{CIE} \\ &= \text{IC} \tan \tfrac{1}{2}(\text{C} + \text{B}), \end{aligned}$$

and

$$\begin{aligned} \text{IH} &= \text{IC} \tan \text{ICH} \\ &= \text{IC} \tan \tfrac{1}{2}(\text{C} - \text{B}); \end{aligned}$$

hence, we have, after omitting the equal factor IC (B. II., P. VII.),

$$\text{EC} : \text{IH} :: \tan \tfrac{1}{2}(\text{C} + \text{B}) : \tan \tfrac{1}{2}(\text{C} - \text{B}).$$

The triangles ECB and IHB being similar (B. IV., P. XXI.),

$$\text{EC} : \text{IH} :: \text{EB} : \text{IB},$$

or, since $\text{EB} = c + b,$

and $\text{IB} = c - b,$

$$\text{EC} : \text{IH} :: c + b : c - b.$$

Combining the preceding proportions, we have

$$c + b : c - b :: \tan \tfrac{1}{2}(\text{C} + \text{B}) : \tan \tfrac{1}{2}(\text{C} - \text{B}); \quad (14.)$$

which was to be proved.

By means of (14), the half difference of the two required angles may be found. Knowing the half sum and the half difference, the greater angle is found by adding the half difference to the half sum, and the less angle is found by subtracting the half difference from the half sum. Then the solution is completed as in Case I.

Examples.

1. Given $c = 540$, $b = 450$, and $\text{A} = 80°$, to find B, C, and a.

Operation.

$$c + b = 990;$$

$$c - b = \ \ 90;$$

$$\begin{aligned} \tfrac{1}{2}(\text{C} + \text{B}) &= \tfrac{1}{2}(180° - 80°) \\ &= 50°. \end{aligned}$$

Applying logarithms to formula (14), we have

$$\log\tan\tfrac{1}{2}(C-B) = (\text{a. c.})\log(c+b) + \log(c-b) + \log\tan\tfrac{1}{2}(C+B) - 10;$$

(a. c.) $\log(c+b)$ · ·	(990)	7.004365
$\log(c-b)$ · ·	(90)	1.954243
$\log\tan\frac{1}{2}(C+B)$	(50°)	10.076187
$\log\tan\frac{1}{2}(C-B)$		9.034795 $\therefore \frac{1}{2}(C-B) = 6° 11'$;

$$C = 50° + 6° 11' = 56° 11';$$

$$B = 50° - 6° 11' = 43° 49'.$$

From formula (13), we have

$$\sin C : \sin A :: c : a;$$

whence,

(a. c.) $\log \sin C$	(56° 11′) ·	0.080492
$\log \sin A$	(80°) · ·	9.993351
$\log c$ ·	(540) · ·	2.732394
$\log a$ · · · · · ·		2.806237 $\therefore a = 640.082$.

Ans. $B = 43° 49'$, $C = 56° 11'$, $a = 640.082$.

2. Given $c = 1686$ yds., $b = 960$ yds., and $A = 128° 04'$, to find B, C, and a.

Ans. $B = 18° 21' 21''$, $C = 33° 34' 39''$, $a = 2400$ yds.

3. Given $a = 18.739$ yds., $c = 7.642$ yds., and $B = 45° 18' 28''$, to find A, b, and C.

Ans. $A = 112° 34' 13''$, $C = 22° 07' 19''$, $b = 14.426$ yds.

4. Given $a = 464.7$ yds., $b = 289.3$ yds., and $C = 87° 03' 48''$, to find A, B, and c.

Ans. $A = 60° 13' 39''$, $B = 32° 42' 33''$, $c = 534.66$ yds.

5. Given $a = 16.9584$ ft., $b = 11.9613$ ft., and $C = 60° 43' 36''$, to find A, B, and c.

Ans. $A = 76° 04' 12''$, $B = 43° 12' 12''$, $c = 15.22$ ft.

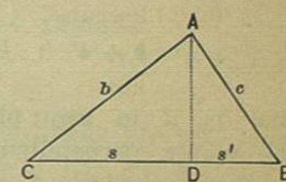
6. Given $a = 3754$, $b = 3277.628$, and $C = 57° 53' 17''$, to find A, B, and c.

Ans. $A = 68° 02' 25''$, $B = 54° 04' 18''$, $c = 3428.512$.

CASE IV.

*Given the three sides of a triangle, to find the remaining parts.**

46. Let ABC represent any plane triangle, of which BC is the longest side. Draw AD perpendicular to the base, dividing it into two segments CD and BD.



[The longest side is taken as the base, to make it certain that the perpendicular from the vertex shall fall on the base, and *not* on the base *produced.*]

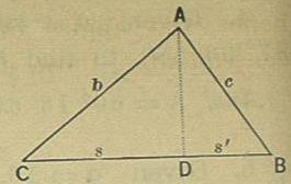
From the right-angled triangles CAD and BAD, we have

$$\overline{AD}^2 = \overline{AC}^2 - \overline{DC}^2,$$

and

$$\overline{AD}^2 = \overline{AB}^2 - \overline{BD}^2.$$

* The angles may be found by formula (A) or (B), Lemma, Art. 97, Mensuration.



