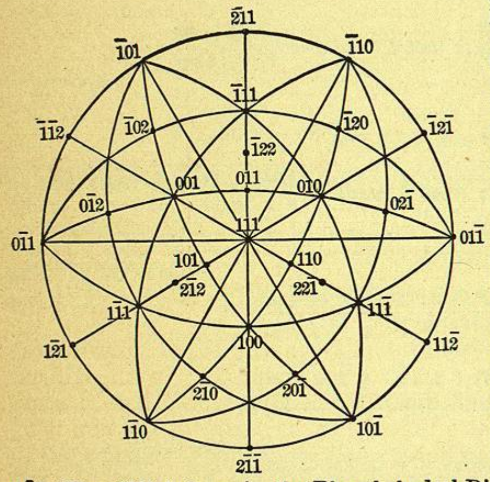
162. Angles.—The angles for some commonly occurring prisms are given in the following table:

		m ($10\bar{1}0$, I)*	a ($11\bar{2}0$, i 2)
$51\bar{6}0$	i-$\frac{6}{5}$	8° 57′	21° 3′
$41\bar{5}0$	i-$\frac{5}{4}$	10 53½	19 6½
$31\bar{4}0$	i-$\frac{4}{3}$	13 54	16 6
$52\bar{7}0$	i-$\frac{7}{5}$	16 6	13 54
$21\bar{3}0$	i-$\frac{3}{2}$	19 6½	10 53½
$32\bar{5}0$	i-$\frac{5}{3}$	23 24¾	6 35¼
$54\bar{9}0$	i-$\frac{9}{5}$	26 19¾	3 40¼

163. The Rhombohedral System of Miller.—The following projection (Fig. 273) is added in order to show the relation of the forms in the hexagonal system as referred by Miller to three equal oblique axes parallel to the faces of the fundamental rhombohedron. The forms are as follows:

273.



The faces having the indices 100, 010, 001 are those of the (plus) fundamental rhombohedron, while the face 111 is the base. The faces $22\bar{1}$, $\bar{1}22$, $2\bar{1}2$ are those of the minus fundamental rhombohedron; with 100, 010, 001 they form the unit hexagonal pyramid.

The faces of the hexagonal unit prism, m ($10\bar{1}0$), have the symbols $2\bar{1}\bar{1}$, $\bar{1}\bar{1}2$, $\bar{1}2\bar{1}$, $\bar{2}11$, $11\bar{2}$, $1\bar{2}1$. Those of the second, or diagonal, hexagonal prism, a ($11\bar{2}0$), have the symbols $10\bar{1}$, $01\bar{1}$, $\bar{1}10$, $\bar{1}01$, $0\bar{1}1$, $1\bar{1}0$.

The dihexagonal pyramid embraces, like the simple hexagonal pyramid, two forms (hkl) and (efg); of these the symbol (hkl) hence belongs to the plus scalenohedron, and (efg) to the minus. In this as in other cases it is true that $e = 2h + 2k - l$, $f = 2h - k + 2l$, $g = -h + 2k + 2l$. For example, the faces $2\bar{1}0$, $20\bar{1}$, $02\bar{1}$, $\bar{1}20$, $\bar{1}02$, $0\bar{1}2$ (Fig. 273) belong in the Rhombohedral Division of this system to the scalenohedron ($21\bar{3}1$). The complementary minus scalenohedron would have the faces $52\bar{4}$, etc. The twenty-four faces of these two forms taken together would embrace all the faces of the dihexagonal pyramid of the Hexagonal Division ($21\bar{3}1$). Cf. Fig. 209, p. 70, and Fig. 252, p. 79, with Fig. 273 given here.

Similarly the dihexagonal prism includes the six faces of the form ($hk0$), and the remaining six of the form ($ef0$).

It is seen at once that the indices given above are those of the isometric system, where the cube corresponds to a rhombohedron of 90°; the projection of Fig. 110, p. 41, is brought into relation with the above if an octahedral axis is placed vertical.

