Mathematical Relations of the Isometric System.

81. Most of the problems arising in the isometric system can be solved at once by the right-angled triangles in the sphere of projection (Fig. 110, p. 41) without the use of any formulas.

It will be remembered that the angles between a cubic face, as 100, and the adjacent face of a tetrahexahedron, 310, 210, 320, etc., can be obtained at once, since the tangent of this angle is equal to $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, or in general $\frac{k}{h}$.

$$\tan\,(hk0 \wedge 100) = \frac{k}{h}.$$

Since all the forms of a given symbol under different species have the same angles, the tables of angles already given are very useful.

These and similar angles may be calculated immediately from the sphere, or often more simply by the formulas given in the following article.

82. Formulas.—(1) The distance of the pole of any face P(hkl) from the cubic faces is given by the following equations. Here Pa is the distance between (hkl) and (100); Pb is the distance between (hkl) and (010); and Pc that between (hkl) and (001).

These equations admit of much simplification in the various special cases, for ($hk0$), (hhl), etc.:

$$\cos^2 Pa = \frac{h^2}{h^2+k^2+l^2}; \qquad \cos^2 Pb = \frac{k^2}{h^2+k^2+l^2}; \qquad \cos^2 Pc = \frac{l^2}{h^2+k^2+l^2}.$$

(2) The distance between the poles of any two faces P(hkl) and Q(pqr) is given by the following equation, which in special cases may also be more or less simplified:

$$\cos PQ = \frac{hp + kq + lr}{\sqrt{(h^2+k^2+l^2)\,(p^2+q^2+r^2)}}$$

(3) The calculation of the supplement interfacial or normal angles for the several forms may be accomplished as follows:

Trisoctahedron.—The angles A and B are, as before, the supplements of the interfacial angles of the edges lettered as in Fig. 88.

$$\cos A = \frac{h^2+2hl}{2h^2+l^2}; \qquad \cos B = \frac{2h^2-l^2}{2h^2+l^2}.$$

For the *tetragonal-tristetrahedron* (Fig. 133), $\cos B = \frac{h^2-2hl}{2h^2+l^2}.$

Trapezohedron (Fig. 92). B and C are the supplement angles of the edges as lettered in the figure.

$$\cos B = \frac{h^2}{h^2+2l^2}; \qquad \cos C = \frac{2hl+l^2}{h^2+2l^2}.$$

For the *trigonal-tristetrahedron* (Fig. 134), $\cos B = \frac{h^2-2l^2}{h^2+2l^2}.$

Tetrahexahedron (Fig. 82).

$$\cos A = \frac{h^2}{h^2+k^2}; \qquad \cos C = \frac{2hk}{h^2+k^2}.$$

For the *pyritohedron* (Fig. 112), $\cos A = \frac{h^2-k^2}{h^2+k^2}$; $\qquad \cos C = \frac{hk}{h^2+k^2}.$

Hexoctahedron (Fig. 102).

$$\cos A = \frac{h^2+2kl}{h^2+k^2+l^2}; \quad \cos B = \frac{h^2+k^2-l^2}{h^2+k^2+l^2}; \quad \cos C = \frac{2hk+l^2}{h^2+k^2+l^2}.$$

For the *diploid* (Fig. 118), $\cos A = \frac{h^2-k^2+l^2}{h^2+k^2+l^2}$; $\quad \cos C = \frac{kl+lh+hk}{h^2+k^2+l^2}.$

For the *hexakistetrahedron* (Fig. 135), $\cos B = \frac{h^2-2kl}{h^2+k^2+l^2}.$

II. TETRAGONAL SYSTEM.

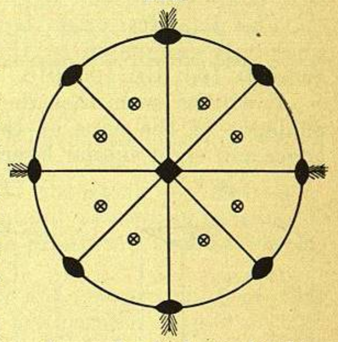
83. The Tetragonal System includes all the forms which are referred to three rectangular axes of which the two lateral axes are equal to each other and the third, the vertical axis, is either shorter or longer. The lateral axes are designated by the letter a; the vertical axis by $\dot{c}$ (see Fig. 149) The length of the vertical axis expresses properly the axial ratio of $a : \dot{c}$, a being uniformly taken as equal to unity.

Seven groups are embraced in this system. Of these the normal group is common and important among minerals; two others have several representatives, and another a single one only. It may be noted that in four of the groups the vertical axis is an axis of tetragonal symmetry; in the remaining three it is an axis of binary symmetry only.

