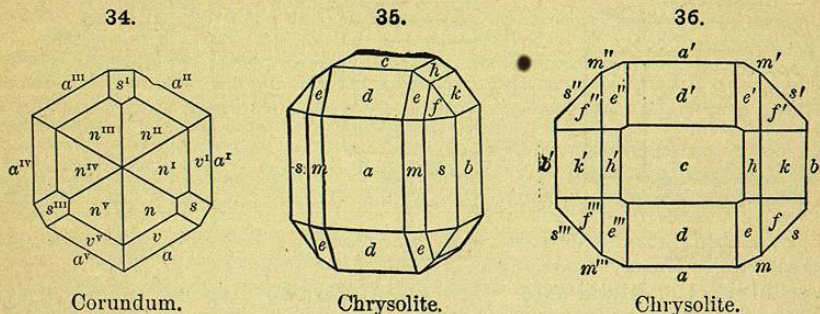


In the *Trigonal* or *Rhombohedral Division* of this system there are three like planes of symmetry intersecting at angles of 60° in the vertical axis. Further, the forms belonging here have a vertical principal axis of trigonal symmetry, and three horizontal axes of binary symmetry, diagonal in position to the crystallographic axes. Fig. 33 shows a typical rhombohedral crystal, and Fig. 34 a basal projection. See also Figs. 226-252.



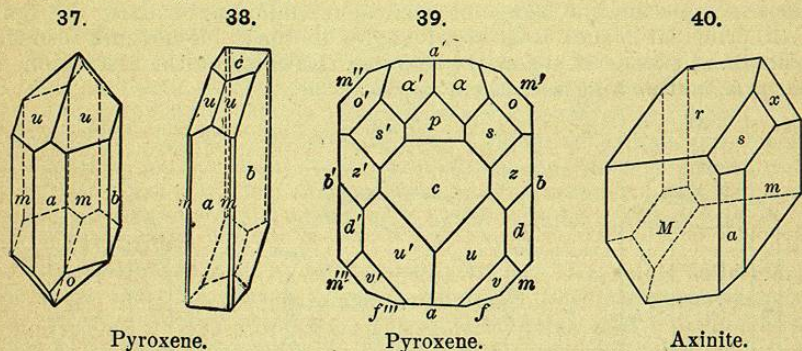
Corundum.

Chrysolite.

Chrysolite.

IV. ORTHORHOMBIC SYSTEM. Three unlike planes of symmetry meeting at 90° , and fixing by their intersection-lines the position of the crystallographic axes. Further, three unlike axes of binary symmetry coinciding with the last-named axes. Fig. 35 shows a typical orthorhombic crystal, and Fig. 36 a basal projection. See also Fig. 44 and Figs. 275-303.

V. MONOCLINIC SYSTEM. One plane of symmetry which contains two of the crystallographic axes. Also one axis of binary symmetry, normal to this plane and coinciding with the third crystallographic axis. See Figs. 37-39; also Fig. 45 and Figs. 312-327.



Pyroxene.

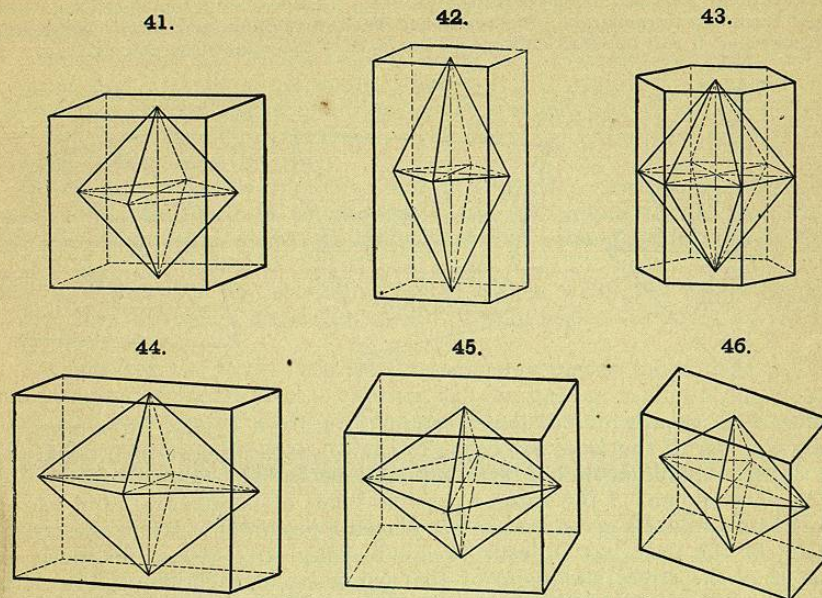
Pyroxene.