The inconvenience of having the faces of the same form (*e.g.*, the dihexagonal prism or pyramid of beryl) represented by two sets of indices is obvious, and this method, introduced by Miller, is now seldom employed. This objection, however, disappears if the axes and indices described are used for rhombohedral forms only, that is, for forms belonging to the groups which are characterized by a vertical axis of trigonal symmetry. This is the method adopted by Groth (1895). It is believed by the author, however, that the mutual relations of all the groups of both divisions of the hexagonal system among themselves (as also to the groups of the tetragonal system), both morphological and physical, are best brought out by keeping throughout the same axes, namely, those of Fig. 193, Art. 115.

IV. ORTHORHOMBIC SYSTEM.

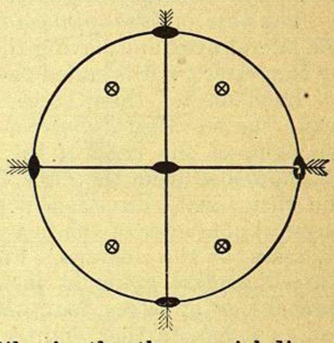
164. The ORTHORHOMBIC SYSTEM includes all the forms which are referred to three unequal axes at right angles to each other.

Of these axes the shorter lateral axis, or *brachy-axis*,* is represented by the letter $\breve{a}$, the longer lateral axis, or *macro-axis*, by $\bar{b}$, and the vertical axis by $\dot{c}$ (cf. Fig. 275). In the statement of the axial ratio $\bar{b}$ is uniformly taken as the unit.

1. NORMAL GROUP (25). BARITE TYPE.

165. Symmetry.—The forms of the *normal group* of the orthorhombic system are characterized by three unlike planes of symmetry, at right angles to each other, and further, coincident with their intersection-lines, there are three axes of binary symmetry, which directions are also those of the crystallographic axes. These axes are consequently fixed in position by the symmetry, but any one of them may be made the vertical axis.

274.



The symmetry of the group is exhibited in the accompanying spherical projection, Fig. 274. This should be compared with Fig. 69 (p. 33) and Fig. 148 (p. 53), representing the symmetry of the normal groups of the isometric and tetragonal systems respectively. It will be seen that while normal isometric crystals are developed alike in the three axial directions, those of the tetragonal type have a like development only in the direction of the two lateral axes, and those of the orthorhombic type are unlike in the three axial directions. Compare also Figs. 70 (p. 34), 149 (p. 54), and 275 (p. 90).

166. Forms.—The various forms possible in this group are as follows:

		Miller.		Naumann.
1.	Macropinacoid or a-pinacoid	…(100)	$\breve{a} : \infty\bar{b} : \infty\dot{c}$	$\infty\bar{P}\infty$ or i-$\bar{i}$, a
2.	Brachypinacoid or b-pinacoid	..(010)	$\infty\breve{a} : \bar{b} : \infty\dot{c}$	$\infty\breve{P}\infty$ or i-$\breve{i}$, b
3.	Base or c-pinacoid	…..(001)	$\infty\breve{a} : \infty\bar{b} : \dot{c}$	$0P$ or O, c
4.	Unit prism	……… (110)	$\breve{a} : \bar{b} : \infty\dot{c}$	∞P or I, m
	Macroprisms	…($hk0$) $h > k$	$\breve{a} : n\bar{b} : \infty\dot{c}$	$\infty\bar{P}\bar{n}$ or i-$\bar{n}$, as (210) i-$\bar{2}$
	Brachyprisms	..($hk0$) $h < k$	$n\breve{a} : \bar{b} : \infty\dot{c}$	$\infty\breve{P}\breve{n}$ or i-$\breve{n}$, as (120) i-$\breve{2}$
5.	Macrodomes	………($h0l$)	$\breve{a} : \infty\bar{b} : m\dot{c}$	$m\bar{P}\infty$ or m-$\bar{i}$, as (201) 2-$\bar{i}$
6.	Brachydomes	………($0kl$)	$\infty\breve{a} : \bar{b} : m\dot{c}$	$m\breve{P}\infty$ or m-$\breve{i}$, as (021) 2-$\breve{i}$
7.	Unit pyramids	…..(hhl)	$\breve{a} : \bar{b} : m\dot{c}$	mP or m, as (221) 2
		(111)	$\breve{a} : \bar{b} : \dot{c}$	P or 1
	Macropyramids	(hkl) $h > k$	$\breve{a} : n\bar{b} : m\dot{c}$	$m\bar{P}\bar{n}$ or m-$\bar{n}$, as (211) 2-$\bar{2}$
	Brachypyramids	(hkl) $h < k$	$n\breve{a} : \bar{b} : m\dot{c}$	$m\breve{P}\breve{n}$ or m-$\breve{n}$, as (121) 2-$\breve{2}$

* The prefixes *brachy-* and *macro-* used in this system (and also in the triclinic system) are from the Greek words βραχύς, *short*, and μακρός, *long*.