1. NORMAL GROUP (6). ZIRCON TYPE.

84. Symmetry.—The forms belonging to the normal group of the tetragonal system (cf. Figs. 149 to 171) have one principal plane of symmetry, the plane of the lateral axes a, a; further, at right angles to this, and meeting each other at angles of 45° in the vertical axis, $\dot{c}$, two pairs of planes of symmetry, like two-and-two. One of these sets, the axial planes, pass through the crystallographic axes, a, a, and are hence parallel to the faces lettered a; the others are diagonal to them, or parallel to the faces m.

148.



Further, the vertical axis, $\dot{c}$, is a principal axis of tetragonal symmetry; there are also four axes of binary symmetry, like two-and-two; one set coincides with the lateral axes a, a; the others are diagonal to them.

The distribution of the faces of the general form, hkl, belonging to this group, is shown in the spherical projection, Fig. 148.

85. Forms.—The various possible forms under the normal group of this system are as follows:

	Miller.		Naumann.
1. Base or basal pinacoid	(001)	$\infty a : \infty a : c$	$0P$ or O, c
2. Diametral prism, or prism of the second order }	(100)	$a : \infty a : \infty c$	$\infty P \infty$ or i-i, a
3. Unit prism, or prism of the first order }	(110)	$a : a : \infty c$	∞P or I, m
4. Ditetragonal prism	($hk0$)	$a : na : \infty c$	∞Pn or i-n
as, (310) i-3; (210) i-2; (320) i-$\frac{3}{2}$, etc.			
5. Pyramids of the diametral or second order }	($h0l$)	$a : \infty a : mc$	$mP\infty$ or m-i
as, (203) $\frac{2}{3}$-i; e (101) 1-i; (201) 2-i, etc.			

	Miller.		Naumann.
6. Pyramids of the unit, or first order,	........(hhl)	$a : a : mc$	mP or m
as, (223) $\frac{2}{3}$; (111) 1; (221) 2, etc.			
7. Ditetragonal pyramids, or Zirconoids,	.......(hkl)	$a : na : mc$	mPn or m-n
as, (421) 4-2; (321) 3-$\frac{3}{2}$; (122) 1-2, etc.			

86. Basal Pinacoid or Base.—The *base* is that form which includes the two similar faces which are parallel to the plane of the lateral axes. These faces have the symbols 001 and $00\bar{1}$ respectively; it is an "open form," as they do not inclose a space, consequently this form can occur only in combination with other forms. Cf. Figs. 149–152, etc. This form is always lettered *c* in this work.

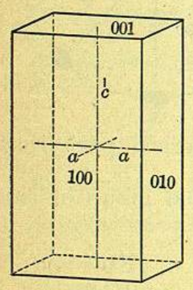
87. Prisms.—Prisms, in systems other than the isometric, have been defined to be forms whose faces are parallel to the vertical axis ($\dot{c}$) of the crystal, while they meet the two lateral axes; in this system the four-faced form whose planes are parallel both to the vertical and a lateral axis is also called a prism. There are hence three types of prisms here included.

88 Diametral Prism.—The *diametral prism* shown* in combination with the base in Fig. 149 includes the four faces which are parallel at once to the vertical and to a lateral axis; it has, therefore, the general symbol (100). It is a square prism, that is, the angle between any two adjacent faces is 90°.

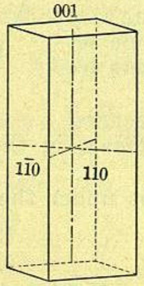
The diametral prism is often called the prism of the second order; it is uniformly designated by the letter *a*, and its faces, taken in order, have the symbols 100, 010, $\bar{1}00$, $0\bar{1}0$.

It will be seen that the combination of this form with the base is the analogue of the cube of the isometric system. It has four similar vertical edges and eight similar lateral edges. It has also eight similar solid angles.

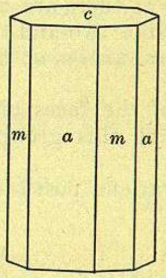
149.



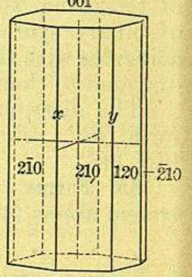
150.



151.



152.



89. Unit Prism.—The *unit prism* includes the four faces which, while parallel to the vertical axis, meet the lateral axes at equal distances; its

* In Figs. 149–152 the dimensions of the form are made to correspond to the assumed length of the vertical axis (here $\dot{c} = 1{\cdot}78$ as in octahedrite) used in Fig. 156. It must be noted, however, that in the case of actual crystals of these forms, while the tetragonal symmetry is usually indicated by the unlike physical character of the face *c* as compared with the faces *a*, *m*. etc., in the vertical prismatic zone, no inference can be drawn as to the relative length of the vertical axis. This last can be determined only when a pyramid is present; it is fixed for the species when a particular pyramid is chosen as fundamental or unit form, as explained later.

general symbol is consequently (110). Like the preceding form, it is a square prism, with interfacial angles of 90°. It is shown in combination with the base in Fig. 150. It is often called the prism of the first order, and is uniformly designated by the letter *m*. The symbols of its faces, taken in order, are 110, $\bar{1}10$, $\bar{1}\bar{1}0$, $1\bar{1}0$.