Axinite.

VI. TRICLINIC SYSTEM. No plane and no axis of symmetry, but symmetry solely with respect to the central point. Fig. 40 and Fig. 46 show typical triclinic crystals. See also Figs. 333-341.

26. The relations of the normal groups of the different systems are further illustrated both as regards the crystallographic axes and symmetry by the accompanying figures, 41-46. The exterior form is here that bounded by faces each of which is parallel to a plane through two of the crystallographic axes indicated by the central broken lines. Further, there is shown, within this, the combination of faces each of which joins the extremities of the unit lengths of the axes.

The full understanding of the subject will not be gained until after a study of the forms of each system in detail. Nevertheless the student will do well to make himself familiar at the outset with the fundamental relations here illustrated.



It will be shown later that the symmetry of the different groups can be most clearly and easily exhibited by the use of the spherical projection explained in Art. 39 *et seq.*

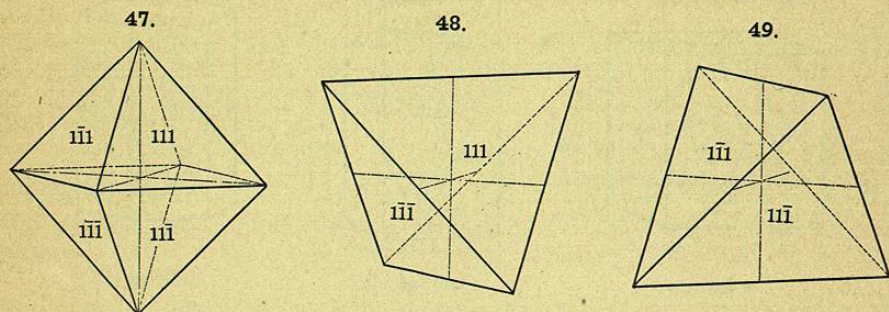
27. Models.—Glass (or transparent celluloid) models illustrating the different systems, having the forms shown in Figs. 41-46, will be very useful to the student, especially in learning the fundamental relations as regards symmetry. They should show within the crystallographic axes, and by colored threads or wires the outlines of one or more simple forms. Models* of wood are also made in great variety and perfection of form; these are indispensable to the student in mastering the principles of crystallography.

28. So-called *Holohedral* and *Hemihedral* Forms.—It will appear later that each crystal form † of the normal group in a given system embraces *all* the faces which have a like geometrical position with reference to the crystallographic axes; such a form is said to be *holohedral* (from $\delta\lambda\omicron\varsigma$ and $\epsilon\delta\rho\alpha$, face). On the other hand, under the groups of lower symmetry, a certain form, while necessarily having all the faces which the symmetry allows, may yet have but *half* as many as the corresponding form of the normal group; these half-faced forms are sometimes called on this account *hemihedral*. Furthermore, it will be seen that, in such cases, to the given holohedral form there correspond two similar and complementary hemihedral forms, called respectively plus and minus (or right and left), which together embrace all of its faces.

* Models can be obtained from Dr. F. Krantz in Bonn, Germany.

† The use of the word *form* is defined in Art. 36.

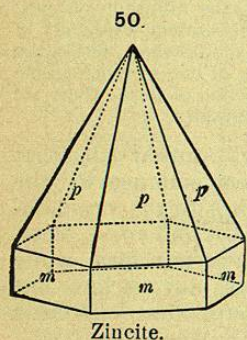
A single example will help to make the above statement intelligible. In the normal group of the isometric system, the octahedron (Fig. 47) is a "holohedral" form with all the possible faces—eight in number—which are alike in that they meet the axes at equal distances. In the tetrahedral group of the same system, the forms are referred to the same crystallographic axes, but the symmetry defined in Art. 19 (and more fully later) calls for but four similar faces having the position described. These yield a four-faced, or "hemihedral," form, the tetrahedron. Figures 48 and 49 show the plus and minus tetrahedron, which together, it will be seen, embrace all the faces of the octahedron, Fig. 47.