In general, as defined on p. 26, a *pinacoid* is a form whose faces are parallel to two of the axes, that is, to an axial plane; a *prism* is one whose faces are parallel to the vertical axis, but intersect the two lateral axes; a *dome** is one whose faces are parallel to one of the lateral axes, but intersect in the vertical axis. A dome is sometimes called a *horizontal prism*; a pyramid is a form whose faces meet all the three axes.

These terms are used in the above sense not only in the orthorhombic system, but also in the monoclinic and triclinic systems; in the last each form consist of two planes only.

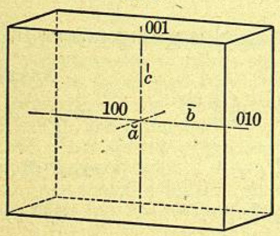
167. Pinacoids.—The *macropinacoid* includes two faces, each of which is parallel both to the macro-axis $\bar{b}$ and to the vertical axis $\dot{c}$; their symbols are respectively 100 and $\bar{1}00$. This form is uniformly designated by the letter a, and is conveniently and briefly called the *a-pinacoid.*

The *brachypinacoid* includes two faces, each of which is parallel both to the brachy-axis $\breve{a}$ and to the vertical axis $\dot{c}$; they have the symbols 010 and $0\bar{1}0$. This form is designated by the letter b; it is called the *b-pinacoid.*

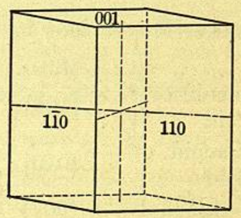
The *base* or *basal pinacoid* includes the two faces parallel to the plane of the lateral axes, and having the symbols 001 and $00\bar{1}$. This form is designated by the letter c; it is called the *c-pinacoid.*

Each one of these three pinacoids is an open-form,† but together they make the so-called *diametral prism,* shown in Fig. 275, a solid which is the analogue of the cube of the isometric system. Geometrically it cannot be distinguished from the cube, but it differs in having the symmetry unlike in the three axial directions; practically this may be shown by the unlike physical character of the faces, a, b, c, for example as to luster, striations, etc.; or, again, by the cleavage. Further, it is proved at once by optical properties. This diametral prism, as just stated, has three pairs of unlike faces. It has three kinds of edges, four in each set, parallel respectively to the axes $\breve{a}$, $\bar{b}$, and $\dot{c}$; it has, further, eight similar solid angles. In Fig. 275 the dimensions are arbitrarily made to correspond to the relative lengths of the axes, but the student will understand that a crystal of this shape gives no suggestions as to these values.

275.



276.



168. Prisms.—The prisms proper include those forms whose faces are parallel to the vertical axis, while they intersect both the lateral axes; their general symbol is, therefore, $(hk0)$. These all belong to one type of *rhombic prism,* in which the interfacial angles corresponding to the two unlike vertical edges have different values.

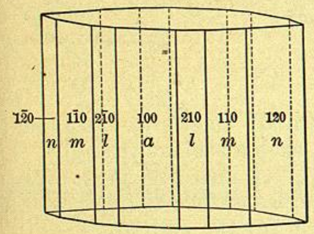
The *unit prism,* (110), is that form whose faces intersect the lateral axes in lengths having a ratio corresponding to the accepted axial ratio of $\breve{a}:\bar{b}$ for the given species; in other words, the angle of this unit prism fixes the unit lengths of the lateral axes. This form is shown in combination with the basal

* From the Latin *domus,* because resembling the roof of a house; cf. Figs. 279, 280.
† See p. 25.

