Finally, note the prominent *zones of planes;* for example, the zone between two cubic faces including a dodecahedral face and the faces of all possible tetrahexahedrons. Again, the zones from a cubic face (as 001) through an octahedral face (as 111) passing through the trisoctahedrons, as 113, 112, 223, and the trapezohedrons 332, 221, 331, etc. Also the zone from one dodecahedral face, as 110, to another, as 101, passing through 321, 211, 312, etc. At the same time compare these zones with the same zones shown on the figures already described.

65. Angles of Common Isometric Forms.*

TETRAHEXAHEDRONS.

Cf. Fig. 82.	Edge A 210 ∧ 201, etc.	Edge C 210 ∧ 120, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
410, *i*-4	19 45	61 $55\frac{3}{4}$	14 $2\frac{1}{4}$	45 $33\frac{3}{4}$
310, *i*-3	25 $50\frac{1}{2}$	53 $17\frac{3}{4}$	18 26	43 $5\frac{1}{4}$
520, *i*-$\frac{5}{2}$	30 27	46 $23\frac{3}{4}$	21 48	41 22
210, *i*-2	36 $52\frac{1}{4}$	36 $52\frac{1}{4}$	26 34	39 14
530, *i*-$\frac{5}{3}$	42 40	28 $4\frac{1}{4}$	30 $57\frac{3}{4}$	37 37
320, *i*-$\frac{3}{2}$	46 $11\frac{1}{4}$	22 $37\frac{1}{4}$	33 $41\frac{1}{2}$	36 $48\frac{1}{2}$
430, *i*-$\frac{4}{3}$	50 $12\frac{1}{2}$	16 $15\frac{1}{2}$	36 $52\frac{1}{4}$	36 $4\frac{1}{4}$
540, *i*-$\frac{5}{4}$	52 $25\frac{3}{4}$	12 $40\frac{3}{4}$	38 $39\frac{1}{2}$	35 $45\frac{1}{2}$

TRISOCTAHEDRONS.

Cf. Fig. 88.	Edge A 221 ∧ 212, etc.	Edge B 221 ∧ 22$\bar{1}$, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
332, $\frac{3}{2}$	17 $20\frac{1}{2}$	50 $28\frac{3}{4}$	50 $14\frac{1}{4}$	10 $1\frac{1}{2}$
221, 2	27 16	38 $56\frac{1}{2}$	48 11	15 $47\frac{1}{2}$
552, $\frac{5}{2}$	33 $33\frac{1}{2}$	31 $35\frac{1}{4}$	47 $7\frac{1}{2}$	19 $28\frac{1}{4}$
331, 3	37 $51\frac{3}{4}$	26 $31\frac{1}{2}$	46 $30\frac{1}{2}$	22 0
772, $\frac{7}{2}$	40 59	22 $50\frac{2}{3}$	46 $7\frac{1}{2}$	23 $50\frac{1}{2}$
441, 4	43 $20\frac{1}{2}$	20 $2\frac{3}{4}$	45 52	25 $14\frac{1}{2}$

TRAPEZOHEDRONS.

Cf. Fig. 92.	Edge B 211 ∧ 2$\bar{1}$1, etc.	Edge C 211 ∧ 121, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
411, 4-4	27 16	60 0	19 $28\frac{1}{4}$	35 $15\frac{3}{4}$
722, $\frac{7}{2}$-$\frac{7}{2}$	30 $43\frac{1}{4}$	55 $50\frac{3}{4}$	22 0	32 44
311, 3-3	35 $5\frac{3}{4}$	50 $28\frac{3}{4}$	25 $14\frac{1}{4}$	29 $29\frac{3}{4}$
522, $\frac{5}{2}$-$\frac{5}{2}$	40 45	43 $20\frac{1}{2}$	29 $29\frac{3}{4}$	25 $14\frac{1}{4}$
211, 2-2	48 $11\frac{1}{2}$	33 $33\frac{1}{2}$	35 $15\frac{3}{4}$	19 $28\frac{1}{4}$
322, $\frac{3}{2}$-$\frac{3}{2}$	58 2	19 45	43 $18\frac{3}{4}$	11 $25\frac{1}{4}$

HEXOCTAHEDRONS.

