As a further simplification of the above equations for the case of prismatic planes $hk0$ and $pq0$, or domes $h0l$ and $p0r$ or $0kl$ and $0qr$, between two pinacoid planes, we have

$$\frac{\tan(100 \wedge hk0)}{\tan(100 \wedge pq0)} = \frac{k}{h} \cdot \frac{p}{q};$$

$$\frac{\tan(001 \wedge h0l)}{\tan(001 \wedge p0r)} = \frac{h}{l} \cdot \frac{r}{p};$$

$$\frac{\tan(001 \wedge 0kl)}{\tan(001 \wedge 0qr)} = \frac{k}{l} \cdot \frac{r}{q}.$$

These equations are the ones ordinarily employed to determine the symbol of any prismatic plane or dome.

The most common and important application of this tangent principle is where the positions of the unit faces 110, 101, 011 are known, then the relation becomes

$$\frac{\tan(100 \wedge hk0)}{\tan(100 \wedge 110)} = \frac{k}{h}, \quad \text{or} \quad \frac{\tan(010 \wedge hk0)}{\tan(010 \wedge 110)} = \frac{h}{k}.$$

Also,

$$\frac{\tan(001 \wedge h0l)}{\tan(001 \wedge 101)} = \frac{h}{l}, \quad \frac{\tan(001 \wedge 0kl)}{\tan(001 \wedge 011)} = \frac{k}{l}.$$

Thus the tangents of angles between the base, 001, and 102, 203, 302, 201, etc., are respectively $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{2}$, 2 times the tangent of the angle between 001 and 101. Again, the tangent of the angle $100 \wedge 120$ is twice the tangent of $100 \wedge 110$ $\left(\text{here } \frac{k}{h} = 2\right)$, and one-half the tangent of $010 \wedge 110$.

48. Formulas for Spherical Triangles.—For convenience, some of the more important formulas for the solution of spherical triangles are here added.

In right-angled spherical triangles $C = 90°$.

$$\sin A = \frac{\sin a}{\sin h}, \qquad \sin B = \frac{\sin b}{\sin h},$$

$$\cos A = \frac{\tan b}{\tan h}, \qquad \cos B = \frac{\tan a}{\tan h},$$

$$\tan A = \frac{\tan a}{\sin b}, \qquad \tan B = \frac{\tan b}{\sin a},$$

$$\sin A = \frac{\cos B}{\cos b}, \qquad \sin B = \frac{\cos A}{\cos a},$$

$$\cos h = \cos a \cos b,$$

$$\cos h = \cot A \cot B.$$

In oblique-angled spherical triangles familiar relations are as follows:

(1) $\sin A : \sin B = \sin a : \sin b$;
(2) $\cos a = \cos b \cos c + \sin b \sin c \cos A$;
(3) $\cot b \sin c = \cos c \cos A + \sin A \cot B$;
(4) $\cos A = -\cos B \cos C + \sin B \sin C \cos a$.

In calculation it is often more convenient to use, instead of the latter formulas, those especially arranged for logarithms, which will be found in any of the many books devoted to mathematical formulas.

I. ISOMETRIC SYSTEM.

49. The Isometric System embraces all the forms which are referred to three equal axes at right angles to each other. Each of these axes is designated by the letter a.

There are five groups here included, of which the normal group,* possessing the highest degree of symmetry for the system and, indeed, for all crystals, is by far the most important. Two of the other groups, the pyritohedral and tetrahedral groups, also have numerous representatives among minerals.

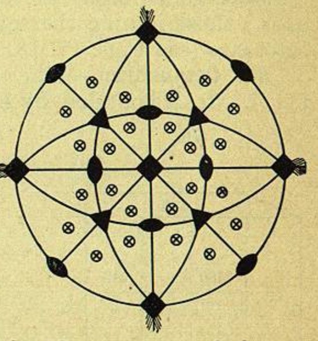
1. NORMAL GROUP (1). GALENA TYPE.

50. Symmetry.—Of each of the types of solids enumerated in the following table, as belonging to this group, as of all their combinations, it is true † that there are three like principal planes of symmetry, whose intersections fix the position of the crystallographic axes (see Fig. 12, p. 9). There are also six other auxiliary planes of symmetry; these are situated diagonally to the others, each two equally inclined (45°) to the adjacent planes of chief symmetry, that is, to the axial planes.

Further, the crystals of this group have three principal axes of tetragonal symmetry, the cubic or crystallographic axes; four axes of trigonal symmetry, the octahedral axes; six axes of binary symmetry, the dodecahedral axes (see Art. **16**, also the following paragraph). These axes are shown in Figs. 17, 18, 19, p. 10.

The accompanying spherical projection (Fig. 69), constructed in accordance with the principles explained in Art. **42**, shows the distribution of the faces of the general form, hkl, and hence represents clearly the symmetry of the group. Compare also the projection given later, Fig. 110, p. 41.

69.



51. Forms.—The various possible forms belonging to this group, and possessing the symmetry defined, may be grouped under seven types of solids. These are enumerated in the following table, commencing with the most simple. The symbols are given in accordance with both the systems of Miller and Naumann; also the full expression showing the general position of the planes with relation to the axes. The last, however, are reduced to the form, corresponding to (2) in Art. **34**, which shows how the Naumann symbols are derived.