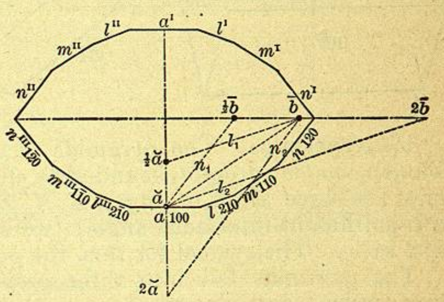
pinacoid in Fig. 276; it is uniformly designated by the letter m. The four faces of the unit prism have the symbols 110, $\bar{1}10$, $\bar{1}\bar{1}0$, $1\bar{1}0$.

The *macroprisms* lie between the macropinacoid, a (100), and the unit prism m (110), and consequently for them the ratio of h to k is greater than 1:1; in other words, the ratio of the intercepts on the axes $\bar{b}$ and $\breve{a}$ is greater than that for the unit prism. Common forms have the symbols (410), (310), (210), (320), (430), etc., given in order from 100 toward 110; cf. the spherical projection of Fig. 303. The face l of Fig. 277 is the macroprism (210); for this form the axial intercepts (see the basal projection, Fig. 278) are in the ratio of $\frac{1}{2}\breve{a}:1\bar{b}$, or $1\breve{a}:2\bar{b}$; a similar relation holds for the other forms (410), etc.

277.



278.



The *brachyprisms* lie between the unit prism and the brachypinacoid b (010), and consequently for them the ratio of the first two indices is less than 1:1, or the ratio of the intercepts on $\bar{b}$, $\breve{a}$ is less than that of the unit prism. Common forms are (340), (230), (120), (130), given in order from 110 toward 010. For the form n (120), shown in Figs. 277, 278, the axial intercepts are in the ratio of $1\breve{a}:\frac{1}{2}\bar{b}$, or $2\breve{a}:\bar{b}$. Other examples of these prisms are given later (see Figs. 296–299).

In Naumann's symbols the number n, the multiple of the lateral axis, is always made greater than unity. Hence while the macroprism, l, of Fig. 277 has the full symbol $\breve{a}:2\bar{b}:\infty\dot{c}$, or briefly $\infty \bar{P}\bar{2}$ (or i-$\bar{2}$), the brachyprism is written $2\breve{a}:\bar{b}:\infty\dot{c}$, or $\infty \breve{P}\breve{2}$ (or i-$\breve{2}$), instead of the equivalent form $\breve{a}:\frac{1}{2}\bar{b}:\infty\dot{c}$. In other words, with the macroprisms (and macropyramids) the value of the brachy-axis is made equal to unity, while with the brachyprisms (and brachypyramids) the macro-axis is taken as the unit.

169. Macrodomes, Brachydomes.—The *macrodomes* are forms whose faces are parallel to the macro-axis, $\bar{b}$, while they intersect the vertical axis $\dot{c}$ and the lateral axis $\breve{a}$; hence the general symbol is $(h0l)$. The angle of the unit macrodome, (101), fixes the ratio of the axes $\breve{a}:\dot{c}$. This form is shown in Fig. 279 combined (since it is an open form) with the brachypinacoid.

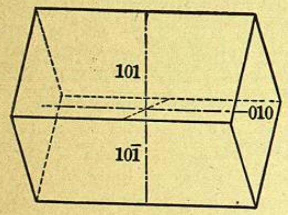
In the macrodome zone between the base c (001) and the macropinacoid a (100) there may be a large number of macrodomes having the symbols, taken in the order named, (103), (102), (203), (101), (302), (201), (301), etc. Cf. Figs. 298 and 302 described later.

The *brachydomes* are forms whose faces are parallel to the brachy-axis, $\breve{a}$, while they intersect the other axes $\dot{c}$ and $\bar{b}$; their general symbol is $(0kl)$. The angle of the unit brachydome, (011), which is shown with a (100) in Fig. 280, determines the ratio of the axes $\bar{b}:\dot{c}$.

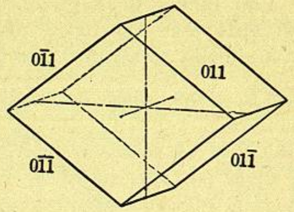
The brachydome zone between c (001) and b (010) includes the forms (013), (012), (023), (011), (032), (021), (031), etc. Cf. Figs. 298 and 302.

Both sets of domes are often spoken of as *horizontal prisms.* The propriety of this expression is obvious, since they are in fact prisms in geometrical form; further, the choice of position for the axes which makes them domes, instead of prisms in the narrower sense, is more or less arbitrary, as already explained elsewhere.

