symmetry, and there are, further, six horizontal axes of binary symmetry. There is no center of symmetry. The distribution of the faces of the typical form ($hk\bar{i}l$) is shown in the spherical projection (Fig. 221). The typical forms, the right and left hexagonal trapezohedrons (see Fig. 222), are enantiomorphous, and the few crystallized salts falling in this group show circular polarization. The symbols of the right form ($21\bar{3}3$) are as follows:

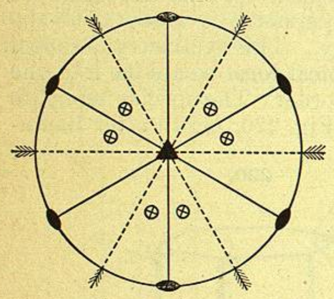
Above $21\bar{3}3$, $\bar{1}3\bar{2}3$, $\bar{3}213$, $\bar{2}\bar{1}33$, $1\bar{3}23$, $3\bar{2}\bar{1}3$.
Below $12\bar{3}\bar{3}$, $\bar{2}3\bar{1}\bar{3}$, $\bar{3}12\bar{3}$, $\bar{1}\bar{2}3\bar{3}$, $2\bar{3}1\bar{3}$, $3\bar{1}\bar{2}\bar{3}$.

B. Trigonal or Rhombohedral Division.

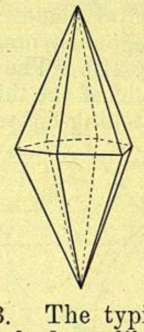
134. General Character.—As stated on p. 65, the groups of this division are characterized by a vertical axis of trigonal, or threefold, symmetry. There are seven groups here included of which the group of the Calcite Type is by far the most important.

135. Trigonotype Group.—The first group (18), that which has strictly the highest grade of symmetry, has no known representatives among crystals, natural or artificial. It has, besides the vertical axis of trigonal symmetry, three horizontal axes of binary symmetry. There are four planes of symmetry, one horizontal, and three others intersecting at angles of 60° in a vertical axis. The characteristic forms are the trigonal prism and pyramid and ditrigonal prism and pyramid. The symmetry is exhibited in Fig. 223. The typical form (Fig. 224) is a double ditrigonal pyramid with terminal edges alike in alternate sets of three each. This form may be compared to a scalenohedron twinned about the vertical axis.

223.



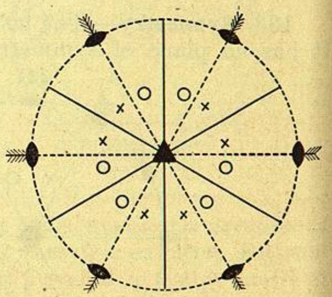
224.



2. RHOMBOHEDRAL GROUP (19). CALCITE TYPE.

136. Typical Forms and Symmetry.—The typical forms of the *rhombohedral group* are the rhombohedron (Fig. 226) and the scalenohedron (Fig. 242). These forms, with the spherical projection, Figs. 225 and 252, illustrate the symmetry characteristic of the group. By comparing Fig. 252 with Fig. 209, p. 70, it will be seen that all the faces in half the sectants are present. This group is hence analogous to the tetrahedral group of the isometric system, and the sphenoidal group of the tetragonal system.

225.



In this group there are three planes of symmetry only; these are diagonal to the crystallographic axes and intersect at angles of 60° in the vertical crystallographic axis. This axis is with these forms an axis of trigonal symmetry; there are, further, three horizontal axes of binary symmetry. Compare Fig. 225, also Fig. 226 *et seq.*

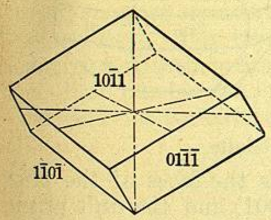
137. Rhombohedron.—Geometrically described, the *rhombohedron* is a solid bounded by six like faces, each a rhomb. It has six like lateral edges forming a zigzag line about the crystal, and six like terminal edges, three above and three in alternate position below. The vertical axis joins the two trihedral solid angles, and the lateral axes join the middle points of the opposite sides, as shown in Fig. 226.