* It is called *normal*, as before stated, since it is the most common and hence by far the most important group under the system; also, more fundamentally, because the forms here included possess the highest grade of symmetry possible in the system. There are five forms in this system, each geometrically a cube, but only that of this normal group actually has the full symmetry as regards molecular structure which its geometrical shape suggests. If a crystal is said to belong to the isometric system, without further qualification, it is to be understood that it is included here. Similar remarks apply to the normal groups of the other systems.

† The symmetry of the normal groups of the different systems has been already briefly explained in Art. **25**.

	Miller.		Naumann.
1. Cube	(100)	$a : \infty a : \infty a$	$\infty O \infty$ or i-i
2. Octahedron	(111)	$a : a : a$	O or 1
3. Dodecahedron	(110)	$a : a : \infty a$	∞O or i
4. Tetrahexahedron	($hk0$)	$a : na : \infty a$	∞On or i-n
	as, (310) i-3; (210) i-2; (320) i-$\frac{3}{2}$, etc.		
5. Trisoctahedron	(hhl)	$a : a : ma$	mO or m
	as, (331) 3; (221) 2; (332) $\frac{3}{2}$, etc.		
6. Trapezohedron	(hll)	$a : ma : ma$	mOm or m-m
	as, (311) 3-3; (211) 2-2; (322) $\frac{3}{2}$-$\frac{3}{2}$, etc.		
7. Hexoctahedron	(hkl)	$a : na : ma$	mOn or m-n
	as, (421) 4-2; (321) 3-$\frac{3}{2}$, etc.		

In the general expression of Miller's symbols, $h > k > l$. In those of Naumann, $m > 1$.

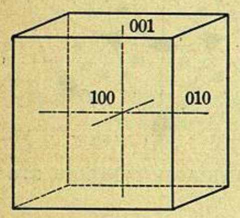
Attention is called to the letters uniformly used in this work and in Dana's System of Mineralogy (1892) to designate certain of the isometric forms.* They are :

Cube: a.
Octahedron: o.
Dodecahedron: d.
Tetrahexahedrons: $e = 210$, i-2; $f = 310$, i-3; $g = 320$, i-$\frac{3}{2}$; $h = 410$, i-4.
Trisoctahedrons: $p = 221$, 2; $q = 331$, 3; $r = 332$, $\frac{3}{2}$; $\rho = 441$, 4.
Trapezohedrons: $m = 311$, 3-3; $n = 211$, 2-2; $\beta = 322$, $\frac{3}{2}$-$\frac{3}{2}$.
Hexoctahedrons: $s = 321$, 3-$\frac{3}{2}$; $t = 421$, 4-2.

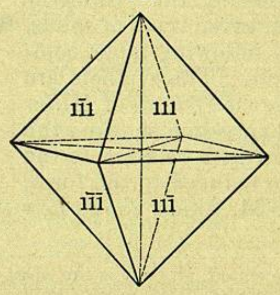
52. Cube.—The cube, whose general symbol is (100), is shown in Fig. 70. It is bounded by six similar faces, each parallel to two of the axes. Each face is a square, and the interfacial angles are all 90°. The faces of the cube are parallel to the principal or axial planes of symmetry. The lines joining the opposite solid angles of the cube are called the octahedral or trigonal interaxes; those joining the middle points of opposite edges are the dodecahedral interaxes (see Figs. 17, 18, p. 10).

53. Octahedron.—The octahedron, shown in Fig. 71, has the general symbol (111). It is bounded by eight similar faces, each meeting the three axes at

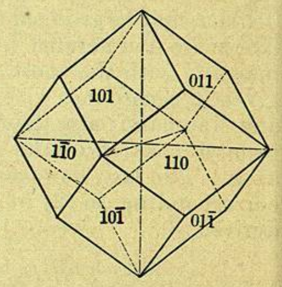
70.



71.



72.



equal distances. Each face is an equilateral triangle with plane angles of 60°. The normal interfacial angle, ($111 \wedge 1\bar{1}1$), is 70° 31′ 44″.

54. Dodecahedron.—The rhombic dodecahedron, shown in Fig. 72, has the general symbol (110). It is bounded by twelve faces, each of which meets two

* The usage followed here (as also in the other systems) is in most cases that of Miller (1852).

of the axes at equal distances and is parallel to the third axis. Each face is a rhomb with plane angles of 70½° and 109½°. The real or interior interfacial angle is 120°, or the angle between two adjacent poles, that is, the normal interfacial angle, is 60°. The faces of the dodecahedron are parallel to the six auxiliary, or diagonal, planes of symmetry.

It will be remembered that, while the forms described are designated respectively by the symbols (100), (111), and (110), each face of any one of the forms has its own symbol. Thus for the cube the six faces have the symbols

100, 010, 001, $\bar{1}00$, $0\bar{1}0$, $00\bar{1}$.

For the octahedron the symbols of the eight faces are :

Above 111, $\bar{1}11$, $\bar{1}\bar{1}1$, 111;
Below $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}1$, $1\bar{1}\bar{1}$.