The faces of the unit prism truncate the edges of the diametral prism and *vice versa*. When both are equally developed, as in Fig. 151, the result is a regular eight-sided prism, which, however, it must be remembered, is a combination of *two* distinct forms.

It is evident that the two prisms described do not differ geometrically from one another, and furthermore, in a given case, the symmetry of this group allows either to be made the unit, and the other the diametral, prism according to the position assumed for the lateral axes. If on crystals of a given species both forms occur together equally developed (or, on the other hand, separately on different crystals) and without other faces than the base, there is no means of telling them apart unless by minor characteristics, as striations or other markings on the surface, etchings, etc.

90. Ditetragonal Prism.—The ditetragonal prism is the form which is bounded by eight similar faces, each one of which is parallel to the vertical axis while meeting the two lateral axes at unequal distances. It has the general symbol ($hk0$). It is shown in Fig. 152, where ($hk0$) = (210). The successive faces have here the symbols 210, 120, $\bar{1}20$, $\bar{2}10$, $\bar{2}\bar{1}0$, $\bar{1}\bar{2}0$, $1\bar{2}0$, $2\bar{1}0$.

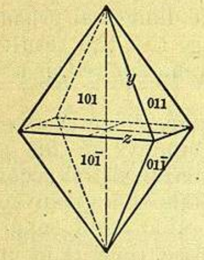
In Fig. 164 a combination is shown of this form (y = 310) with the diametral prism, the edges of which it bevels. In Fig. 168 it bevels (h = 210) the edges of the unit prism *m*. In Fig. 169 it is combined (l = 310) with both the square prisms.

91. Pyramids.—There are three types of pyramids in this group, corresponding, respectively, to the three prisms which have just been described. As already stated, the name *pyramid* is given (in systems other than the isometric) to a form whose planes meet all three of the axes; in this system the form whose planes meet the axis $\dot{c}$ and one lateral axis while parallel to the other is also a pyramid. The pyramids of this group are strictly double pyramids.

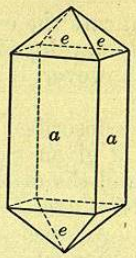
92. Diametral Pyramid.—The *diametral pyramid*, or pyramid of the second order, is the form, Fig 153, whose faces are parallel to one of the lateral axes, while meeting the other two axes. The general symbol is ($h0l$). These faces replace the basal edges of the diametral prism (Fig. 154), and the solid angles of the unit prism (cf. Fig. 155). It is a square pyramid (also called a square octahedron), since its basal section is a square, and the interfacial angles over the four terminal edges, above and below, are equal. The successive faces of the form (101) are as follows: Above 101, 011, $\bar{1}01$, $0\bar{1}1$; below $10\bar{1}$, $01\bar{1}$, $\bar{1}0\bar{1}$, $0\bar{1}\bar{1}$.

If the ratio of the intercepts on the lateral and vertical axes is the assumed axial ratio of the species, the symbol is (101), and the form is designated by the letter *e*. This ratio can be deduced from the measurement of either one of the interfacial angles (y or z) over the terminal or basal edges, as explained later. In the case of a given species, a number of diametral pyramids may occur, varying in the ratio of the axes a and $\dot{c}$. Hence there is possible an indefinite number of such forms whose symbols may be, for example, (104), (103), (102), (101), (302), (201), (301), etc. Those mentioned first come nearest to the base (001), those last to the diametral prism (100); the base is therefore the limit of these pyramids ($h0l$) when $h = 0$, and the diametral prism (100) when $h = \infty$ and $l = 1$; or, what is the same thing, when $h = 1$ and $l = 0$. Fig. 165 shows the three diametral pyramids *u* (105), *e* (101), *q* (201).

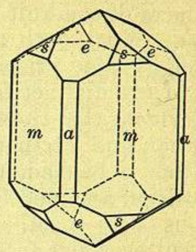
153.



154.



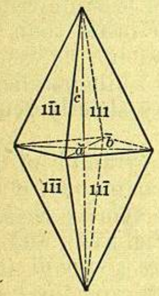
155.



Rutile.