The general symbol of the rhombohedron is ($h0\bar{h}l$), and the successive faces of the unit form ($10\bar{1}1$) have the symbols:

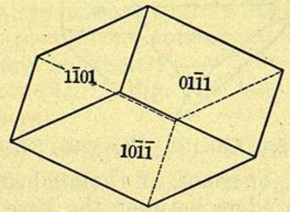
Above, $10\bar{1}1$, $\bar{1}101$, $0\bar{1}11$; below, $01\bar{1}\bar{1}$, $\bar{1}01\bar{1}$, $1\bar{1}0\bar{1}$.

The geometrical shape of the rhombohedron varies widely as the angles change, and consequently the relative length of the vertical axis $\dot{c}$ (expressed in terms of the lateral axes, $a = 1$). As the vertical axis diminishes, the rhombohedrons become more and more obtuse or flattened; and as it increases they become more and more acute. A cube placed with an octahedral axis vertical is obviously the limiting case between the obtuse and acute forms where the interfacial angle is 90°. In Fig. 226 of calcite the normal rhombohedral angle is 74° 55′ and $\dot{c} = 0{\cdot}854$, while for Fig. 228 of hematite this angle is 94° and $\dot{c} = 1{\cdot}366$. Further, Figs. 229–234 show other rhombohedrons of calcite, namely, e ($01\bar{1}2$), ϕ ($05\bar{5}4$), f ($02\bar{2}1$), M ($40\bar{4}1$), and ν ($13\cdot0\cdot\bar{13}\cdot1$), ρ ($16\cdot0\cdot\bar{16}\cdot1$), here the vertical axes are in the ratio of $\frac{1}{2}$, $\frac{5}{4}$, 2, 4, 13, 16, to that of the fundamental (cleavage) rhombohedron of Fig. 226, whose angle determines the value of $\dot{c}$.

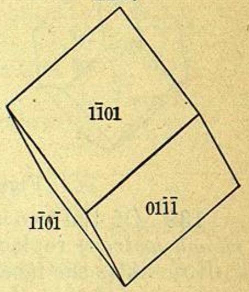
226.



227.



228.



138. Plus and Minus Rhombohedrons.—To every plus rhombohedron there may be an inverse and complementary form, identical geometrically, but bounded by faces falling in the alternate sectants. Thus the minus form of the unit rhombohedron ($01\bar{1}1$) shown in Fig. 227 has the faces:

Above, $01\bar{1}1$, $\bar{1}011$, $1\bar{1}01$; below, $\bar{1}10\bar{1}$, $0\bar{1}1\bar{1}$, $10\bar{1}\bar{1}$.

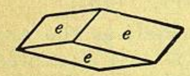
The position of these in the spherical projection (Fig. 252) should be carefully studied; see also Fig. 262. Of the figures already referred to, Figs. 226, 228, 232, 233 are plus, and Figs. 227, 229, 230, 231 minus, rhombohedrons; Fig. 234 shows both forms.

It will be seen that the two complementary plus and minus rhombohedrons of given axial length, that is, of given angle, *e.g.*, $10\bar{1}1$ ($+R$) and $01\bar{1}1$ ($-R$), together embrace all the like faces of the double six-sided pyramid. When these two rhombohedrons are equally developed the form is geometrically identical with this pyramid. This is illustrated by Fig. 237 of gmelinite r ($10\bar{1}1$), ρ ($01\bar{1}1$) and by Figs. 266, 267, p. 83, of quartz, r ($10\bar{1}1$), z ($01\bar{1}1$).*

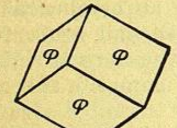
* Quartz serves as a convenient illustration in this case, none the less so notwithstanding the fact that it belongs to the trapezohedral group of this division.

