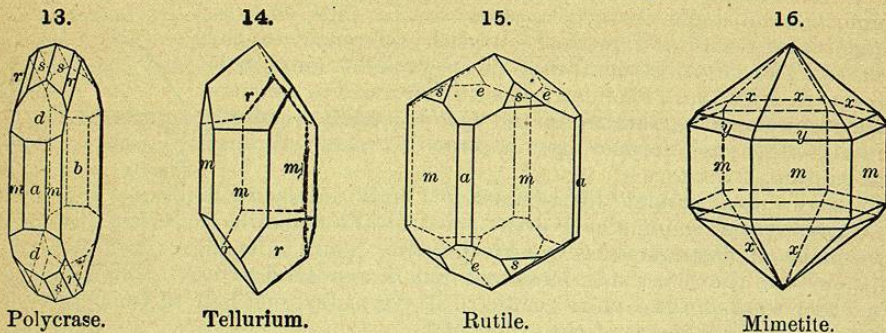


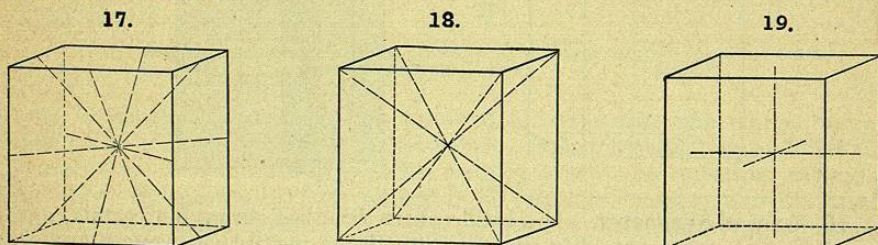
(c) A crystal has an axis of *tetragonal*, or fourfold, symmetry when a revolution of  $90^\circ$  is called for; in other words, when it repeats itself four times



in a complete revolution. The vertical axis in the crystal shown in Fig. 15 is such an axis.

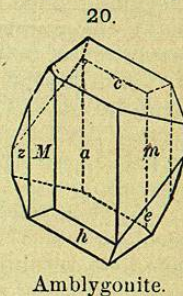
(d) Finally, a crystal has an axis of *hexagonal*, or sixfold, symmetry when a revolution of  $60^\circ$  is called for; in other words, when it repeats itself six times in a complete revolution. This is illustrated by Fig. 16.

The cube\* illustrates three of the four possible kinds of symmetry with respect to axes of symmetry. It has six axes of binary symmetry joining the middle points of opposite edges (Fig. 17). It has four axes of trigonal symmetry, joining the opposite solid angles (Fig. 18). It has, finally, three axes of tetragonal symmetry joining the middle points of opposite faces (Fig. 19).



**17. Center of Symmetry.**—Most crystals, besides planes and axes of symmetry, have also a center of symmetry. On the other hand, a crystal, though possessing neither plane nor axis of symmetry, may yet be symmetrical with reference to a point, its center. This last is true of the triclinic crystal shown in Fig. 20, in which it follows that every face, edge, and solid angle has a face, edge, and angle similar to it in the opposite half of the crystal.

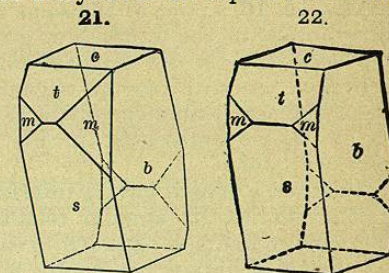
There is another method of viewing the symmetry in this last case, which is adopted by some authors. If the crystal be thought of as divided into two similar halves by a plane parallel to any one of its faces, and one half be revolved  $180^\circ$  about an axis normal to this face, this half would be brought into a position in which it would be the mirror-image of the remaining half. This symmetry is hence described as *compound symmetry* with reference to an axis of binary symmetry and a plane normal to it, both taken together. This method is not followed here since, though having certain theoretical advantages, it is likely to confuse the student meeting the problems of crystallography for the first time.



Amblygonite.