93. Unit Pyramid.—A *unit pyramid*, or pyramid of the first order, is a form whose eight similar faces intersect the two lateral axes at equal distances and also intersect the vertical axis. It has the general symbol (*hhl*). Like the diametral pyramid, it is a square pyramid (or square octahedron) with equal interfacial angles over the terminal edges, and the faces replace the lateral, or basal, edges of the unit prism. If the ratio of the vertical to the lateral axis for a given unit pyramid is the assumed axial ratio for the species, the form is called the *fundamental form*, and it has the symbol (111) as in Fig. 156. Its faces mentioned in order as before are: Above 111, $\bar{1}11$, $\bar{1}\bar{1}1$, $1\bar{1}1$; below $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}\bar{1}$, $1\bar{1}\bar{1}$.

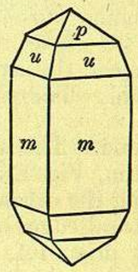
156.



157.

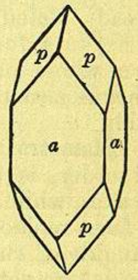
Zircon.

158.



Zircon.

159.



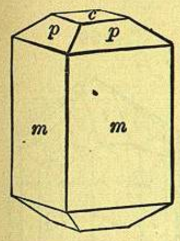
Apophyllite

Obviously the angles of the unit pyramid, and hence its geometrical aspect, vary widely with the length of the vertical axis. For Fig. 156 (octahedrite) $\dot{c} = 1{\cdot}78$; for Fig. 161 $p = (111)$ and $\dot{c} = 0{\cdot}64$.

For a given species there may be a number of unit pyramids, varying in position according to the ratio of the vertical to the lateral axis. Their symbols, passing from the base (001) to the unit prism (110), may thus be (115), (113), (223), (111), (332), (221), (441), etc. In the general symbol of these forms (*hhl*), as *h* diminishes, the form approximates more and more nearly to the base (001), for which $h = 0$; as *h* increases, the form passes toward the unit prism, for which $h = \infty$ if $l = 1$, that is, for which $h = 1$ if $l = 0$. In Fig. 158 two pyramids of this order are shown, *p* (111) and *u* (331).

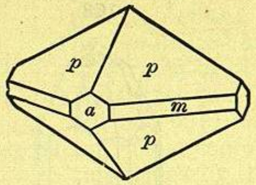
The faces of the unit pyramids replace the terminal edges of the unit prism (Figs. 157, 160) and the solid angles of the diametral prism (Fig. 159).

160.



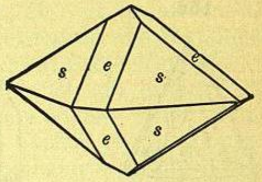
Vesuvianite.

161.



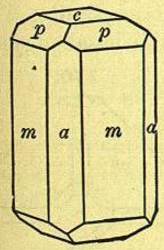
Vesuvianite.

162.



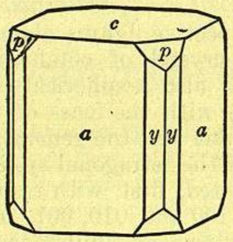
Cassiterite.

163.



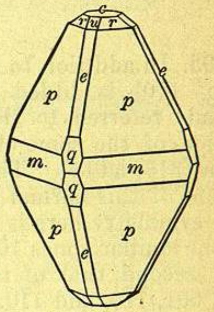
Vesuvianite.

164.



Apophyllite.

165.



Octahedrite.

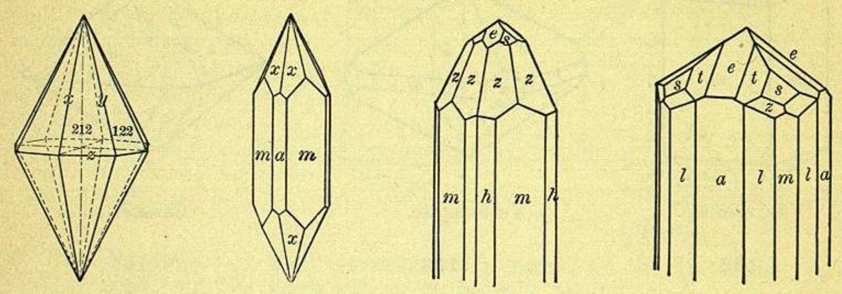
The application of the zonal relations proves that a diametral pyramid truncating the pyramidal edges of a given unit pyramid has the *same* ratio as it has for *h* to *l*. Thus (101) truncates the terminal edge of (111); (201) of (221), etc. Again, if a unit pyramid truncates the pyramidal edges of a given diametral pyramid, its ratio for *h* to *l* is *half* that of the other form; that is, (112) truncates the pyramidal edges of (101); (111) of (201), etc. These relations are exhibited by Fig. 165, and the basal and spherical projections (Figs. 170, 171) corresponding to it. Here *e* (101) and *u* (105) truncate the terminal edges of *p* (111) and *r* (115), respectively, while *p* (111) truncates the edges of *q* (201).