In each case the form, which is geometrically an hexagonal pyramid (in Fig. 237 with *c* and *m*), is in fact a combination of the two unit rhombohedrons, plus and minus. Commonly a difference in size between the two forms may be observed, as in Figs. 236 and 268, where the form taken as the plus rhombohedron predominates. But even if this distinction cannot be established, the two rhombohedrons can always be distinguished by etching, or, as in the case of quartz, by pyro-electrical phenomena.

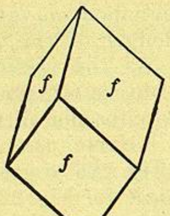
229.



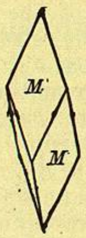
230.



231.



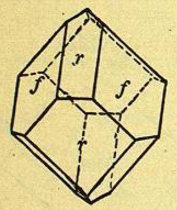
232.



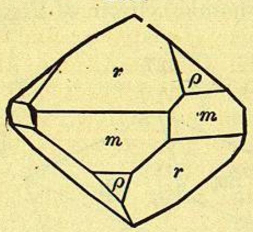
233.

234.

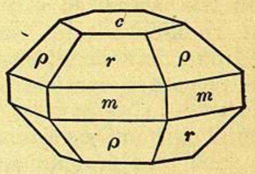
235.



236.



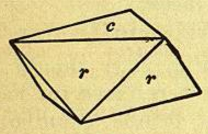
237.



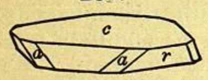
Figs. 229–235, Calcite. Figs. 236, 237, Gmelinite.

139. Of the two series, or zones, of rhombohedrons the faces of the *plus rhombohedrons* replace the edges between the base (0001) and the unit prism ($10\bar{1}0$). Also the faces of the *minus rhombohedrons* replace the alternate edges of the same forms, that is, the edges between (0001) and ($01\bar{1}0$), (compare Figs. 236, 237, etc.). Fig. 238 shows the rhombohedron in combination with the base. Fig. 239 the same with the prism *a* ($11\bar{2}0$). When the angle between the two forms happens to approximate to 70° 32′ the crystal simulates the aspect of a regular octahedron. This is illustrated by Fig. 240; here $co = 69° 42'$, also $oo = 71° 22'$, and the crystal resembles closely an octahedron with truncated edges (cf. Fig. 77, p. 36).

238.

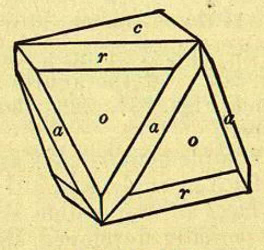


239.



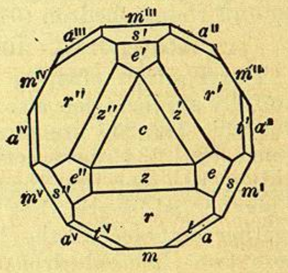
Figs 238, 239, Hematite.

240.



Coquimbite.

241.

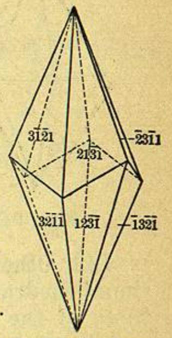


Eudialyte.