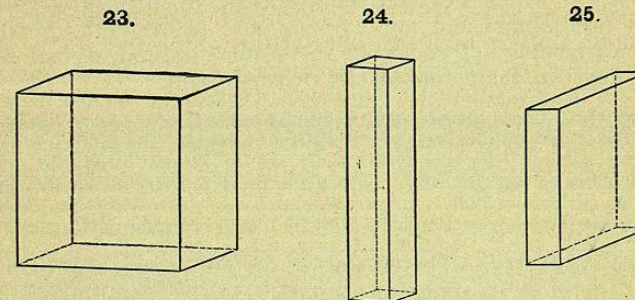
\* This is again the cube of the normal group of the isometric system.

**18. Relation of Geometrical to Crystallographic Symmetry.**—Since the symmetry in the arrangement of the faces of a crystal is an expression of the internal molecular structure, which in general is alike in all parallel directions, the *relative size* of the faces and their *distance* from the plane or axis of symmetry are of no moment, their *angular position* alone is essential. Hence Fig. 21 has as truly a vertical plane of symmetry (parallel to *b*) as Fig. 22 if the faces have exactly the same angular position as in that, although the strict geometrical definition\* could not be applied to it.



Heulandite.

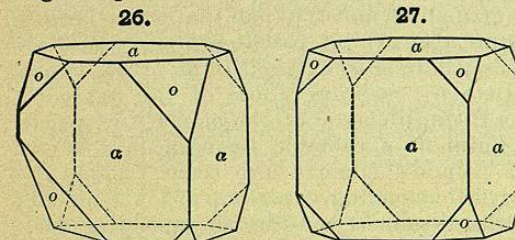
Also in a normal cube (Fig. 23) the three central planes parallel to each pair of cubic faces are like planes of symmetry, as stated in Art. 15. But a crystal is still crystallographically a cube, though deviating widely from the requirements of the strict geometrical definition, as shown in Figs. 24, 25, if only it can be proved, *e.g.* by cleavage, the



physical nature of the faces, or by optical means, that the three pairs of faces are like faces, independently of their size, or, in other words, that the molecular structure is the same in the three directions normal to them.

Further, in the case of a normal cube, a face of an octahedron on any solid angle requires, as explained beyond, similar faces on the other angles. It is *not* necessary, however, that these eight faces should be of equal size, for in the crystallographic sense Fig. 26 is as truly symmetrical with reference to the planes named as Fig. 27.

**19.** On the other hand, the molecular and hence the crystallographic symmetry is not always that which the geometrical form would suggest. Thus, deferring for the moment the consideration of pseudo-symmetry, an illustration of the fact stated is afforded by the cube.



\* It is to be noted that the perspective figures of crystals always show the geometrically ideal form, in which like faces, edges, and angles have the same shape, size, and position. In other words, the ideal crystal is uniformly represented as having the symmetry called for by the strict geometrical definition.

It has already been implied and will be fully explained later that while the cube of the normal group of the isometric system has the symmetry described in Arts. 15, 16, a cube of the same geometrical form but belonging molecularly, for example, to the tetrahedral group, has no planes of symmetry parallel to the faces, only the six diagonal planes; further, though the four axes shown in Fig. 18 are still axes of trigonal symmetry, the cubic axes (Fig. 19) are axes of binary symmetry only, and there are no axes of symmetry corresponding to those represented in Fig. 17. Other more complex cases will be described later.

Further, a crystal having interfacial angles of  $90^\circ$  is not necessarily a cube; in other words, the angular relations of the faces do not show in this case whether the figure is bounded by six like faces; or whether only two are alike and the third unlike; or, finally, whether there are three pairs of unlike faces. The question must be decided, in such cases, by the molecular structure as indicated by the physical nature of the surfaces, by the cleavage, or by other physical characters, as pyro-electricity, those connected with light phenomena, etc.

Still again, the student will learn later that the decision reached in regard to the symmetry to which a crystal belongs, based upon the *distribution* of the faces, is only preliminary and approximate, and before being finally accepted it must be confirmed, first, by accurate measurements, and, second, by a minute study of the physical characters as just insisted upon.

The method based upon the physical characters, which gives most conclusive results and admits of the widest application, is the skillful etching of the surface of the crystal by some appropriate solvent. By this means there are, in general, produced upon it minute depressions the shape of which always conforms to the symmetry in the arrangement of the molecules. This process, which is in part essentially one involving the dissection of the molecular structure, is more particularly discussed in the chapter on Physical Mineralogy.