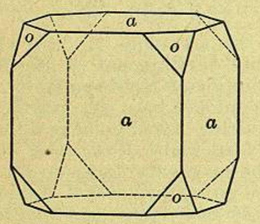
For the dodecahedron, the symbols of the twelve faces are :

110, $\bar{1}10$, $\bar{1}\bar{1}0$, $1\bar{1}0$,
101, $\bar{1}01$, $\bar{1}0\bar{1}$, $10\bar{1}$,
011, $0\bar{1}1$, $0\bar{1}\bar{1}$, $01\bar{1}$.

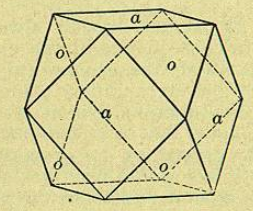
These should be carefully studied with reference to the figures (and to models), and also to the spherical projection (Fig. 110). The student should become thoroughly familiar with these individual symbols and the relations to the axes which they express, so that he can give at once the symbol of any face required.

55. Combinations of the Cube, Octahedron, and Dodecahedron.—Figs. 73, 74, 75 represent combinations of the cube and octahedron ; Figs. 76, 79, of the cube and dodecahedron ; Figs. 77, 78, of the octahedron and dodecahedron ; finally, Figs. 80, 81 show combinations of the three forms. The predominating

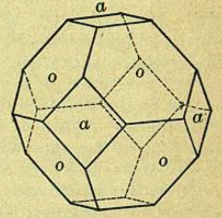
73.



74.



75.



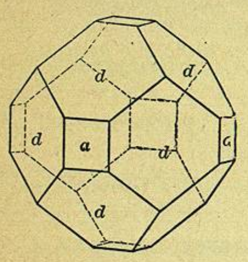
form, as the cube in Fig. 73, the octahedron in Fig 75, etc., is usually said to be *modified* by the faces of the other forms. In Fig. 74 the cube and octahedron are said to be "in equilibrium," since the faces of the octahedron meet at the middle points of the edges of the cube.

It should be carefully noticed, further, that the octahedral faces replace the solid angles of the cube, as regular triangles equally inclined to the adjacent cubic faces, as shown in Fig. 73. Again, the square cubic faces replace the six solid angles of the octahedron, being equally inclined to the adjacent octahedral faces (Fig. 75). The faces of the dodecahedron *truncate* * the twelve

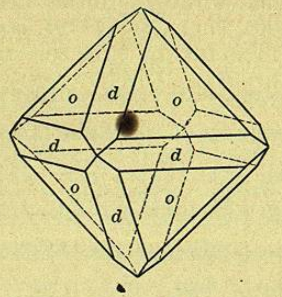
* The words *truncate, truncation,* are used only when the modifying face makes equal angles with the adjacent similar faces.

similar edges of the cube, as shown in Fig. 79. They also truncate the twelve edges of the octahedron (Fig. 77). Further, in Fig. 76 the cubic faces replace the six tetrahedral solid angles of the dodecahedron, while the octahedral faces replace its eight trihedral solid angles (Fig. 78).

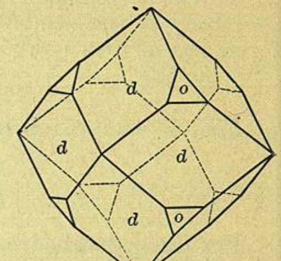
76.



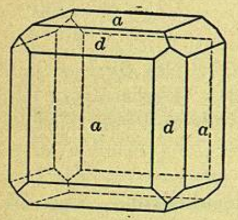
77.



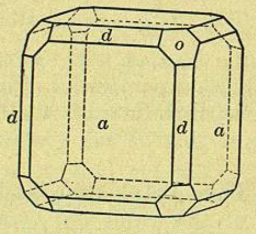
78.



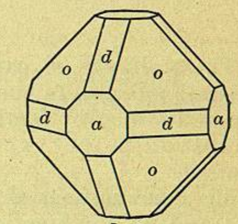
79.



80.



81.



The normal interfacial angles for adjacent faces are as follows :

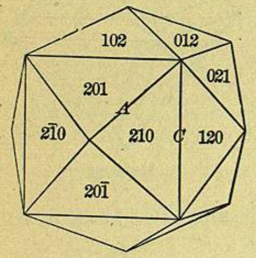
Cube on octahedron,	*ao*, $100 \wedge 111 = 54° \ 44' \ 8''$.
Cube on dodecahedron,	*ad*, $100 \wedge 110 = 45° \ 0' \ 0''$.
Octahedron on dodecahedron,	*od*, $111 \wedge 110 = 35° \ 15' \ 52''$.

56. As explained in Art. **18**, actual crystals always deviate more or less widely from the ideal solids figured, in consequence of the unequal development of like faces. Such crystals, therefore, do not satisfy the geometrical definition of right symmetry relatively to the three principal and the six auxiliary planes mentioned on p. 33, but they do conform to the conditions of crystallographic symmetry, requiring like angular position for similar faces. Again, it will be noted that in a combination form many of the faces do not actually meet the axes within the crystal, as, for example, the octahedral face *o* in Fig. 73. It is still true, however, that this face would meet the axes at equal distances if produced; and since the *axial ratio* is the essential point in the case of each form, and the *actual lengths* of the axes are of no importance, it is not necessary that the faces of the different formsi n a crystal should be referred to the same actual axial lengths. The above remarks will be seen to apply also to all the other forms and combinations of forms described in the pages following.