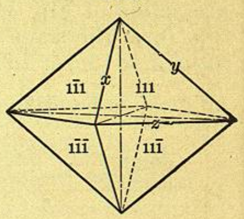
279.



280.



281.



170. Pyramids.—The pyramids in this system all belong to one type, the double *rhombic pyramid,* bounded by eight faces, each a scalene triangle. This form has three kinds of edges, X, Y, Z (Fig. 281; cf. also Fig. 290), each set with a different interfacial angle; two of these angles suffice to determine the axial ratio. The symbol for this, the general form for the system, is (*hkl*).

The pyramids fall into three groups corresponding respectively to the three prisms just described, namely, unit pyramids, macropyramids, and brachypyramids.

The *unit pyramids* are characterized by the fact that their intercepts on the lateral axes have the same ratio as those of the unit prism; that is, the assumed axial ratio ($\breve{a}:\bar{b}$) for the given species. For them, therefore, the general symbol becomes (*hhl*).

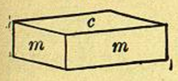
For different unit pyramids on crystals of the same species the vertical axes may have different lengths bearing usually some simple numerical ratio to each other (and always commensurate), and these form a *zone* of faces lying between the base *c* (001) and the unit prism *m* (110). This zone, for example as shown in the basal projection of a sulphur crystal given in Fig. 302, includes the forms *ψ* (119), *ω* (117), *t* (115), *o* (114), *s* (113), *y* (112), *p* (111). Cf. also Fig. 66, p. 30, of the same species, and the spherical projection, Fig. 303. In the symbol of all of the forms of this zone $h = k$, and the lengths of the vertical axes are hence, in the example given, $\frac{1}{9}, \frac{1}{7}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ of the vertical axis $\dot{c}$ of the pyramid *p* (Fig. 290), which in this species is taken as the unit pyramid. The axial ratio for sulphur is given on p. 22.

The *macropyramids* and *brachypyramids* are related to each other and to the unit pyramids, as were the macroprisms and brachyprisms to themselves and to the unit prism. Further, each vertical zone of macropyramids (or brachypyramids), having a common ratio for the lateral axes (or of $h:k$ in the symbol), belongs to a particular macroprism (or brachyprism) characterized by the same ratio. Thus the macropyramids (214), (213), (212), (421), etc., all belong in a common vertical zone between the base (001) and the prism (210). Similarly the brachypyramids (123), (122), (121), (241), etc., fall in a common vertical zone between (001) and (120). Cf. Fig. 299, where *u* and *o* are the brachypyramids (134), (131), falling in the same vertical zone as the brachyprism *d* (130). See also the basal projection, Fig. 302, and the spherical projection, Fig. 303, both of sulphur, noting the relation of the

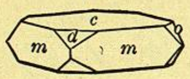
macropyramid (315) to the macroprism (310) and the brachypyramids (135), (133), (131) to the brachyprism (130).

171. Illustrations.—The following figures of barite (282–289) give excellent illustrations of crystals of a typical orthorhombic species, and show also how the habit of one and the same species may vary. The axial ratio for this species is given on p. 96. Here *d* is the macrodome (102) and *o* the unit brachydome (011); *m* is, as always, the unit prism (110). Figs. 282–285 and

282.



283.



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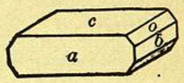

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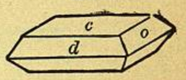
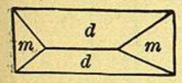
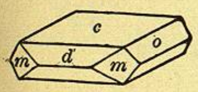


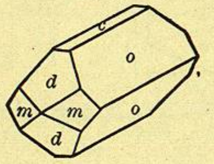
286.



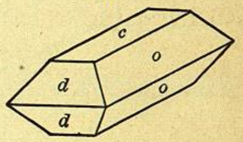
287.



288.



289.

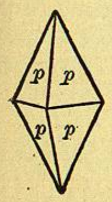


Barite Crystals.

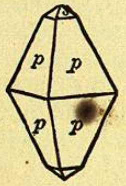
287 are described as tabular ∥ *c*; Fig. 286 is prismatic in habit in the direction of the macro-axis ($\bar{b}$), and 288, 289 prismatic in that of the brachy-axis ($\breve{a}$).