In certain groups of still lower symmetry a given crystal form may have but *one-quarter* of the faces belonging to the corresponding normal form, and, after the same method, such a form is sometimes called *tetartohedral*.

The development of the various possible kinds of hemihedral (and tetartohedral) forms under a given system has played a prominent part in the crystallography of the past, but it leads to much complexity and is distinctly less simple than the direct statement of the symmetry in each case. The latter method is systematically followed in this work, and the subject of hemihedrism is dismissed with the brief (and incomplete) statements of this and the following paragraphs.

29. Hemimorphic Forms.—In several of the systems, forms occur under the groups of lower symmetry than that of the normal group which are characterized by this: that there is no transverse plane of symmetry, but the faces present are only those belonging to one extremity of an axis of symmetry (and crystallographic axis). Such forms are conveniently called *hemimorphic* forms. A simple example under the hexagonal system is given in Fig. 50. It is obvious that hemimorphic forms have no center of symmetry.



Zincite.

30. Molecular Networks.—Müchlight has recently been thrown upon the relations existing between the different types of crystals, on the one hand, and of these to the physical properties of crystals, on the other, by the consideration of the various possible methods of grouping of the molecules of which the crystals are supposed to be built up. This subject, very early treated by Haüy and others (including J. D. Dana), was discussed at length by Frankenheim and later by Bravais. More recently it has been extended and elaborated by Sohncke, Wulff, Schönfliess, Fedorow, Barlow, and others.*

All solid bodies, as stated in Art. 7, are believed to be made up of definite

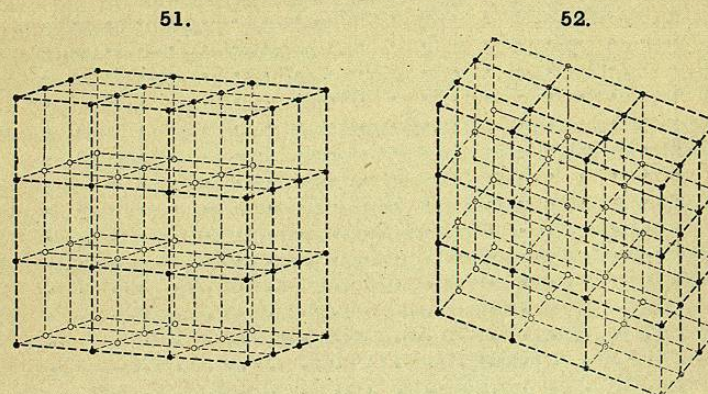
* See the literature following Art. 32.

physical units, called the physical, or crystal, molecules. Of the form of the molecules nothing is definitely known, and though theory has something to say about their size, it is enough here to understand that they are almost infinitely small, so small that the surface of a solid—*e.g.* of a crystal—may appear to the touch and to the eye, even when assisted by a powerful microscope, as perfectly smooth.

The molecules are further believed to be not in contact but separated from one another—if in contact, it would be impossible to explain the motion to which the sensible heat of the body is due, or the transmission of radiation (radiant heat and light) through the mass by the wave motion of the ether, which is believed to penetrate the body.

When a body passes from the state of a liquid or a gas to that of a solid, under such conditions as to allow perfectly free action to the forces acting between the molecules, the result is a crystal of some definite type as regards symmetry. The simplest hypothesis which can be made assumes that the form of the crystal is determined by the way in which the molecules group themselves together in a position of equilibrium under the action of the intermolecular forces.

As, however, the forces between the molecules vary in magnitude and direction from one type of crystal to another, the resultant grouping of the molecules must also vary, particularly as regards the distance between them and the angles between the planes in which they lie. This may be simply represented by a series of geometrical diagrams, showing the hypothetical groupings of



points which are strictly to be regarded as the centers of gravity of the molecules themselves. Such a grouping is named a *network*, or point-system, and it is said to be *regular* when it is the same for all parallel lines and planes, however they be taken. For the fundamental observed fact, true in all simple crystals, that they have like physical properties in all parallel directions, leads to the conclusion that the grouping of the molecules must be the same about each one of them (or at least about each unit group of them), and further the same in all parallel lines and planes.