94. Ditetragonal Pyramid, or Zirconoid.—The *ditetragonal pyramid*, or double eight-sided pyramid, is the form each of whose sixteen similar faces meets the three axes at unequal distances. This is the most general case of the symbol (*hkl*), where *h*, *k*, *l* are all unequal and no one is equal to 0. That there are sixteen faces in a single form is evident. Thus, for example, for the form (212) the face 212 is similar to 122, the two lateral axes being equal (not, however, to 221). Hence there are two like faces in each octant. Similarly the symbols of all the faces in the successive octants are, therefore, as follows:

Above	212	122	$\bar{1}22$	$\bar{2}12$	$\bar{2}\bar{1}2$	$\bar{1}\bar{2}2$	$1\bar{2}2$	$2\bar{1}2$
Below	$21\bar{2}$	$12\bar{2}$	$\bar{1}2\bar{2}$	$\bar{2}1\bar{2}$	$\bar{2}\bar{1}\bar{2}$	$\bar{1}\bar{2}\bar{2}$	$1\bar{2}\bar{2}$	$2\bar{1}\bar{2}$

This form is common with the species zircon, and is hence often called a *zirconoid*. It is shown in Fig. 166. It is not observed alone, though sometimes, as in Figs 167 ($x = 311$) and 168 ($z = 321$), it is the predominating form. In Fig. 169 two zirconoids occur, namely, t (313) and z (321). Cf. also Figs. 6, 8, 9 of zircon on p. 8.

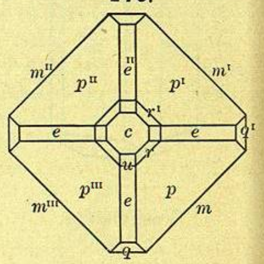
166. 167. 168. 169.



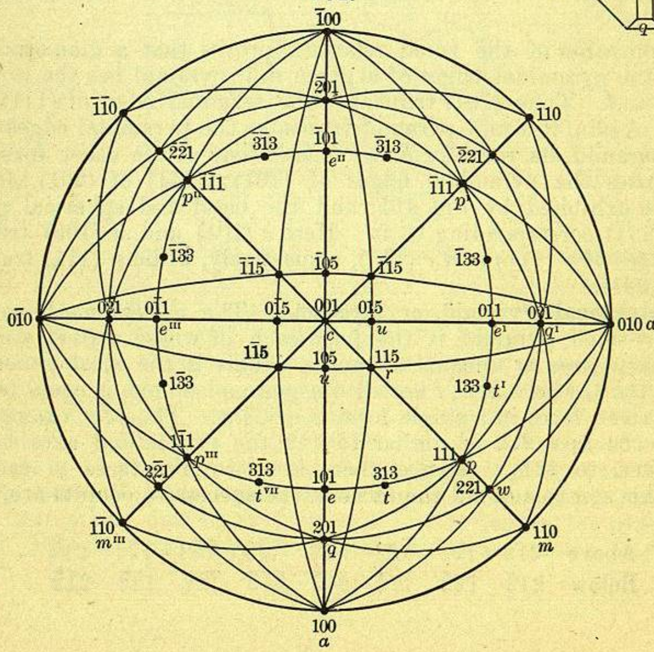
Zircon. Cassiterite. Rutile

95. In addition to the perspective figures already given, a basal projection (Fig 170) is added of the crystal of octahedrite already referred to (Fig. 165); also a spherical projection of the same (Fig. 171) with the faces of the form (313) added. These exhibit well the general relations of this normal group of the tetragonal system. The symmetry here is to be noted, first, with respect to the similar zones 100, 001, $\bar{1}00$ and 010, 001, $0\bar{1}0$; also, second, that of the other pair of similar zones, 110, 001, $\bar{1}\bar{1}0$, and $1\bar{1}0$, 001, $\bar{1}10$.

170.



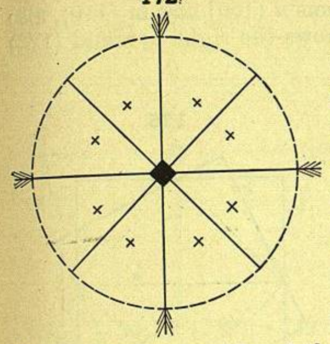
171.



2. HEMIMORPHIC GROUP (7).

96. Symmetry.—This group differs from the normal group only in having no plane of symmetry through the plane of the transverse axes; hence the forms are hemimorphic as defined in Art. **29**. It is not known to be represented among minerals, and is sufficiently illustrated by the spherical projection (Fig. 172). Here the two basal planes are distinct forms, 001 and $00\bar{1}$; the prisms do not differ geometrically from those of the normal group, though distinguished by their molecular structure; further, the pyramids are no longer double pyramids, but each form is represented by one half of Figs. 153, 156, 166 (cf Fig. 50, p 18). There are hence six distinct pyramidal forms, corresponding to the upper and lower halves of the unit pyramid, the diagonal pyramid and the ditetragonal pyramid.