Figs. 290–294 of native sulphur show a series of crystals of pyramidal habit with the unit domes *e* (101), *n* (011), and the unit pyramids *p* (111), *s* (113). Note that *e* and *n* truncate respectively the terminal edges of the fundamental pyramid *p*. In general it should be remembered that a macrodome truncating the edge of a pyramid must have the same ratio of $h:l$; thus, (201) truncates the edge of (221), etc. Similarly of the brachydomes: (021) truncates the edge of (221), etc. Cf. Figs. 302 and 303.

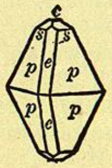
290



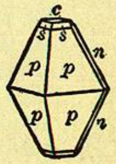
291.



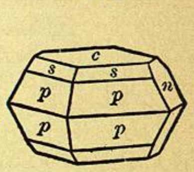
292.



293.



294.



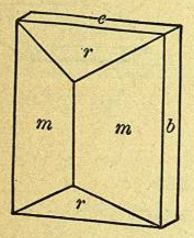
Sulphur Crystals.

Again, Fig. 295, of staurolite, shows the pinacoids *b* (010), *c* (001), the unit prism *m* (110), and the unit macrodome *r* (101).

Figs. 296–298 are prismatic crystals of topaz. Here *m* is the unit prism (110); *l* and *n* are the brachyprisms (120), (140); *d* and *ρ* are the macrodomes (201) and (401); *f* and *y* are the brachydomes (021) and (041); *i*, *u*, and *o* are the unit pyramids (223), (111), (221).

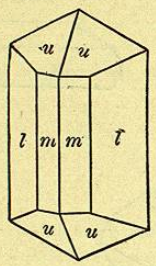
In Fig. 299, of iolite, *s* and *r* are the unit pyramids (112), (111); *d* is the brachyprism (130), and *u*, *o* are the corresponding brachypyramids (134), (131). Fig. 300, of brookite, simulates a tetragonal crystal since the prismatic angle is not very far from 90°; here $z = (112)$. In Fig. 301 of the same species, *e* is the brachypyramid (122); this crystal closely resembles an hexagonal pyramid with its axis placed horizontal since the angles *me* ($110 \wedge 122$) and *ee'* ($122 \wedge \bar{1}22$) are approximately equal.

295.

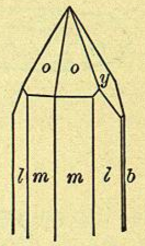


Staurolite.

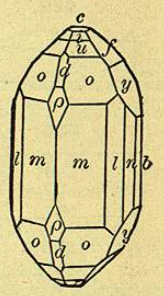
296.



297.

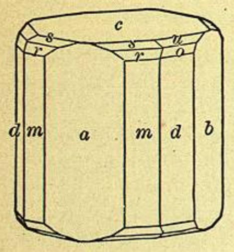


298.



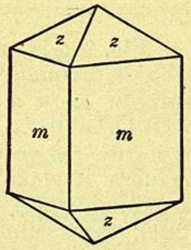
296–298 Topaz.

299.



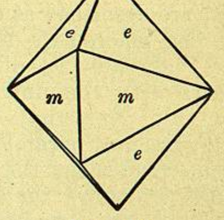
Iolite.

300.



Brookite.

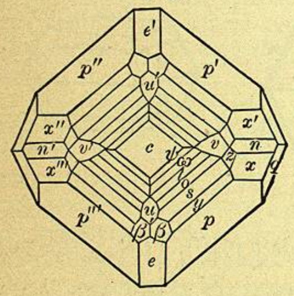
301.



Brookite.