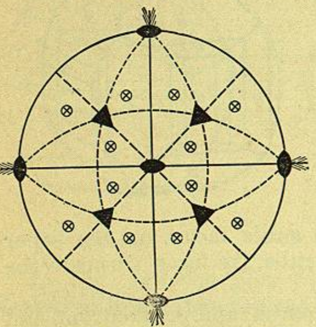
Cf. Fig. 102.	Edge A 321 ∧ 312, etc.	Edge B 321 ∧ 32$\bar{1}$, etc.	Edge C 321 ∧ 231, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
421, 4-2	17 $45\frac{1}{4}$	25 $12\frac{1}{2}$	35 57	29 $12\frac{1}{4}$	28 $6\frac{1}{2}$
18·10·5, $\frac{18}{5}$-$\frac{9}{5}$	19 $12\frac{1}{2}$	27 $17\frac{3}{4}$	30 58	31 $50\frac{3}{4}$	25 $57\frac{1}{4}$
18·10·1, 18-$\frac{9}{5}$	35 $57\frac{3}{4}$	5 $33\frac{3}{4}$	31 $51\frac{1}{4}$	29 $10\frac{1}{2}$	35 $41\frac{1}{2}$
531, 5-$\frac{5}{3}$	27 $39\frac{1}{2}$	19 $27\frac{3}{4}$	27 $39\frac{1}{2}$	32 $18\frac{3}{4}$	28 $33\frac{3}{4}$
321, 3-$\frac{3}{2}$	21 $47\frac{1}{4}$	31 $0\frac{1}{4}$	21 $47\frac{1}{4}$	36 42	22 $12\frac{1}{2}$
432, 2-$\frac{4}{3}$	15 $5\frac{1}{2}$	43 $36\frac{1}{4}$	15 $5\frac{1}{2}$	42 $1\frac{3}{4}$	15 $13\frac{1}{2}$
431, 4-$\frac{4}{3}$	32 $12\frac{1}{4}$	22 $37\frac{1}{4}$	15 $56\frac{1}{2}$	38 $19\frac{3}{4}$	25 4

* A fuller list is given in the Introduction to Dana's System of Mineralogy, pp. xx–xxiii.

2. PYRITOHEDRAL GROUP (2). PYRITE TYPE.

66. Typical Forms and Symmetry.—The typical forms of the pyritohedral group are the *pyritohedron,* or pentagonal dodecahedron, Figs. 112, 113, and the *diploid,* or dyakis-dodecahedron, Fig. 118. The symmetry of these forms, as of the group as a whole, is as follows: There are but three planes of symmetry; these are parallel to the cubic faces and coincide with the planes of the cubic axes. The three crystallographic axes are axes of binary symmetry only; there are also four axes of trigonal symmetry coinciding with the octahedral axes.

111.

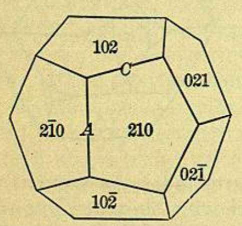


The spherical projection in Fig. 111 shows the distribution of the faces of the general form (hkl) and thus exhibits the symmetry of the group. This should be carefully compared with the corresponding projection (Fig. 69) for the normal group, so that the lower grade of symmetry here present be thoroughly understood. In studying the forms described and illustrated in the following pages, this matter of symmetry, especially in relation to that of the normal group, should be continually before the mind.

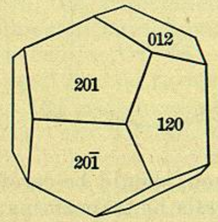
It will be observed that the faces of both the pyritohedron (Fig. 112) and the diploid (Fig. 118) are arranged in parallel pairs, and on this account these forms have been sometimes called *parallel hemihedrons.* Further, those authors who prefer to describe these forms as cases of hemihedrism call this type parallel-faced hemihedrism or pentagonal hemihedrism.

67. Pyritohedron.—The pyritohedron (Fig. 112) is so named because it is a typical form with the common species, pyrite. It is a solid bounded by twelve faces, each of which is a pentagon, but with one edge (A, Fig. 112) longer than the other four similar edges (C). It is often called a pentagonal dodecahedron, and indeed it resembles closely the regular dodecahedron of geometry, in which the faces are regular pentagons. This latter form is, however, as already noted (Art. **35**), an impossible form in crystallography.

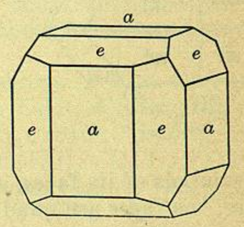
112.



113.



114.