172.

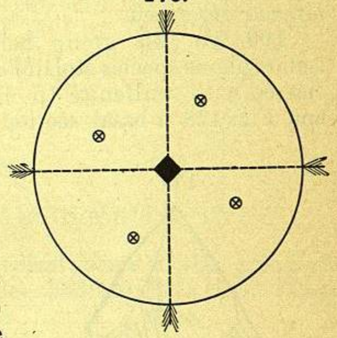


3. PYRAMIDAL GROUP (8). SCHEELITE TYPE.

97. Typical Forms and Symmetry.—The forms here included have one plane of symmetry only, that of the transverse axes, and one axis of tetragonal symmetry (the vertical axis) normal to it. The distinct forms are the tetragonal prism ($hk0$) and pyramid (hkl) of the *third order*, shown in Figs. 174, 175.

The distribution of the faces of the general form (hkl) on the spherical projection, Fig. 173, exhibits the symmetry of the group. Comparing this, as well as the figures immediately following, with those of the normal group, it is seen that this group differs from it in the absence of the vertical planes of symmetry and the horizontal axes of symmetry. Further, half the faces, belonging to each octant, of the normal form (hkl) shown in Fig. 166 only are present, and these are the faces situated in a vertical zone, from 001 to $00\bar{1}$.

173.



98. Prism and Pyramid of the Third Order.—The typical forms of the group, as above stated, are a square prism and a square pyramid, which are distinguished respectively from the square prisms a (100) and m (110), shown in Figs. 149 and 150, and from the square pyramids ($h0l$) and (hhl) of Figs. 153 and 156 by the name "third order."

There are two complementary forms in each case, designated *left* and *right*, which together include all the faces of the ditetragonal prism (Fig. 152) and ditetragonal pyramid (Fig. 166) of the normal group

The faces of the two complementary prisms, as (210), are:

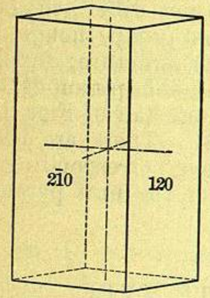
Left: 210, $\bar{1}20$, $\bar{2}\bar{1}0$, $1\bar{2}0$.
Right: 120, $\bar{2}10$, $\bar{1}\bar{2}0$, $2\bar{1}0$.

The faces of the corresponding pyramids, as (212), are:

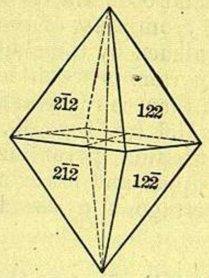
Left: above 212, $\bar{1}22$, $\bar{2}\bar{1}2$, $1\bar{2}2$; below $21\bar{2}$, $\bar{1}2\bar{2}$, $\bar{2}\bar{1}\bar{2}$, $1\bar{2}\bar{2}$.
Right: above 122, $\bar{2}12$, $\bar{1}\bar{2}2$, $2\bar{1}2$; below $12\bar{2}$, $\bar{2}1\bar{2}$, $\bar{1}\bar{2}\bar{2}$, $2\bar{1}\bar{2}$.

Fig. 176 gives a transverse section of the prisms *a* (100) and *m* (110), also the prism of the third series (120). Fig. 175 shows the right pyramid (122) corresponding to the same prism.

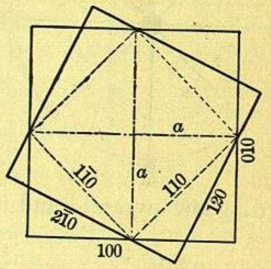
174.



175.



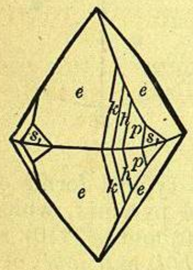
176.



99. Other Forms.—The other forms of this group, that is, the base *c* (001); the other square prisms, *a* (100) and *m* (110); also the square pyramids (*h0l*) and (*hhl*) are geometrically like the corresponding forms of the normal group already described.

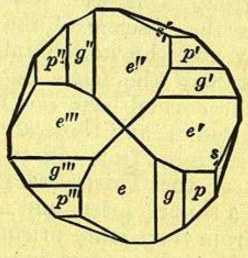
100. To this group belongs the important species scheelite; also the isomorphous species stolzite and powellite, unless it be that they are rather to be classed with wulfenite (p. 61). Fig. 177 shows a typical crystal of scheelite, and Fig. 178 a basal section of one similar; these illustrate well the characteristics of the group.

177.



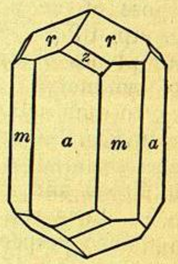
Scheelite.