The subject may be illustrated by Figs. 51, 52 for two typical cases, which are easily understood. In Fig. 51 the most special case is represented where the points are grouped at equal distances, in planes at right angles to each other. The structure in this case obviously corresponds in symmetry to the

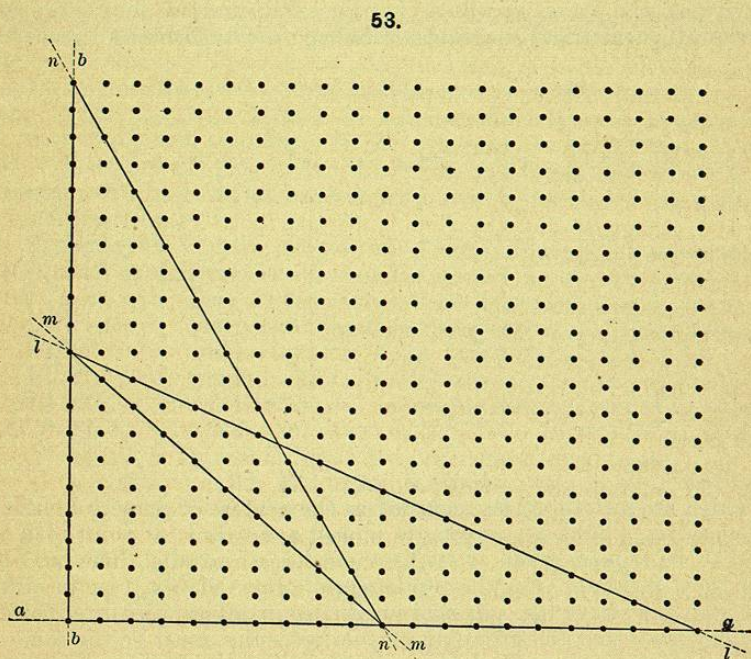
cube described in Arts. 15 and 16, or, in other words, to the normal group of the isometric system. Again, in Fig. 52, the general case is shown where the molecules are unequally grouped in the three directions, and further these directions are oblique. The symmetry is here that of the normal group of the triclinic system.

If, in each of these cases, the figure be bounded by the simplest possible arrangement of eight points, the result is an *elementary parallelepiped*, which obviously defines the molecular structure of the whole. In the grouping of these parallelepipeds together, as described, it is obvious that in whatever direction a line be drawn through them, the points (molecules) will be spaced alike along it, and the grouping about any one of these points will be the same as about any other.

31. Certain important conclusions can be deduced from a consideration of such regular molecular networks as have been spoken of, which will be enumerated here though it is impossible to attempt a full explanation.

(1) The prominent crystalline faces must be such as include the largest number of points, that is, those in which the points are nearest together.

Thus in Fig. 53, which represents a section of a network conforming in symmetry to the structure of a normal orthorhombic crystal, the common crystalline faces would be expected to be those having the position bb , aa , mm , then



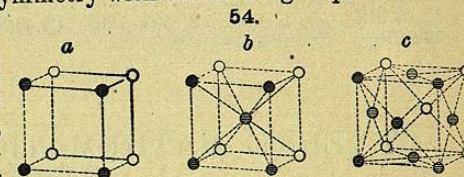
ll , nn , and so on. This is found to be true in the study of crystals, for the common forms are, in nearly all cases, those whose position bears some simple relation to the assumed axes; forms whose position is complex are usually present only as small faces on the simple predominating forms, that is, as modifications of them. So-called *vicinal* forms, that is, forms taking the place

of the simple fundamental forms to which they approximate very closely in angular position, are exceptional.

(2) When a variety of faces occur on the same crystal, the numerical relation existing between them (that which fixes their position) must be rational and, as stated in (1), a simple numerical ratio is to be expected in the common cases. This, as explained later, is found by experience to be a fundamental law of all crystals. Thus in Fig. 53, starting with a face meeting the section in mm , ll would be a common face, and for it the ratio is 1:2 in the directions b and a ; nn would be also common with the ratio 2:1.

(3) If a crystal shows the natural easy fracture, called cleavage, due to a minimum of cohesion, the cleavage surface must be a surface of relatively great molecular crowding, that is, one of the common or fundamental faces. This follows (and thus gives a partial, though not complete, explanation of cleavage) since it admits of easy proof that that plane in which the points are closest together is farthest separated from the next molecular plane. Thus in Fig. 53 compare the distance separating two adjoining planes parallel to bb or aa ; then two parallel to mm , ll , nn , etc. Illustrations of the above will be found under the special discussion of the subject of cleavage.