Equating these values of $\overline{AD}^2$, we have,

$$\overline{AC}^2 - \overline{DC}^2 = \overline{AB}^2 - \overline{BD}^2;$$

whence, by transposition,

$$\overline{AC}^2 - \overline{AB}^2 = \overline{DC}^2 - \overline{BD}^2.$$

Hence (B. IV., P. X), we have

$$(AC + AB)(AC - AB) = (DC + BD)(DC - BD).$$

Converting this equation into a proportion (B. II., P. II.), we have

$$DC + BD : AC + AB :: AC - AB : DC - BD;$$

or, denoting the greater segment by s and the less segment by s', and the sides of the triangle by a, b, and c,

$$s + s' : b + c :: b - c : s - s'; \quad \cdot \quad (15.)$$

that is, if in any plane triangle, a line be drawn from the vertex perpendicular to the base, dividing it into two segments; then,

The sum of the two segments, or the whole base, is to the sum of the two other sides, as the difference of these sides is to the difference of the segments.

The half difference of the segments added to the half sum gives the greater segment, and the half difference subtracted from the half sum gives the less segment. [The greater segment is, of course, adjacent to the greater side.] We shall then have two right-angled triangles, in each of which we know the hypothenuse and the base; hence, the angles of these triangles may be found, and consequently, those of the given triangle.

Examples.

1. Given $a = 40$, $b = 34$, and $c = 25$, to find A, B, and C.

Operation.

Applying logarithms to formula (15), we have

$$\log(s - s') = (\text{a. c.}) \log(s + s') + \log(b + c) + \log(b - c) - 10;$$

$$\begin{array}{llll}
(\text{a. c.}) \log(s + s') & \cdot\ \cdot\ (40) & \cdot\ \cdot & 8.397940 \\
\log(b + c) & \cdot\ \cdot\ (59) & \cdot\ \cdot & 1.770852 \\
\log(b - c) & \cdot\ \cdot\ (9) & \cdot\ \cdot & \underline{0.954243} \\
\log(s - s') & \cdot\ \cdot\ \cdot\ \cdot & \cdot & \underline{1.123035} \;\therefore\; s - s' = 13.275.
\end{array}$$

$$s = \tfrac{1}{2}(s + s') + \tfrac{1}{2}(s - s') = 26.6375.$$

$$s' = \tfrac{1}{2}(s + s') - \tfrac{1}{2}(s - s') = 13.3625.$$

From formula (11), we find

$$\begin{array}{lll}
\log \cos C = \log s + (\text{a. c.}) \log b & \therefore & C = 38^\circ\ 25'\ 20'', \text{ and} \\
\log \cos B = \log s' + (\text{a. c.}) \log c & \therefore & B = \underline{57^\circ\ 41'\ 25''} \\
 & & \quad\ \ \underline{96^\circ\ 06'\ 45''}
\end{array}$$

$$A = 180^\circ - 96^\circ\ 06'\ 45'' = 83^\circ\ 53'\ 15''.$$

2. Given $a = 6$, $b = 5$, and $c = 4$, to find A, B and C.

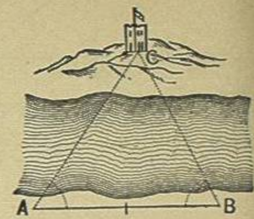
Ans. $A = 82^\circ\ 49'\ 09''$, $B = 55^\circ\ 46'\ 16''$, $C = 41^\circ\ 24'\ 35''$.

3. Given $a = 71.2$ yds., $b = 64.8$ yds., and $c = 37$ yds., to find A, B, and C.

Ans. $A = 84^\circ\ 01'\ 53''$, $B = 64^\circ\ 50'\ 51''$, $C = 31^\circ\ 07'\ 16''$.

PROBLEMS.

1. Knowing the distance AB, equal to 600 yards, and the angles BAC = 57° 35′, ABC = 64° 51′, find the two distances AC and BC.

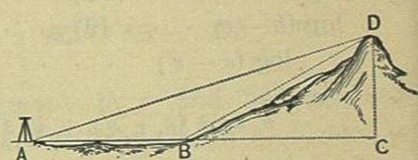


Ans. $\begin{cases} AC = 643.49 \text{ yds.,} \\ BC = 600.11 \text{ yds.} \end{cases}$

2. At what horizontal distance from a column, 200 feet high, will it subtend an angle of 31° 17′ 12″?

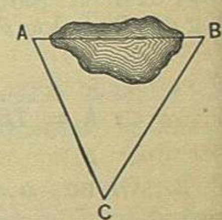
Ans. 329.114 ft.

3. Required the height of a hill D above a horizontal plane AB, the distance between A and B being equal to 975 yards, and the angles of elevation at A and B being respectively 15° 36′ and 27° 29′.