140. There is a very simple relation between the plus and minus rhombohedrons which it is important to remember. The form of one series which truncates the terminal edges of a given form of the other has half the length of the vertical axis, and this ratio is expressed in the values of the indices of the two forms. Thus ($01\bar{1}2$), or $-\frac{1}{2}$R, truncates the terminal edges of the plus unit rhombohedron ($10\bar{1}1$), or R; ($10\bar{1}4$), or $+\frac{1}{4}$R, truncates the terminal edges of ($01\bar{1}2$), or $-\frac{1}{2}$R, $10\bar{1}5$ of ($20\bar{2}5$). Again ($10\bar{1}1$), or + R, truncates the edges of ($02\bar{2}1$), or − 2R; ($40\bar{4}1$), or + 4R, of $02\bar{2}1$, or − 2R, etc. This is illustrated by Fig. 235 with the forms *r* ($10\bar{1}1$) and *f* ($02\bar{2}1$). Also in Fig. 241 a basal projection, *z* ($10\bar{1}4$) truncates the edges of *e* ($01\bar{1}2$), *e* ($01\bar{1}2$) of *r* ($10\bar{1}1$); *r* ($1\bar{1}01$) of *s* ($02\bar{2}1$).

141. Scalenohedron.—The *scalenohedron*, shown in Fig. 242, is the general form for this group corresponding to the symbol $hk\bar{i}l$. It is a solid, bounded by twelve faces, each a scalene triangle. It has roughly the shape of a double six-sided pyramid, but there are two sets of terminal edges, one more obtuse than the other, and the lateral edges form a zigzag edge around like that of the rhombohedron. Like the rhombohedrons, the scalenohedrons may be either plus or minus according as to whether the faces are symmetrical to the zone-circle $10\bar{1}0$, 0001, or to $01\bar{1}0$, 0001. The former plus forms correspond in position to the plus rhombohedrons and conversely.

242.



Scalenohedron.

The plus scalenohedron ($21\bar{3}1$), Fig. 242) has the following symbols for the several faces:

Above $21\bar{3}1$, $\bar{2}3\bar{1}1$, $\bar{3}211$, $\bar{1}\bar{2}31$, $1\bar{3}21$, $3\bar{1}\bar{2}1$.
Below $12\bar{3}\bar{1}$, $\bar{1}3\bar{2}\bar{1}$, $\bar{3}12\bar{1}$, $\bar{2}\bar{1}3\bar{1}$, $2\bar{3}4\bar{1}$, $3\bar{2}\bar{1}\bar{1}$.

For the complementary minus scalenohedron ($12\bar{3}1$) the symbols of the faces are:

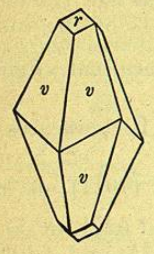
Above $12\bar{3}1$, $\bar{1}3\bar{2}1$, $\bar{3}121$, $\bar{2}\bar{1}31$, $2\bar{3}11$, $3\bar{2}\bar{1}1$.
Below $\bar{2}3\bar{1}\bar{1}$, $\bar{3}21\bar{1}$, $\bar{1}\bar{2}3\bar{1}$, $1\bar{3}2\bar{1}$, $3\bar{1}\bar{2}\bar{1}$, $21\bar{3}\bar{1}$.

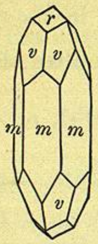
142. Relation of Scalenohedrons to Rhombohedrons.—It was noted above that the scalenohedron in general has a series of zigzag lateral edges like the rhombohedron. It is obvious, further, that for every rhombohedron there will be a series or zone of scalenohedrons having the *same* lateral edges. This is shown in Fig. 245, where the scalenohedron *v* ($21\bar{3}1$) bevels the lateral edges of the fundamental rhombohedron *r* ($10\bar{1}1$); the same would be true of the scalenohedron ($32\bar{5}1$), etc. Further, in Fig. 246, the minus scalenohedron *x* ($13\bar{4}1$) bevels the lateral edges of the minus rhombohedron *f* ($02\bar{2}1$). The relation of the indices which must exist in these cases may be shown to be, for example, for the rhombohedron *r* ($10\bar{1}1$), $h = k + l$; again for *f* ($02\bar{2}1$), $h + 2l = k$, etc. See also the projection. Fig. 252.

