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AN  
 ELEMENTARY TREATISE  
 ON  
 ANALYTIC GEOMETRY,  
 EMBRACING  
 PLANE GEOMETRY  
 AND AN  
 INTRODUCTION TO GEOMETRY OF THREE DIMENSIONS.  
 WITH NUMEROUS EXAMPLES.  
 BY  
 EDWARD A. BOWSER, LL.D.,  
 PROFESSOR OF MATHEMATICS AND ENGINEERING IN RUTGERS COLLEGE.  
 TWENTY-SECOND EDITION.

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## PREFACE.

THE present work on Analytic Geometry is designed as a text-book for Colleges and Scientific Schools. The object has been to exhibit the subject in a clear and simple manner, especially for the use of beginners, and at the same time to include all that students usually require in the regular undergraduate course.

It is thought that among the merits of this book are the presentations of the symmetrical and normal forms of the equations of the right line and of the plane, the equations of the ellipsoid and of the plane tangent to the ellipsoid, and the formulæ for the distances of a point from a line and from a plane. These equations and formulæ are not usually given in our American elementary text-books; and yet they are so important in their applications, they enable us to abridge and to simplify the solution of examples to so great an extent, that they should always be taught, even though considerable else may have to be omitted.

To make the student familiar with the principles of the subject, a large number of examples is given at the ends of the chapters, with hints for the solution of the more difficult ones.

In preparing this book, I have consulted freely what works were available to me. In the geometry of two dimensions I am indebted chiefly to the works of Salmon, O'Brien, Todhunter, Puckle, Howison, and Biot. In the geometry of



three dimensions my chief indebtedness is to Gregory's Solid Geometry, Salmon's Analytic Geometry of Three Dimensions, and Howison's Analytic Geometry. The chapter on Higher Plane Curves was taken substantially from Salmon's Higher Plane Curves and Gregory's Examples, with some aid from Price's Calculus. For the Chordel I am indebted to Mr. J. Bruen Miller, of the Class of '79 of this College.

I have to thank my friend and former pupil, Mr. R. W. Prentiss, B. S., of the Class of '78, now a Fellow in Mathematics at the Johns Hopkins University, for his kind aid in reading the MS. and for valuable suggestions.

E. A. B.

RUTGERS COLLEGE,  
NEW BRUNSWICK, N. J., Jan., 1880.

## PREFACE TO SEVENTEENTH EDITION.

As this book has passed through Sixteen Editions, it has been thought advisable to make a few changes suggested by its use in the class-room. Accordingly, some of the demonstrations have been shortened and simplified, a few propositions have been added, several diagrams have been inserted, and quite a number of notes and about two hundred additional examples have been distributed throughout the book. It is hoped that these changes will commend themselves to those who use the work, and increase its value as a text-book.

E. A. B.

RUTGERS COLLEGE,  
NEW BRUNSWICK, N. J., Jan., 1893.

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