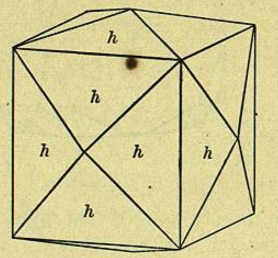
57. Tetrahexahedron.—The tetrahexahedron (Figs. 82, 83, 84) is bounded by twenty-four faces, each of which is an isosceles triangle. Four of these faces together occupy the position of one face of the cube (hexahedron) whence the name commonly applied to this form. The general symbol is (*hk*0), hence each face is parallel to one of the axes while it meets the other two axes at unequal distances. There are two kinds of edges, lettered *A* and *C* in Fig. 82 ; the interfacial angle of either edge is sufficient to determine the symbol of a given form (see below). The angles of some of the common forms are given on a later page (p. 42).

There may be an indefinite number of tetrahexahedrons, as the ratio of the intercepts of the two axes, and hence of *h* to *k* varies ; for example, (410), (310), (210), (320), etc. The form (210) is shown in Fig. 82 ; (410) in Fig. 83, and (530) in Fig. 84. All the tetrahexahedrons fall in a zone with a cubic face and a dodecahedral face. As *h* increases relatively to *k* the form approaches

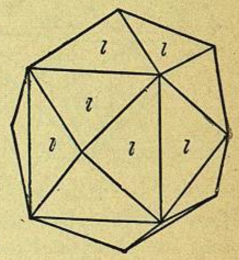
82.



83.

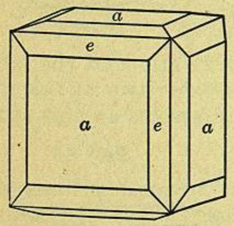


84.

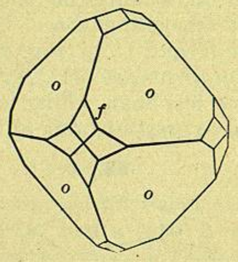


the cube (in which $h : k = \infty : 1$ or $1 : 0$), while as it diminishes and becomes more and more nearly equal to *k* in value it approaches toward the dodecahedron ; for which $h = k$. Compare Fig. 83 and Fig. 84; also Fig. 110. The special symbols belonging to each face of the tetrahexahedron should be carefully noted.

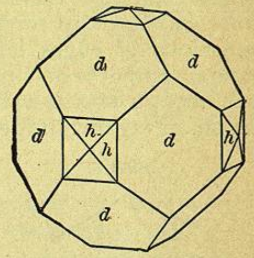
85.



86.



87.



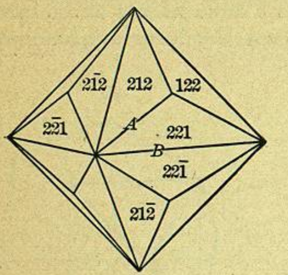
The faces of the tetrahexahedron bevel * the twelve similar edges of the cube, as in Fig. 85; they replace the solid angles of the octahedron by four faces inclined on the edges (Fig. 86), and also the tetrahedral solid angles of the dodecahedron by four faces inclined on the faces (Fig. 87).

58. Trisoctahedron.—The trisoctahedron (Fig. 88), or, more definitely, the trigonal trisoctahedron, is bounded by twenty-four similar faces; each of these is an isosceles triangle, and three together occupy the position of an octahedral face, whence the common name. Further, to distinguish it from the trapezohedron or tetragonal trisoctahedron, it is sometimes called the trigonal trisoctahedron. There are two kinds of edges, lettered *A* and *B* in Fig. 88, and the interfacial angle corresponding to either is sufficient for the determination of the special symbol.

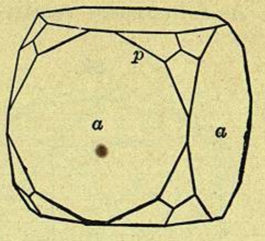
* The word *bevel* is used when two like faces replace the edge of a form and hence are inclined at equal angles to its adjacent similar faces.

The general symbol is (*hhl*) ; common forms are (221), (331), etc. Each

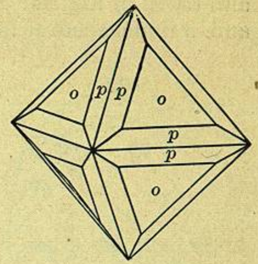
88.



89.



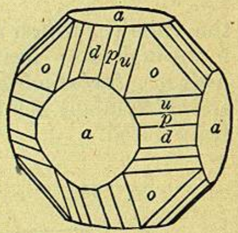
90.



Galena.

face of the trisoctahedron meets two of the axes at a distance less than unity and the third at the unit length, or (which is an identical expression *) it meets two of the axes at the unit length and the third at a distance greater than unity. The symbols belonging to each face should be carefully noted. The normal interfacial angles for some of the more common forms are given on a later page.