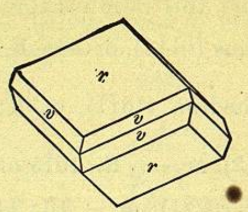
Further, the position of the scalenohedron may be defined with reference to its parent rhombohedron. For example, in Fig. 245 the scalenohedron *v* ($21\bar{3}1$) has three times the vertical axis of the unit rhombohedron *r* ($10\bar{1}1$). Again in Fig 246 *x* ($13\bar{4}1$) has twice the vertical axis of *f* ($02\bar{2}1$). Hence the system of symbols devised by Naumann to express this relation, written in general *m*R*n* or (in Dana's System m^n), where the *n* expresses the multiple value of the vertical axis corresponding to the rhombohedron *m*R. The symbol of v is on this system R3 or 1^3, and of $x - 2R_2$, or -2^2. If mPn is the symbol of a scalenohedron on the hexagonal type and m_0Rn_0 that referred in this way, it may be shown that

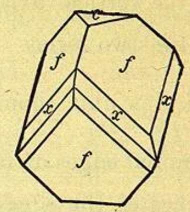
$$m_0 = \frac{m(2-n)}{n}, \quad n_0 = \frac{n}{2-n}. \quad \text{Also } m = m_0n_0, \quad n = \frac{2n}{n_0+1}.$$

243. 244. 245. 246.



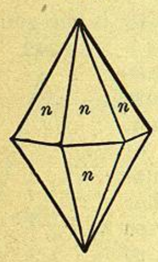


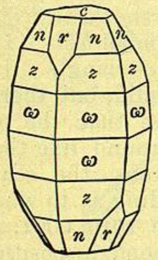


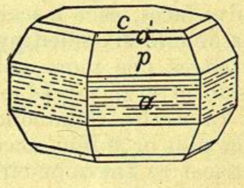


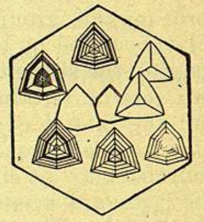
Figs. 243–246, Calcite.

247. 248. 249. 250.





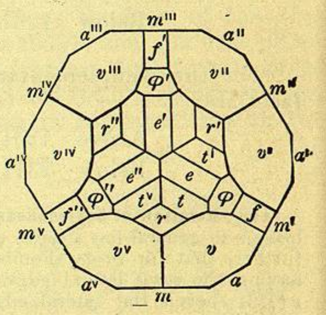




Figs. 247, 248, Corundum. Figs. 249, 250, Spangolite.*

143. Other Forms.—The remaining forms of the normal group of the rhombohedral division are geometrically like those of the corresponding group of the hexagonal system—viz., the base c (0001); the prisms m $(10\bar{1}0)$, a $(11\bar{2}0)$, $(hk\bar{i}0)$; also the diagonal pyramids, as $(11\bar{2}1)$. Some of these forms are shown in the accompanying figures. For further illustrations reference may be made to typical rhombohedral species, as calcite, hematite, etc.

251.