178.



Scheelite.

179.



Meionite.

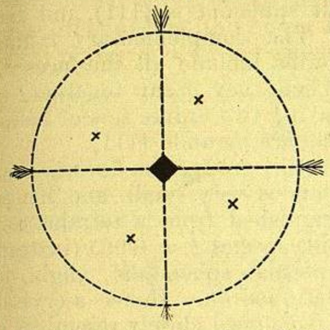
Here the forms are *e* (101), *p* (111), and the third-order pyramids *k* (515), *h* (313), *g* (212), $s_{\prime}$ (131). Fig. 179 represents a meionite crystal with *r* (111) and the third-order pyramid *z* (311) See also Figs. 181, 182, in which the third-order prism is shown.

The forms of this group are sometimes described (see Art. 28) as showing *pyramidal hemihedrism;* hence the name here given.

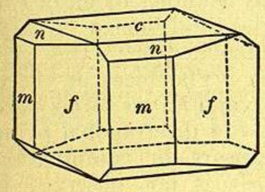
4. PYRAMIDAL-HEMIMORPHIC GROUP (9). WULFENITE TYPE.

101. Symmetry.—The fourth group of the tetragonal system is closely related to the group just described. It has the same vertical axis of tetragonal symmetry, but there is no transverse plane of symmetry. The forms are, therefore, hemimorphic in the distribution of the faces (cf. Fig. 180). The species wulfenite of the Scheelite Group among mineral species probably belongs here, although the crystals do not always show the difference between the pyramidal faces, above and below, which would characterize distinct complementary forms. Figs. 181, 182 could, therefore, serve as illustrations of the preceding group, but in Fig. 183 a characteristic distinction is exhibited. In these figures the forms are *u* (102), *e* (101), *n* (111); also *f* (230), *k* (210), *z* (432), *x* (311).

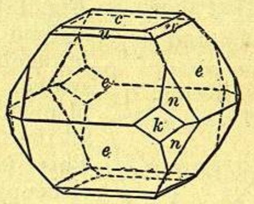
180.



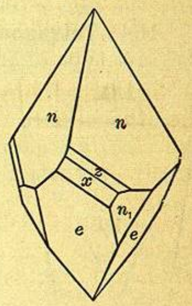
181.



182.



183.

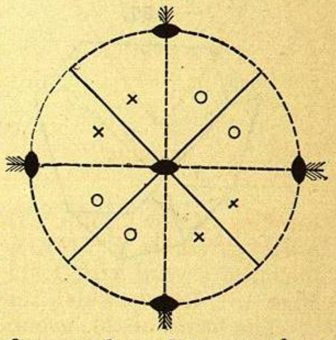


Figs. 181–183, Wulfenite.

5. SPHENOIDAL GROUP (10). CHALCOPYRITE TYPE.

102. Typical Forms and Symmetry.—The typical forms of this group are the sphenoid (Fig. 185) and the tetragonal scalenohedron (Fig. 186). They and all the combinations of this group are characterized by the presence of two vertical planes of symmetry; these are diagonal to the crystallographic axes and intersect at angles of 90° in the vertical axis, which is an axis of binary symmetry only. Further, the two horizontal crystallographic axes are axes of binary symmetry.

184.



This symmetry is exhibited in the distribution of the faces of the general form (*hkl*) in the spherical projection (Fig. 184). It is seen here that the faces are present in the alternate octants only, and it will be remembered that this same statement was made of the tetrahedral group under the isometric system. There is hence a close analogy between these two groups. The symmetry of this group should be carefully compared with that of the first and third groups of this system already described.

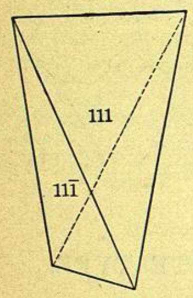
103. Sphenoid.—The sphenoid, shown in Fig. 185, is a four-faced solid, resembling a tetrahedron, but each face is an isosceles (not an equilateral) triangle. The general symbol of the plus unit sphenoid is (111), and the faces have the symbols: 111, $\bar{1}\bar{1}1$, $\bar{1}1\bar{1}$, $1\bar{1}\bar{1}$. The complementary minus sphenoid has the symbol ($1\bar{1}1$), and these two forms include all the faces of the unit pyramid (111) of the normal group. When they occur together, if equally developed, the resulting solid, though having two unlike sets of faces, cannot be distinguished geometrically from the square pyramid (111).

In the species chalcopyrite, which belongs to this group, the deviation in angle and in axial ratio from the isometric system is very small, and hence the unit sphenoid cannot by the eye be distinguished from a tetrahedron (compare Fig. 187 with Fig. 128, p. 47). For this species $\dot{c} = 0{\cdot}985$ (instead of 1, as in the isometric system), and the normal sphenoidal angle is 108° 40′, instead of 109° 28′, the angle of the tetrahedron. Hence a crystal with both the plus and minus sphenoids equally developed closely resembles a regular octahedron.