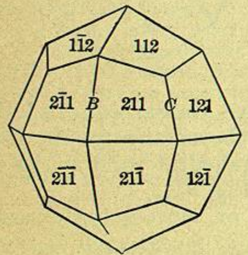
91.



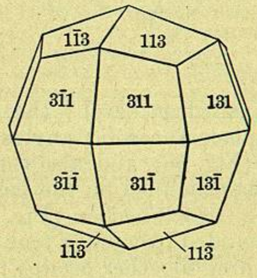
Galena.

59. Trapezohedron.—The trapezohedron † (Figs. 92, 93) is bounded by twenty-four similar faces, each of them a quadrilateral or trapezium. It also bears in appearance a certain relation to the octahedron, whence the name, sometimes employed, of tetragonal trisoctahedron. There are two kinds of edges, lettered *B* and *C*, in Fig. 92. The general symbol is *hll*; common forms are (311), (211), (322), etc. Of the faces, each cuts an axis at a distance less than unity, and the other two at the unit length, or (again, an identical expression) one of them intersects an axis at the

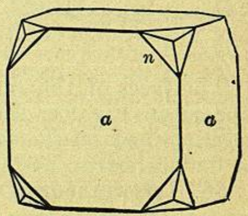
92.



93.



94.



Analcite.

unit length and the other two at distances greater than unity. The symbols belonging to each face should be carefully noted. The normal interfacial

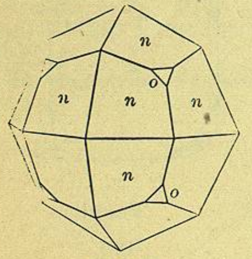
* Since $\frac{1}{2}a : \frac{1}{2}b : \frac{1}{1}c = 1a : 1b : 2c$. The student should read again carefully the explanations in Art. **34**.

† It will be seen later that the name trapezohedron is also given to other solids whose faces are trapeziums, conspicuously to the tetragonal trapezohedron and the trigonal trapezohedron.

angles for some of the common forms are given on a later page. Another name for this form is icositetrahedron.

60. The combinations of these forms with the cube, octahedron, etc., should be carefully studied. It will be seen (Fig. 89) that the faces of the trisoctahedron replace the solid angles of the cube as three faces equally inclined on the *edges*. The faces of the trapezohedron appear as three equal triangles equally inclined to the cubic *faces* (Fig. 94).

95.



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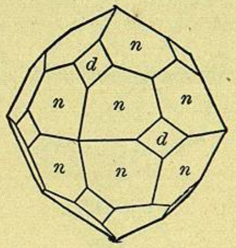
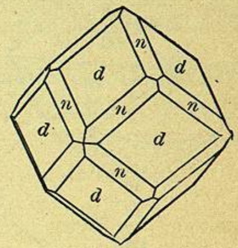
96.



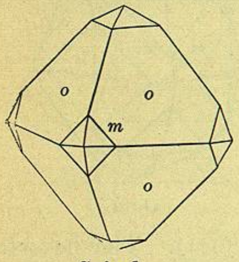
Garnet.

97.



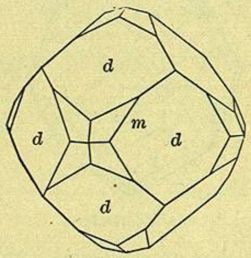
Garnet.

98.



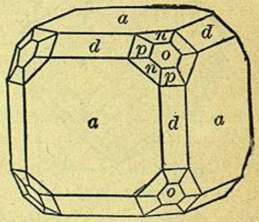
Spinel.

99.



Magnetite.

100.



Amalgam.

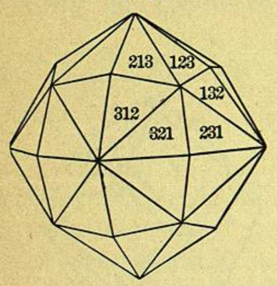
Again, the faces of the trisoctahedron bevel the edges of the octahedron (Fig. 90, also Fig 91, with *p* (221) and *u* (554)), while those of the trapezohedron are triangles inclined to the faces at the extremities of the cubic axes (Fig. 98). Still again, the faces of the trapezohedron (211) truncate the edges of the dodecahedron (110), as shown in Fig. 97; this can be proved to follow at once from the zonal relations (Arts. **43**, **44**), cf. also Fig. 110. The position of the faces of the form (311), in combination with *o*, is shown in Fig. 98; with *d* in Fig. 99. Fig. 100 shows both the trisoctahedron *p* (221) and the trapezohedron *n* (211) with *a*, *o*, and *d*.

It should be added that the trapezohedron *n* (211) is a common form both alone and in combination; *m* (311) is common in combination. The trisoctahedron alone is rarely met with, though in combination (Figs. 90, 91, 100) it is not uncommon.

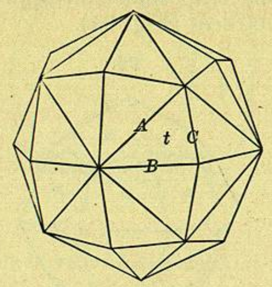
61. Hexoctahedron.—The hexoctahedron, Figs. 101, 102, is the general form in this system ; it is bounded by forty-eight similar faces, each of which is a scalene triangle, and each intersects the three axes at unequal distances. The general symbol is (*hkl*); common forms are (321), shown in Fig. 101, and (421), in Fig. 102. The symbols of the individual faces, as shown in Fig. 101 and more fully in the projection (Fig. 110), should be carefully studied.