32. **Kinds of Molecular Groupings.**—The discussion on the basis just described shows that there are fourteen possible types of arrangement of the molecules. These agree as to their symmetry with the seven groups defined in Art. 25 as representing respectively the normal groups of the six systems with also that of the trigonal (or the rhombohedral) division of the hexagonal system. Of the fourteen, three groupings belong to the isometric system (these are shown, for sake of illustration, in Fig. 54, a , b , c , from Groth); two to the tetragonal; one each to the hexagonal and the rhombohedral; four to the orthorhombic system; two to the monoclinic, and one to the triclinic.



In its simplest form, as above outlined, the theory fails to explain the existence of the groups under the several systems of a symmetry lower than that of the normal group. It has been shown, however, by Sohncke and later by Fedorow, Schönflies and Barlow, that the theory admits of extension. The idea supposed by Sohncke is this: that, instead of the simple form shown, the network may consist of a double system, one of which may be conceived of as having a position relative to the other (1) as if pushed to one side, or (2) as if rotated about an axis, or finally (3) as if both rotated as in (2) and displaced as in (1). The complexity of the subject makes it impossible to develop it here. It must suffice to say that with this extension Sohncke concludes that there are 65 possible groups. This number has been further extended to 230 by the other authors named, but it still remains true that these fall into 32 distinct types as regards symmetry, and thus all the observed groups of forms among crystals, described under the several systems, have a theoretical explanation.

Literature.—A complete understanding of this subject can only be gained by a careful study of the many papers devoted to it, a partial list of which is added below. Further references particularly to the early literature are given in Sohncke's work (see below). An excellent and very clear summary of the whole subject is given by Groth in the third edition of his *Physikalische Krystallographie*, 1895.

Early papers:

Frankenheim. De Crystallorum Cohæsione, 1829; also in Baumgartner's Zeitschrift für Physik, 9, 94, 194, 1831. Die Lehre von der Cohäsion, Breslau, 1835. Ueber die Anordnung der Moleküle im Krystall; Pogg., 97, 337, 1856.

Hessel. Article 'Krystall' in Gehler's physikal. Wörterbuch, 5, 1830 (see Sohncke, Zs. Kryst., 18, 486).

Bravais. Mémoire sur les systèmes formes par des points distribués régulièrement sur un plan ou dans l'espace, Paris, 1850; and in Études cristallographiques, Paris, 1866.

Gadolin. Act. Soc. Fennicæ, 9, 1, 1871 (republished in Ostwald's Klassiker d. exakten Wissenschaften, No. 75).

Later works and papers:

Barlow. Nature, 29, 186, 205, 1883; Zs. Kryst., 23, 1, 1894; 25, 86, 1895; Min. Mag., 11, 119, 1896, and Zs. Kryst., 27, 449, 468, 1896; R. Dublin Soc., 8, 527, 1897, and Zs. Kryst., 29, 433, 1898.

Curie. Bull. Soc. Min., 7, 89, 418, 1884.

Fedorow. Zs. Kryst., 20, 25, 259; 24, 209, 1894; 25, 113, 1895; 28, 36, 232, 468, 1897.

Goldschmidt. Zs. Kryst., 28, 1, 414, 1897.

Kelvin. Proc. R. Soc. Edinb., 16, 693, 1888; Proc. Roy. Soc., 55, 1, 1894.

Minnegerode. Nachr. Ges. Göttingen, 1884; Jb. Min. Beil.-Bd., 5, 145, 1887.

Schönflies. Nachr. Ges. Göttingen, 483, 1888; 239, 1890. Also as a separate work, Krystallsysteme und Krystallstruktur, Leipzig, 1891.

Sohncke. Die Gruppierung der Moleküle in den Krystallen, Pogg. Ann., 132, 75, 1867. Also Wied. Ann., 16, 489, 1882; Zs. Kryst., 13, 209, 214, 1887; 14, 417, 426, 1888; 18, 486, 1890. Entwicklung einer Theorie der Krystallstruktur, Leipzig, 1879.

Viola. Zs. Kryst., 27, 1, 1896; 28, 452; 29, 1, 234, 1897.

L. Wulff. Zs. Kryst., 13, 503, 1887; 15, 366, 1889; 18, 174, 1890.

Wülfing. For title see p. 2.