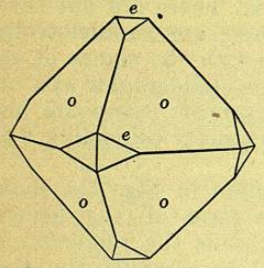


The general symbol is ($hk0$) or like that of the tetrahexahedron of the normal group. Hence each face is parallel to one of the axes and meets the other two axes at unequal distances. Common forms are (410), (310), (210), (320), etc. Besides the *plus* pyritohedron, as (210), there is also the complementary *minus* form* shown in Fig. 113; the symbol is here (120). Other common forms are (250), (230), (130), etc.

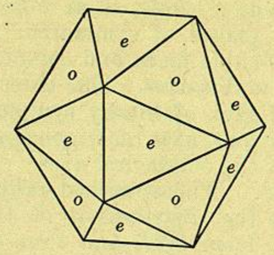
The plus and minus pyritohedrons together embrace twenty-four faces, having the same position as the twenty-four like faces of the tetrahexahedron of the normal group.

68. Combinations.—The faces of the pyritohedron replace the edges of the

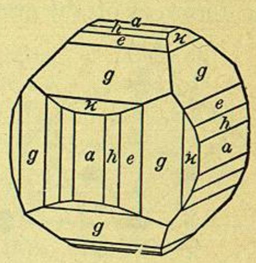
115.



116.



117.

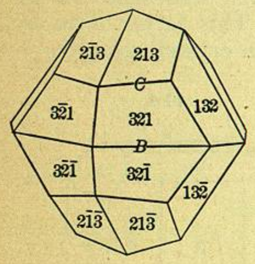


cube, but make unequal angles with two adjacent cubic faces; on the other hand, when the pyritohedron is modified by the cube, its faces truncate the longer edges of the pentagons. Cf. Fig. 114.

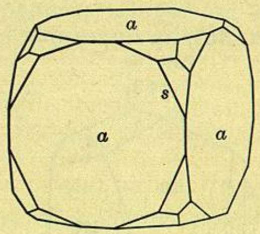
Fig. 115 shows the combination of the pyritohedron and octahedron, and in Fig. 116 these two forms are equally developed. The resulting combination bears a close similarity to the icosahedron, or regular twenty-faced solid, of geometry (see Art. **35**). Here, however, of the twenty faces, the eight octahedral are equilateral triangles, the twelve others belonging to the pyritohedron are isosceles triangles. Fig. 117 shows a number of pyritohedrons with the cube (*a*), namely, *h* (410), *e* (210), *g* (320), and the minus form *κ* (450).

69. Diploid.—The diploid is bounded by twenty-four similar faces, each meeting the axes at unequal distances; its general symbol is hence (*hkl*), and common forms are (321), (421), etc. The form (321) is shown in Fig. 118;

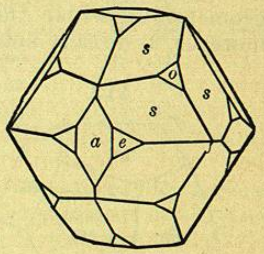
118.



119.



120.

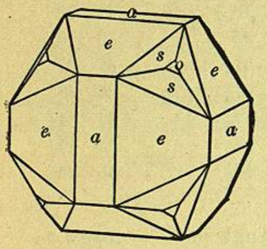


the symbols of its faces, as given, should be carefully studied. As seen in the figure, the faces are quadrilaterals or trapeziums; moreover, they are grouped in pairs, hence the common name diploid. It is also called a dyakis-dodecahedron.

* The minus forms in this and similar cases have sometimes distinct letters, sometimes the same as the plus form, but distinguished by a subscript accent, as *e* (210) and *e*, (120).

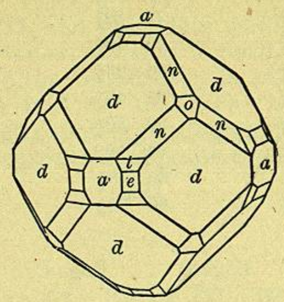
The complementary minus form bears to Fig. 118 the same relation as the minus to the plus pyritohedron. Its faces have the symbols 312, 231, 123, in the front octant, and similarly with the proper negative signs in the others. The plus and minus forms together obviously embrace all the faces of the hexoctahedron of the normal group.

121.



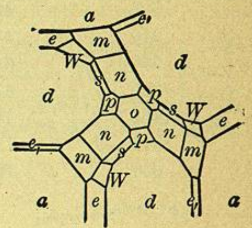
Pyrite.