The hexoctahedron has three kinds of edges lettered *A*, *B*, *C* (longer, middle, shorter) in Fig. 102; the angles of two of these edges are needed to fix the symbol unless the zonal relations can be made use of. In Fig. 104 the faces of the hexoctahedron bevel the dodecahedral edges, and hence for this

101.

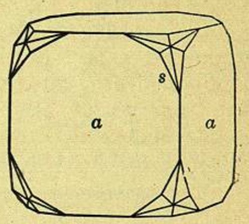


102.



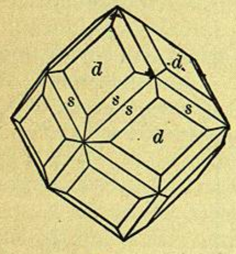
$t = (421)$.

103.



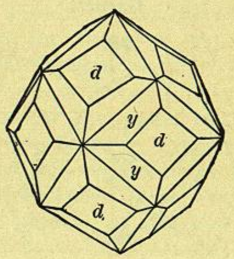
Fluorite.

104.



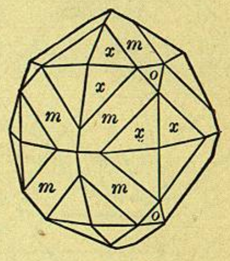
Garnet.

105.



Copper.

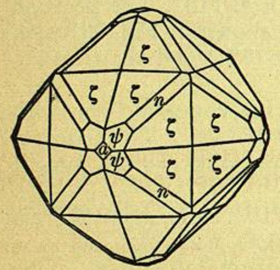
106.



Gold.

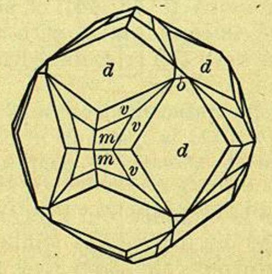
form $h = k + l$; the form *s* has the special symbol (321). The hexoctahedron alone is a very rare form, but it is seen in combination with the cube (Fig. 103, fluorite) as six small faces replacing each solid angle. Fig. 104 is common with garnet; Fig. 105 shows a combination observed in native copper ($y = 18 \cdot 10 \cdot 5$), and Fig. 106 with native gold ($x = 18 \cdot 10 \cdot 1$). The angles of some common hexoctahedrons are given on p. 42.

107.



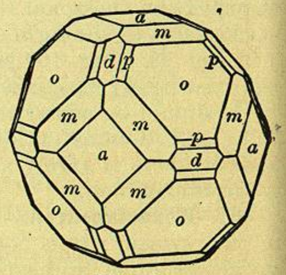
Garnet.

108.



Magnetite.

109.



Microlite.

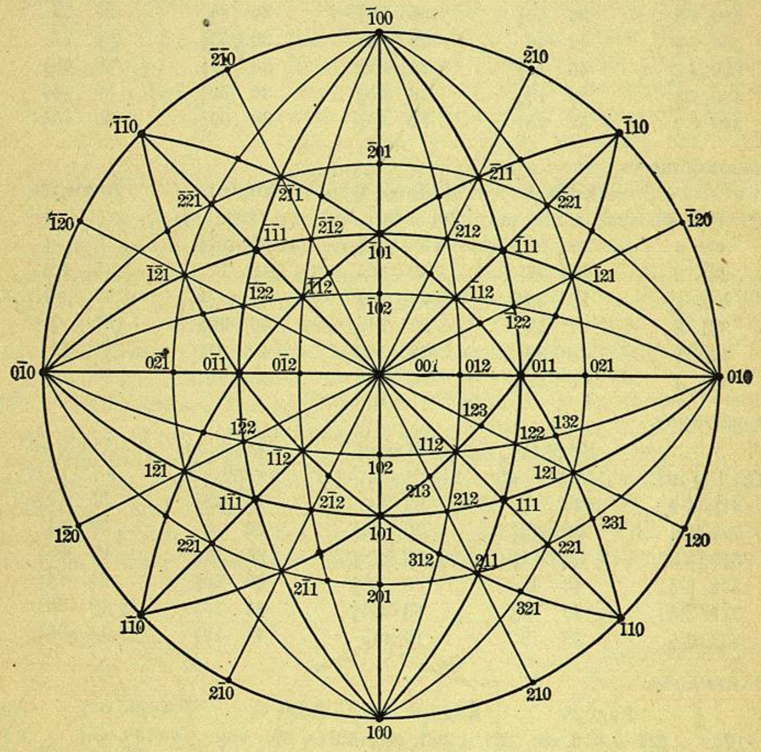
62. Some further examples of isometric forms are given in Figs. 107, 108, 109. In Fig. 107, ψ is the trapezohedron (722); ζ is the hexoctahedron

($64 \cdot 63 \cdot 1$), this last being called a *vicinal form*, since it deviates but slightly in angular position from the simple form ordinarily occurring (*d*, 110); hence the complex indices. In Fig. 108, *v* is the hexoctahedron (531). In Fig. 109, $m = (3\bar{1}1)$, $p = (221)$, etc.