GENERAL MATHEMATICAL RELATIONS OF CRYSTALS.

33. Axial Ratio, Axial Plane.—The crystallographic axes have been defined (Art. 22) as certain lines, usually determined by the symmetry, which are used in the description of the faces of crystals, and in the determination of their position and angular inclination. With these objects in view, certain lengths of these axes are assumed as units to which the occurring faces are referred.

The axes are, in general, lettered a, b, c , to correspond to the scheme in Fig. 55. To aid the memory, the letters may be further distinguished; as \check{c} (vertical axis); \check{a}, \check{b} (shorter and longer lateral axes), etc.

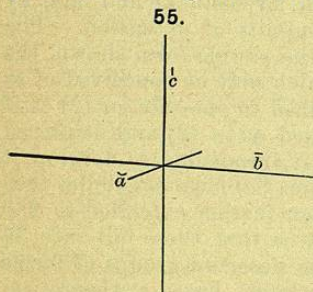
If two of the axes are equal, they are designated a, a, \check{c} ; if three, a, a, a . In one system, the hexagonal, there are four axes, lettered a, a, a, \check{c} .

Further, in the systems other than the isometric, one of the lateral axes is taken as the unit to which the other axes are referred; hence the lengths of the axes express strictly the *axial ratio*. Thus for sulphur (orthorhombic, see Fig. 57) the axial ratio is

$$\check{a} : \check{b} : \check{c} = 0.8131 : 1 : 1.9034.$$

For rutile (tetragonal) it is

$$a : \check{c} = 1 : 0.64415, \text{ or, simply, } \check{c} = 0.64415.$$



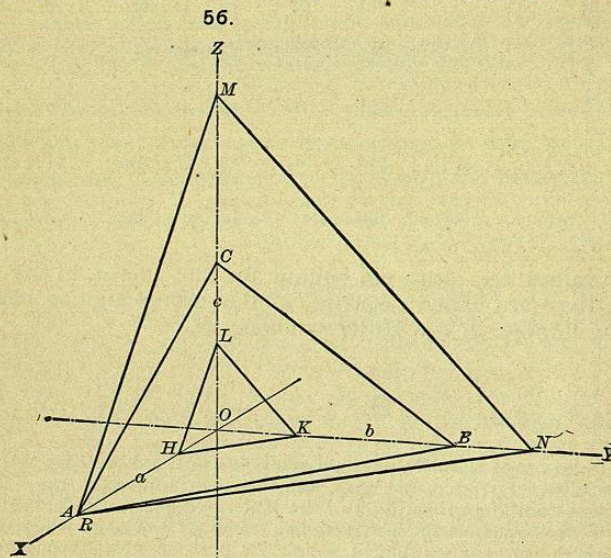
The plane of any two of the axes is called an *axial plane*, and the space included by the three axial planes is an *octant*, since the total space about the center is thus divided by the three axes into eight parts. In the hexagonal system, however, where there are three lateral axes, the space about the center is divided into 12 parts, or *sectants*.

34. Parameters, Symbol.—The *parameters* of a plane are its intercepts on the assumed axes. The *symbol* expresses, often in abbreviated form, the relation of these intercepts to certain lengths of the axes taken as units.

For example, in Fig. 56 let the lines OX, OY, OZ be taken as the directions of the crystallographic axes, and let OA, OB, OC represent the unit lengths, designated (always in the same order) by the letters a, b, c . Then the parameters for the plane (1) HKL are OH, OK, OL ; for the plane (2) RNM they are OR, ON, OM . But in terms of the unit lengths these are, respectively,

$$(1) \frac{1}{4}a : \frac{1}{3}b : \frac{1}{2}c, \quad \text{or} \quad (2) 1a : \frac{4}{3}b : 2c.$$

These two expressions are *identical*, since the two planes HKL, MNR are parallel and hence crystallographically the same. Obviously each of the above expressions may be changed into the other by multiplying (or dividing) by 4.



It will be noted that in (1) the numerators of the fractional numbers expressing the relation to the axes are all unity; while in (2) the number referring to one of the lateral axes (a) is made unity. The significance of this distinction will appear at once.

The general expression for any plane referred to these axes, written after the same method, will be

$$(1) \frac{1}{h}a : \frac{1}{k}b : \frac{1}{l}c, \quad \text{or} \quad (2) 1a : nb : mc.$$