122.



Pyrite.

123.

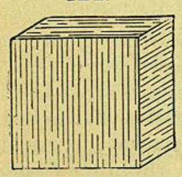


Pyrite.

In Fig. 119 the plus diploid is shown in combination with the cube. Here the three faces replace each of its solid angles. This combination form resembles that of Fig. 89, but the three faces are here unequally inclined upon two adjacent cubic faces. Other combinations of the diploid with the cube, octahedron, and pyritohedron are given in Figs. 120 and 121.

70. Other Forms.—If the pyritohedral type of symmetry be applied to planes each parallel to two of the axes, it is seen that this symmetry calls for six of these, and the resulting form is obviously a cube. This cube cannot be distinguished geometrically from the cube of the normal group, but it has its own characteristic molecular symmetry. Corresponding to this it is common to find cubes of pyrite with fine lines (striations) parallel to the alternate edges, as indicated in Fig. 124. These are due to the partial development of pyritohedral faces (210). On a normal cube such striations, if present, must be parallel to both sets of edges on each cubic face.

124.



Similarly to the cube, the remaining forms of this pyritohedral group, namely, (111), (110), (*hhl*), (*hll*), have the same geometrical form, respectively, as the octahedron, dodecahedron, the trisoctahedrons and trapezohedrons of the normal group. In molecular structure, however, these forms are distinct, each having the symmetry described in Art. **66**.

71. Other combinations of pyritohedral forms are shown in Figs. 122, 123, both of the species pyrite. Fig. 122 is dodecahedral in habit, with the diploid *t* (421), the trapezohedron *n* (211), also *a* (100), *o* (111), *e* (210). In Fig. 123, a single angle of a pyrite crystal is represented with *a* (100), *o* (111), *d* (110); the two pyritohedrons *e* (210) and *e*, (120); the trisoctahedron *p* (221); the trapezohedrons *n* (211), *m* (311); the diploids *s* (321), *W* (851).

This species illustrates well the complexity that may be observed among the crystals of a given mineral. Not only is there wide variation in habit, but the occurring forms are also very numerous. Thus some thirty-five pyritohedrons (+ and −) have been noted and a like number of diploids; also five trisoctahedrons and eleven trapezohedrons.

72. Angles.—The following tables contain the angles of some common forms:

PYRITOHEDRONS.

Cf. Fig. 112.	Edge A $210 \wedge 2\bar{1}0$, etc.	Edge C $210 \wedge 102$, etc.	Angle on *a* (100, *i-i*)	Angle on *o* (111, 1)
410	28° 4¼′	76° 23½′	14° 2¼′	45° 33¾′
310	36 52¼	72 32½	18 26	43 5¼
520	43 36¼	69 49¾	21 48	41 22
210	53 7¾	66 25¼	26 34	39 14
530	61 55¾	63 49¼	30 57¾	37 37
320	67 22¾	62 30¾	33 41½	36 48½
430	73 44½	61 19	36 52¼	36 4¼
540	77 19¼	60 48¼	38 39½	35 45½
650	79 36¾	60 32½	39 48¼	35 35¾

DIPLOIDS.

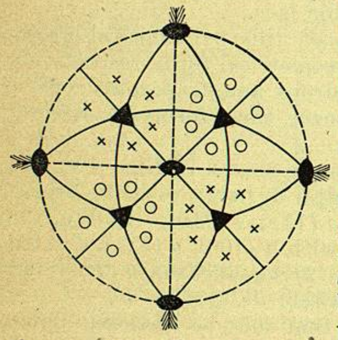
Cf. Fig. 118.	Edge A $321 \wedge 3\bar{2}1$, etc.	Edge B $321 \wedge 32\bar{1}$, etc.	Edge C. $321 \wedge 213$, etc.	Angle on *a* (100, *i i*)	Angle on *o* (111, 1)
421	51° 45¼′	25° 12½	48° 11½′	29° 12¼′	28° 6½′
532	58 14½	37 51¾	35 20	35 47¾	20 30¾
531	60 56½	19 27¾	19 27¾	32 18¾	28 33¾
851	63 36¾	12 6	53 55¼	32 30¾	31 34
321	64 37½	31 0¼	38 12¼	36 42	22 12½
432	67 42½	43 36¼	26 17½	42 1¾	15 13½
431	72 4¾	22 37¼	43 3	38 19¾	25 4

3. TETRAHEDRAL GROUP (3). TETRAHEDRITE TYPE.

73. Typical Forms and Symmetry.—The typical form of this group, and that from which it derives its name, is the *tetrahedron*, shown in Figs. 126, 127. There are also three other distinct forms, shown in Figs. 133, 134, 135.