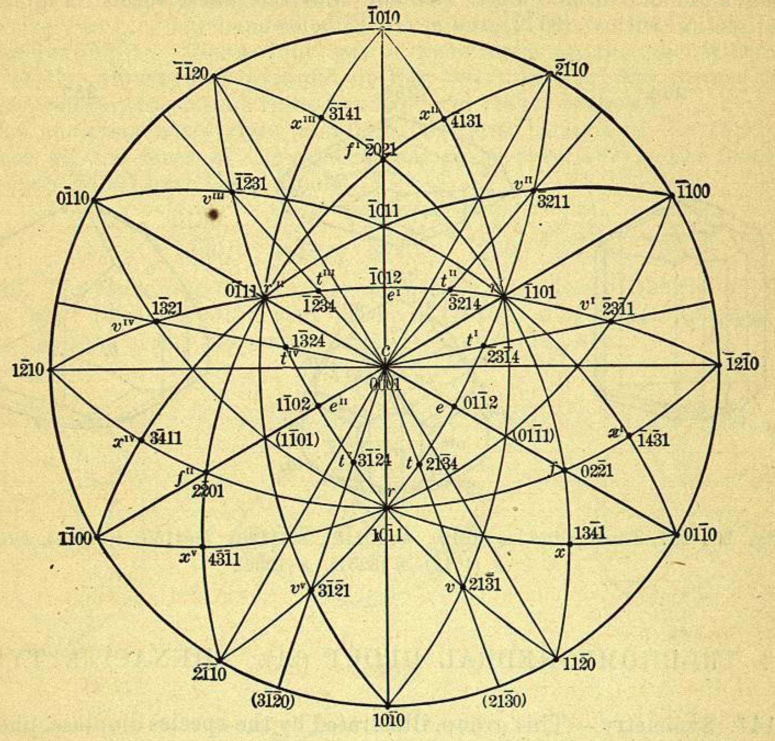
With respect to the diagonal pyramid, it is interesting to note that if it occurs alone (as in Fig. 247, $n = 22\bar{4}3$) it is impossible to say, on geometrical grounds, whether it has the trigonal symmetry of the rhombohedral type or the hexagonal symmetry of the hexagonal type. In the latter case, the form might be made a unit pyramid by exchanging the axial and diagonal planes of symmetry. The true symmetry, however, is often indicated, as with corundum, by the occurrence on other crystals of rhombohedral faces, as r $(10\bar{1}1)$ in Fig. 248 (here $z = 22\bar{4}1$, $\omega = 14 \cdot 14 \cdot \bar{28} \cdot 3$). Even if these are absent (Fig. 249), the etching-figures (Fig. 250) will often serve to reveal the true trigonal molecular symmetry; here $o = (11\bar{2}4)$, $p = (11\bar{2}2)$.

144. A basal projection of a somewhat complex crystal of calcite is given in Fig. 251, and a spherical projection for the same species in Fig. 252; both

* Spangolite belongs properly to the next (hemimorphic) group, but this fact does not destroy the value of the illustration.

show well the symmetry in the distribution of the faces. Here the forms are: prisms, a $(11\bar{2}0)$, m $(10\bar{1}0)$; rhombohedrons, plus, r $(10\bar{1}1)$, minus e $(01\bar{1}2)$, ϕ $(05\bar{5}4)$, f $(02\bar{2}1)$; scalenohedrons, plus, v $(21\bar{3}1)$, t $(21\bar{3}4)$, minus, x $(13\bar{4}1)$.

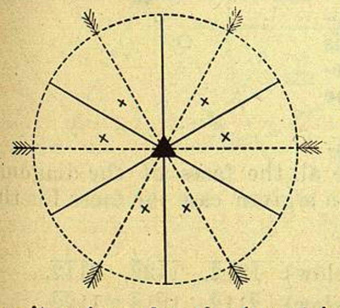
252.



3. RHOMBOHEDRAL-HEMIMORPHIC GROUP (20). TOURMALINE TYPE.

145. Symmetry.—A number of prominent rhombohedral species, as tourmaline, pyrargyrite, proustite, belong to a hemimorphic group under this division. For them the symmetry in the grouping of the faces differs at the two extremities of the vertical axis. The forms have the three diagonal planes of symmetry meeting at angles of 60° in the vertical axis, which is an axis of trigonal symmetry. There are, however, no horizontal axes of symmetry, as in the rhombohedral group, and there is no center of symmetry. Cf. Fig. 253.

253.



146. Typical Forms.—In this group the basal planes (0001) and $(000\bar{1})$ are distinct forms. The other characteristic forms are the two trigonal prisms m $(10\bar{1}0)$ and m_1 $(01\bar{1}0)$ of the unit series; also the four trigonal unit pyramids, corresponding respectively