**20. Pseudo-symmetry.**—The crystals of certain species approximate closely in angle, and therefore in apparent symmetry, to the requirements of a system higher in symmetry than that to which they actually belong: they are then said to exhibit *pseudo-symmetry*. Numerous examples are given under the different systems. Thus the micas have been shown to be truly monoclinic in crystallization, though in angle they seem to be in some cases rhombohedral, in others orthorhombic.

It will be shown later that compound, or twin, crystals may also simulate by their regular grouping a higher grade of symmetry than that which belongs to the single crystal. Such crystals also exhibit pseudo-symmetry and are specifically called *mimetic*. Thus aragonite is an example of an orthorhombic species, whose crystals often imitate by twinning those of the hexagonal system.\* Again, a highly complex twinned crystal of the monoclinic species, phillipsite, may have nearly the form of a rhombic dodecahedron of the isometric system. This kind of pseudo-symmetry also occurs among the groups of a single system, since a crystal belonging to a group of low symmetry may by twinning gain the geometrical symmetry of the corresponding form of the normal group. This is illustrated by a twinned crystal of scheelite like that figured (Fig. 378) in the chapter on twin crystals.

Pseudo-symmetry of still another kind, where there is an imitation of the symmetry of another system of lower grade, is particularly common in crystals of the isometric system (*e.g.*, gold, copper). The result is reached in

\* The terms *pseudo-hexagonal*, etc., used in this and similar cases explain themselves.

such cases by an abnormal development or "distortion" in the direction of certain axes of symmetry. This subject is discussed and illustrated on a later page.

**21. Possible Groups of Symmetry.**—The theoretical consideration of the different kinds of symmetry possible among crystals built up of like molecules, as explained in Arts. 30–32, has led to the conclusion that there are thirty-two (32) types in all, differing with respect to the combination of the different symmetry elements just described. Of these thirty-two natural groups among crystals based upon their symmetry, seven groups include by far the larger number of crystallized minerals. Besides these, some thirteen or fourteen others are distinctly represented, though several of these are of rare occurrence. Further, eight or nine others, making in all twenty-nine or thirty, are known among crystallized salts made in the laboratory. The characters of each of the thirty-two groups are given under the discussion of the several crystalline systems.

**22. Crystallographic Axes.**—In the description of the form of a crystal, especially as regards the position of its faces, it is found convenient to assume, after the methods of analytical geometry, definite lengths of certain lines passing through the center of the ideal crystal, as a basis of reference. (See further Art. 33 *et seq.*)

These lines are called the *crystallographic axes*. Their direction is to a greater or less extent fixed by the symmetry of the crystals, for an axis of symmetry is in almost all cases\* a possible crystallographic axis. Further, their unit lengths are fixed sometimes by the symmetry, sometimes by the faces assumed as fundamental, *i.e.*, the unit forms in the sense defined later. The dotted lines shown in Fig. 19 are the crystallographic axes to which the cubic faces are referred.

**23. Systems of Crystallization.**—The thirty-two possible crystalline groups, distinguished from one another by their symmetry, are classified in this work under six systems, each characterized by the relative lengths and inclinations of the assumed crystallographic axes. These are as follows:

I. ISOMETRIC SYSTEM. Three equal axes at right angles to each other.

II. TETRAGONAL SYSTEM. Three axes at right angles to each other, two of them—the lateral axes—equal, the third—the vertical axis—longer or shorter.

III. HEXAGONAL SYSTEM. Four axes, three equal lateral axes in one plane intersecting at angles of  $60^\circ$ , and a vertical axis at right angles to this plane and longer or shorter.

IV. ORTHORHOMBIC SYSTEM. Three axes at right angles to each other, but all of different lengths.

V. MONOCLINIC SYSTEM. Three axes unequal in length, and having one of their intersections oblique, the two other intersections equal to  $90^\circ$ .

VI. TRICLINIC SYSTEM. Three unequal axes with mutually oblique intersections.

The systems of crystallization have been variously named by different authors, as follows:

ISOMETRIC. *Tessular* of Mohs and Haidinger; *Isometric* of Hausmann; *Tesseral* of Naumann; *Regular* of Weiss and Rose; *Cubic* of Dufrénoy, Miller, Des Cloizeaux; *Monometric* of the earlier editions of Dana's System of Mineralogy.

\* Exceptions are found in the isometric system, where the axes must necessarily be the axes of tetragonal symmetry (Fig. 19), and cannot be those of binary or trigonal symmetry (Figs. 17, 18).