63. Pseudo-symmetry in the Isometric System.—Isometric forms, by development in the direction of one of the cubic axes, simulate tetragonal forms. More common, and of greater interest, are forms simulating those of rhombohedral symmetry by extension, or flattening, in the direction of an octahedral axis. Both these cases are illustrated later. Conversely, certain rhombohedral forms resemble an isometric octahedron in angle and complex twinning.

64. Spherical Projection.—The spherical projection, Fig. 110, shows the

110.



positions of the poles of the faces of the cube (100), octahedron (111), and dodecahedron (110); also the tetrahexahedron (210), the trisoctahedron (221), the trapezohedron (211), and the hexoctahedron (321).

The student should study this projection carefully, noting the symmetry marked by the zone-circles 100, 001, $\bar{1}00$, and 100, 010, $\bar{1}00$; also by 110, 001, $\bar{1}\bar{1}0$; $1\bar{1}0$, 001, $\bar{1}10$; 010, 101, $0\bar{1}0$; 010, $\bar{1}01$, $0\bar{1}0$. Note further that the faces of a given form are symmetrically distributed about a cubic face, as 001; a dodecahedral face, as 101; an octahedral face, as 111.

Note further the symbols that belong to the individual faces of each form, comparing the projection with the figures which precede.

Finally, note the prominent *zones of planes;* for example, the zone between two cubic faces including a dodecahedral face and the faces of all possible tetrahexahedrons. Again, the zones from a cubic face (as 001) through an octahedral face (as 111) passing through the trisoctahedrons, as 113, 112, 223, and the trapezohedrons 332, 221, 331, etc. Also the zone from one dodecahedral face, as 110, to another, as 101, passing through 321, 211, 312, etc. At the same time compare these zones with the same zones shown on the figures already described.

65. Angles of Common Isometric Forms.*

TETRAHEXAHEDRONS.

Cf. Fig. 82.	Edge A 210 ∧ 201, etc.	Edge C 210 ∧ 120, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
410, *i*-4	19 45	61 $55\frac{3}{4}$	14 $2\frac{1}{4}$	45 $33\frac{3}{4}$
310, *i*-3	25 $50\frac{1}{2}$	53 $17\frac{3}{4}$	18 26	43 $5\frac{1}{4}$
520, *i*-$\frac{5}{2}$	30 27	46 $23\frac{3}{4}$	21 48	41 22
210, *i*-2	36 $52\frac{1}{4}$	36 $52\frac{1}{4}$	26 34	39 14
530, *i*-$\frac{5}{3}$	42 40	28 $4\frac{1}{4}$	30 $57\frac{3}{4}$	37 37
320, *i*-$\frac{3}{2}$	46 $11\frac{1}{4}$	22 $37\frac{1}{4}$	33 $41\frac{1}{2}$	36 $48\frac{1}{2}$
430, *i*-$\frac{4}{3}$	50 $12\frac{1}{2}$	16 $15\frac{1}{2}$	36 $52\frac{1}{4}$	36 $4\frac{1}{4}$
540, *i*-$\frac{5}{4}$	52 $25\frac{3}{4}$	12 $40\frac{3}{4}$	38 $39\frac{1}{2}$	35 $45\frac{1}{2}$

TRISOCTAHEDRONS.

Cf. Fig. 88.	Edge A 221 ∧ 212, etc.	Edge B 221 ∧ 22$\bar{1}$, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
332, $\frac{3}{2}$	17 $20\frac{1}{2}$	50 $28\frac{3}{4}$	50 $14\frac{1}{4}$	10 $1\frac{1}{2}$
221, 2	27 16	38 $56\frac{1}{2}$	48 11	15 $47\frac{1}{2}$
552, $\frac{5}{2}$	33 $33\frac{1}{2}$	31 $35\frac{1}{4}$	47 $7\frac{1}{2}$	19 $28\frac{1}{4}$
331, 3	37 $51\frac{3}{4}$	26 $31\frac{1}{2}$	46 $30\frac{1}{2}$	22 0
772, $\frac{7}{2}$	40 59	22 $50\frac{2}{3}$	46 $7\frac{1}{2}$	23 $50\frac{1}{2}$
441, 4	43 $20\frac{1}{2}$	20 $2\frac{3}{4}$	45 52	25 $14\frac{1}{2}$

TRAPEZOHEDRONS.

Cf. Fig. 92.	Edge B 211 ∧ 2$\bar{1}$1, etc.	Edge C 211 ∧ 121, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
411, 4-4	27 16	60 0	19 $28\frac{1}{4}$	35 $15\frac{3}{4}$
722, $\frac{7}{2}$-$\frac{7}{2}$	30 $43\frac{1}{4}$	55 $50\frac{3}{4}$	22 0	32 44
311, 3-3	35 $5\frac{3}{4}$	50 $28\frac{3}{4}$	25 $14\frac{1}{4}$	29 $29\frac{3}{4}$
522, $\frac{5}{2}$-$\frac{5}{2}$	40 45	43 $20\frac{1}{2}$	29 $29\frac{3}{4}$	25 $14\frac{1}{4}$
211, 2-2	48 $11\frac{1}{2}$	33 $33\frac{1}{2}$	35 $15\frac{3}{4}$	19 $28\frac{1}{4}$
322, $\frac{3}{2}$-$\frac{3}{2}$	58 2	19 45	43 $18\frac{3}{4}$	11 $25\frac{1}{4}$

HEXOCTAHEDRONS.