The symmetry of these forms is that which is characteristic of the entire group. There are six planes of symmetry, parallel respectively to the faces of a rhombic dodecahedron, but no planes of symmetry parallel to the cubic faces. The three cubic axes are axes of binary symmetry only, and the four octahedral axes are axes of trigonal symmetry. There is no center of symmetry.

125.



The spherical projection (Fig. 125) shows the distribution of the faces of the general form (*hkl*) and thus exhibits the symmetry of the group. It will be seen at once that the like faces are *all grouped in the alternate octants*, and this will be seen to be characteristic of all the forms peculiar to this group. The relation between the symmetry here described and that of the normal group must be carefully studied.

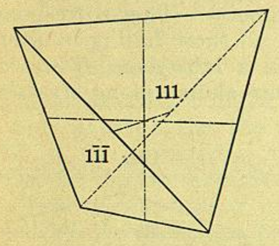
In distinction from the pyritohedral forms whose faces were in parallel pairs, the faces of the tetrahedron and the analogous solids are inclined to

each other, and hence they are sometimes spoken of as *inclined hemihedrons*, and the type of so-called hemihedrism here illustrated is then called inclined or tetrahedral hemihedrism.

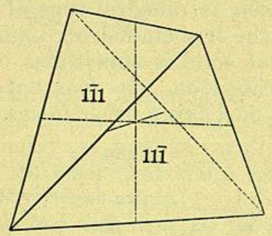
74. Tetrahedron.—The tetrahedron,* as its name indicates, is a four-faced solid, bounded by planes meeting the axes at equal distances. Its general symbol is (111), and the four faces of the plus form (Fig. 126) have the symbols 111, $\bar{1}\bar{1}1$, $\bar{1}1\bar{1}$, $1\bar{1}\bar{1}$. These are four of the faces of the octahedron of the normal group (Fig. 71), and those four which belong to the alternate octants as required by the symmetry already defined.

Each of the four faces of the tetrahedron is an equilateral triangle; the (normal) interfacial angle is 109° 29′ 16″. The tetrahedron is the regular triangular pyramid of geometry, but crystallographically it must be so placed that the axes join the middle points of opposite edges, and one axis is vertical.

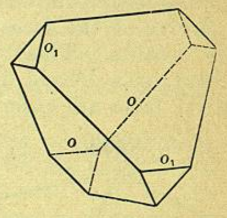
126.



127.

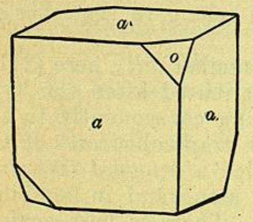


128.

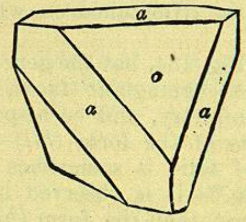


There are two possible tetrahedrons: the *plus* tetrahedron (111), designated by the letter *o*, which has already been described, and the *minus* tetrahedron, having the same geometrical form and symmetry, but the symbols of its four faces are $\bar{1}11$, $1\bar{1}1$, $11\bar{1}$, $\bar{1}\bar{1}\bar{1}$. This second form is shown in Fig. 127; it is usually designated by the letter $o_{,}$. These two forms are, as stated above, identical in geometrical shape, but they may be distinguished in many cases by the tests which serve to reveal the molecular structure, particularly the etching-figures. It is probable that the plus and minus tetrahedrons of

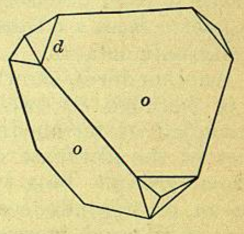
129.



130.



131.



sphalerite (see that species) have a constant difference in this particular, which makes it possible to distinguish them on crystals from different localities and of different habit.

If both tetrahedrons are present together, the form in Fig. 128 results. This is geometrically an octahedron when they are equally developed, but

* This is one of the five regular solids of geometry, which include also the cube, octahedron, the regular pentagonal dodecahedron, and the icosahedron; the last two are impossible forms among crystals.