Ans. DC = 587.61 yds.

4. The distances AC and BC are found by measurement to be respectively, 588 feet and 672 feet, and their included angle 55° 40′. Required the distance AB.

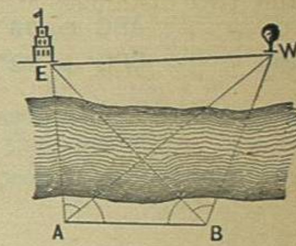


Ans. 592.967 ft.

5. Being on a horizontal plane, and wanting to ascertain the height of a tower, standing on the top of an inaccessible hill, there were measured, the angle of elevation of the top of the hill 40°, and of the top of the tower 51°; then measuring in a direct line 180 feet farther from the hill, the angle of elevation of the top of the tower was 33° 45′; required the height of the tower.

Ans. 83.998 ft.

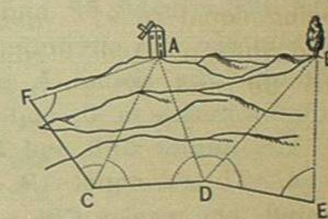
6. Wanting to know the horizontal distance between two inaccessible objects E and W, the following measurements were made:



viz.: $\begin{cases} AB = 536 \text{ yards} \\ BAW = 40° \ 16' \\ WAE = 57° \ 40' \\ ABE = 42° \ 22' \\ EBW = 71° \ 07' \end{cases}$

Required the distance EW. *Ans.* 939.617 yds.

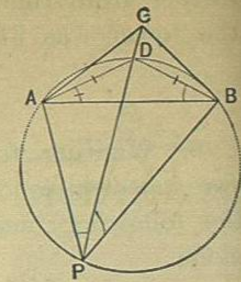
7. Wanting to know the horizontal distance between two inaccessible objects A and B, and not finding any station from which both of them could be seen, two points C and D were chosen at a distance from each other equal to 200 yards; from the former of these points, A could be seen, and from the latter, B; and at each of the points C and D a staff was set up. From C a distance CF was measured, not in the direction DC, equal to 200 yards, and from D, a distance DE equal to 200 yards, and the following angles taken:



AFC = 83° 00′, BDE = 54° 30′, ACD = 53° 30′,

BDC = 156° 25′, ACF = 54° 31′, BED = 88° 30′.

Required the distance AB. *Ans.* 345.459 yds.



8. The distances AB, AC, and BC, between the points A, B, and C, are known; viz.: AB = 800 yds., AC = 600 yds., and BC = 400 yds. From a fourth point P, the angles APC and BPC are measured; viz.:

APC = 33° 45′,

and BPC = 22° 30′.

Required the distances AP, BP, and CP.

Ans. $\begin{cases} AP = 710.198 \text{ yds.} \\ BP = 934.289 \text{ yds.} \\ CP = 1042.524 \text{ yds.} \end{cases}$

This problem is used in locating the position of buoys in maritime surveying, as follows. Three points, A, B, and C, on shore are known in position. The surveyor stationed at a buoy P, measures the angles APC and BPC. The distances AP, BP, and CP, are then found as follows:

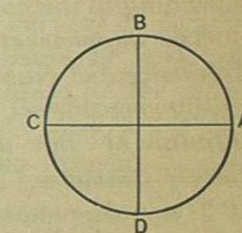
Suppose the circumference of a circle to be described through the points A, B, and P. Draw CP, cutting the circumference in D, and draw the lines DB and DA.

The angles CPB and DAB, being inscribed in the same segment, are equal (B. III., P. XVIII., C. 1); for a like reason, the angles CPA and DBA are equal: hence, in the triangle ADB, we know two angles and one side; we may, therefore, find the side DB. In the triangle ACB, we know the three sides, and we may compute the angle B. Subtracting from this the angle DBA, we have the angle DBC. Now, in the triangle DBC, we have two sides and their included angle, and we can find the angle DCB. Finally, in the triangle CPB, we have two angles and one side, from which data we can find CP and BP. In like manner, we can find AP.

ANALYTICAL TRIGONOMETRY.

47. Analytical Trigonometry is that branch of Mathematics which treats of the general properties and relations of trigonometrical functions.

DEFINITIONS AND GENERAL PRINCIPLES.



48. Let ABCD represent a circle whose radius is 1, and suppose its circumference to be divided into four equal parts, by the diameters AC and BD drawn perpendicular to each other. The horizontal diameter AC is called the *initial diameter;* the vertical diameter BD is called the *secondary diameter;* the point A, from which arcs are usually reckoned, is called the *origin of arcs,* and the point B, 90° distant, is called the *secondary origin.* Arcs estimated from A, around toward B, that is, in a direction contrary to that of the motion of the hands of a watch, are considered *positive;* consequently, those reckoned in a contrary direction must be regarded as *negative.*

The arc AB, is called the *first quadrant;* the arc BC, the *second quadrant;* the arc CD, the *third quadrant;* and the arc DA, the *fourth quadrant.* The point at which