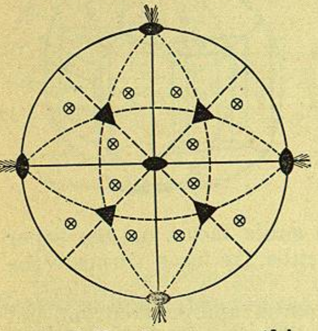
Cf. Fig. 102.	Edge A 321 ∧ 312, etc.	Edge B 321 ∧ 32$\bar{1}$, etc.	Edge C 321 ∧ 231, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
421, 4-2	17 $45\frac{1}{4}$	25 $12\frac{1}{2}$	35 57	29 $12\frac{1}{4}$	28 $6\frac{1}{2}$
18·10·5, $\frac{18}{5}$-$\frac{9}{5}$	19 $12\frac{1}{2}$	27 $17\frac{3}{4}$	30 58	31 $50\frac{3}{4}$	25 $57\frac{1}{4}$
18·10·1, 18-$\frac{9}{5}$	35 $57\frac{3}{4}$	5 $33\frac{3}{4}$	31 $51\frac{1}{4}$	29 $10\frac{1}{2}$	35 $41\frac{1}{2}$
531, 5-$\frac{5}{3}$	27 $39\frac{1}{2}$	19 $27\frac{3}{4}$	27 $39\frac{1}{2}$	32 $18\frac{3}{4}$	28 $33\frac{3}{4}$
321, 3-$\frac{3}{2}$	21 $47\frac{1}{4}$	31 $0\frac{1}{4}$	21 $47\frac{1}{4}$	36 42	22 $12\frac{1}{2}$
432, 2-$\frac{4}{3}$	15 $5\frac{1}{2}$	43 $36\frac{1}{4}$	15 $5\frac{1}{2}$	42 $1\frac{3}{4}$	15 $13\frac{1}{2}$
431, 4-$\frac{4}{3}$	32 $12\frac{1}{4}$	22 $37\frac{1}{4}$	15 $56\frac{1}{2}$	38 $19\frac{3}{4}$	25 4

* A fuller list is given in the Introduction to Dana's System of Mineralogy, pp. xx–xxiii.

2. PYRITOHEDRAL GROUP (2). PYRITE TYPE.

66. Typical Forms and Symmetry.—The typical forms of the pyritohedral group are the *pyritohedron,* or pentagonal dodecahedron, Figs. 112, 113, and the *diploid,* or dyakis-dodecahedron, Fig. 118. The symmetry of these forms, as of the group as a whole, is as follows: There are but three planes of symmetry; these are parallel to the cubic faces and coincide with the planes of the cubic axes. The three crystallographic axes are axes of binary symmetry only; there are also four axes of trigonal symmetry coinciding with the octahedral axes.

111.

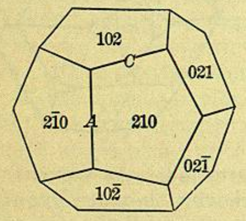


The spherical projection in Fig. 111 shows the distribution of the faces of the general form (*hkl*) and thus exhibits the symmetry of the group. This should be carefully compared with the corresponding projection (Fig. 69) for the normal group, so that the lower grade of symmetry here present be thoroughly understood. In studying the forms described and illustrated in the following pages, this matter of symmetry, especially in relation to that of the normal group, should be continually before the mind.

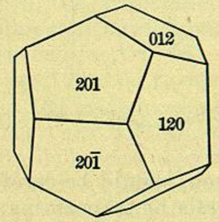
It will be observed that the faces of both the pyritohedron (Fig. 112) and the diploid (Fig. 118) are arranged in parallel pairs, and on this account these forms have been sometimes called *parallel hemihedrons.* Further, those authors who prefer to describe these forms as cases of hemihedrism call this type parallel-faced hemihedrism or pentagonal hemihedrism.

67. Pyritohedron.—The pyritohedron (Fig. 112) is so named because it is a typical form with the common species, pyrite. It is a solid bounded by twelve faces, each of which is a pentagon, but with one edge (*A*, Fig. 112) longer than the other four similar edges (*C*). It is often called a pentagonal dodecahedron, and indeed it resembles closely the regular dodecahedron of geometry, in which the faces are regular pentagons. This latter form is, however, as already noted (Art. **35**), an impossible form in crystallography.

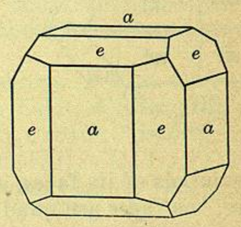
112.



113.



114.



The general symbol is (*hk*0) or like that of the tetrahexahedron of the normal group. Hence each face is parallel to one of the axes and meets the other two axes at unequal distances. Common forms are (410), (310), (210), (320), etc. Besides the *plus* pyritohedron, as (210), there is also the comple-