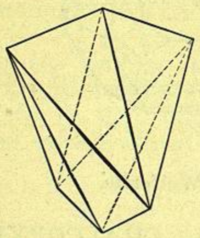
In Fig 188 the diametral pyramids *e* (101) and *z* (201) are also present, also the base *c* (001).

104. Tetragonal Scalenohedron.—The sphenoidal symmetry yields another distinct type of form, that shown in Fig. 186. It is bounded by eight similar scalene triangles, and hence is called a tetragonal scalenohedron; the general symbol is (hkl). The faces of the complementary plus and minus forms embrace all the faces of the ditetragonal pyramid This form appears in combination in chalcopyrite, but is not observed independently. In Fig. 189 the form *s* (531) is the plus tetragonal scalenohedron.

185.

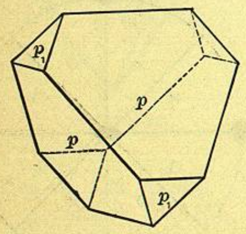


186.

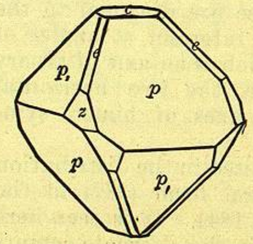


105. Other Forms.—The other forms of the group, namely, the two square prisms, the ditetragonal prism, and the two square pyramids (hhl) and ($h0l$), are geometrically like those of the normal group. The lower symmetry in the molecular structure is only revealed by special investigation, as by etching.

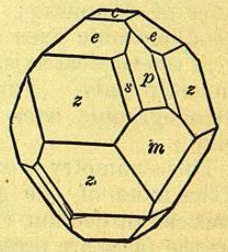
187.



188.



189.

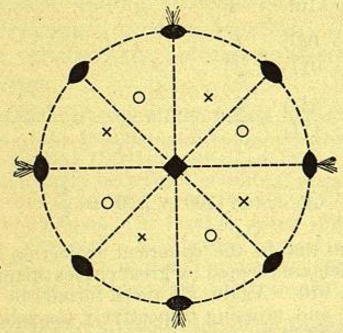


Figs. 187–189, Chalcopyrite.

6. TRAPEZOHEDRAL GROUP (11).

106. The trapezohedral group is analogous to the plagihedral group under the isometric system; it is characterized by the absence of any plane or center of symmetry; the vertical axis, however, is an axis of tetragonal symmetry, and perpendicular to this there are four axes of binary symmetry. The distribution of the faces of the general form (hkl) is shown in the spherical projection,

190.



191.

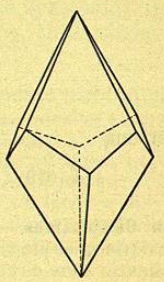


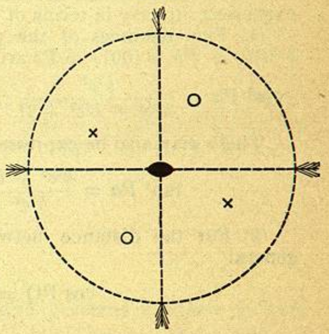
Fig. 190, and Fig. 191 gives the resulting solid, a *tetragonal trapezohedron.* The complementary right- and left-handed forms embrace all the faces of the ditetragonal pyramid of the normal group. These two forms are enantiomorphous, and the salts belonging to this group show circular polarization.

Phosgenite (p. 364) probably belongs to this group (Goldschmidt).

7. TETARTOHEDRAL GROUP (12).

107. Symmetry.—The seventh and last possible group under this system has no plane nor center of symmetry, but the vertical axis is an axis of binary symmetry. The distribution of the faces of the general form (hkl) is shown on the sphere of projection (Fig. 192), and the solid resulting is a sphenoid of the third order. There are also three other possible forms complementary to this, and the four are respectively distinguished as right (+ and −) and left (+ and −). These four together embrace all the sixteen faces of the ditetragonal pyramid. The other characteristic forms of this group are the prism of the third order ($hk0$), the plus and minus sphenoids of the first order (111), and also those of the second order (101). This group has no known representative.

192.



Mathematical Relations of the Tetragonal System.

108. Choice of Axes.—It appears from the discussion of the symmetry of the seven groups of this system that with all of them the position of the vertical axis is fixed. In groups 1, 2, however, where there are two sets of vertical planes of symmetry, either set may be made the axial planes and the other the diagonal planes. The choice between these two possible positions of the lateral axes is guided particularly by the habit of the occurring crystals and the relations of the given species to others of similar form.