172. Projections.—Basal and spherical projections of a typical orthorhombic species have already been given in Figs. 63 and 65 on pp. 27, 28. The subject is so important, however, that others are given here (Figs. 302, 303) for the species sulphur, cf. Figs. 290–294, also Fig. 66, p. 30. In Fig. 303 besides the pinacoids *a* (100), *b* (010), *c* (001), the positions of the prisms* (310), *m* (110), (130) are shown; the macrodomes *u* (103), *e* (101) and the brachydomes *v* (013), *n* (011); the remarkable zone of unit pyramids *ψ* (119), *ω* (117), *t* (115), *o* (114), *s* (113), *y* (112), *p* (111); finally the macropyramid *β* (315) and the brachypyramids *z* (135) and *x* (133). Both projections exhibit clearly the symmetry

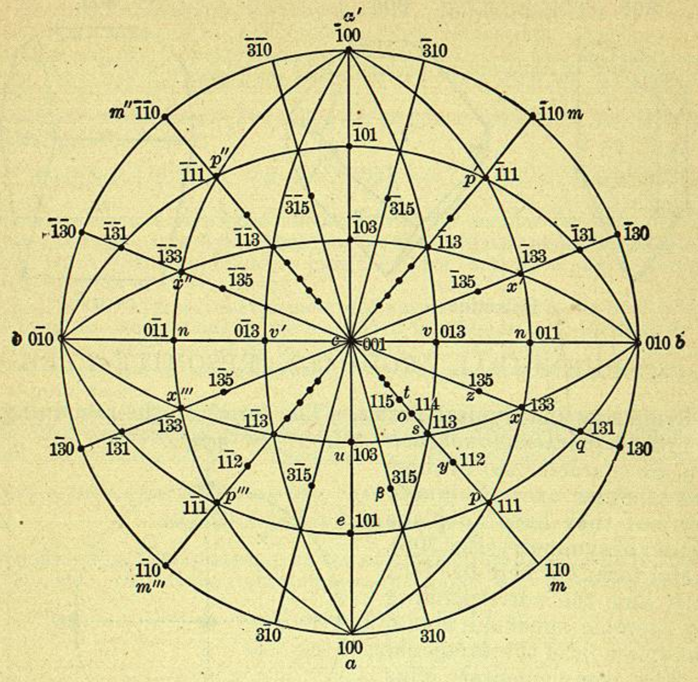
302.



*The prism *m* is not shown in Fig. 302, but is added here for sake of completeness; also (310), (130), forms not yet observed on this species.

characteristic of the group; the prominent zones, already spoken of, should also be noted.

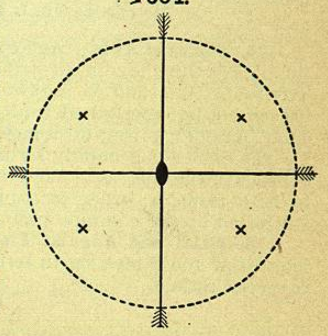
303.



2. HEMIMORPHIC GROUP (26). CALAMINE TYPE.

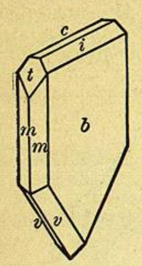
173. Symmetry and Typical Forms.—The forms of the *orthorhombic-hemimorphic* group are characterized by two unlike planes of symmetry and one axis of binary symmetry, the line in which they intersect; there is no center of symmetry. The forms are therefore hemimorphic, as defined in Art. **29**. For example, if, as is usually the case, the vertical axis is made the axis of symmetry, the two planes of symmetry are parallel to the pinacoids *a* (100) and *b* (010). The prisms are then geometrically like those of the normal group, as are also the macropinacoid and brachypinacoid; but the two basal planes become two independent forms, (001) and (00$\bar{1}$). There are also two macrodomes, (101) and (10$\bar{1}$), or in general ($h0l$) and ($h0\bar{l}$); and similarly two sets, for a given symbol, of brachydomes and pyramids.

304.

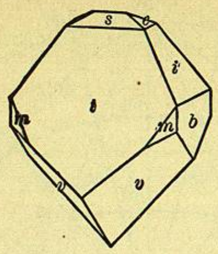


The general symmetry of the group is shown in the spherical projection of Fig. 304. Further, Figs. 305, 306, of calamine, and 307, of struvite, represent typical crystals of this group. In Figs. 305, 306 the forms present are t (301), s (101), i (031), e (011), v $(12\bar{1})$; in Fig. 307 they are s $(10\bar{1})$, s_1 $(10\bar{1})$, q $(0\bar{1}1)$.

305.

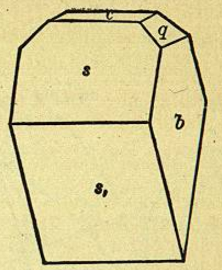


306.



Calamine.

307.

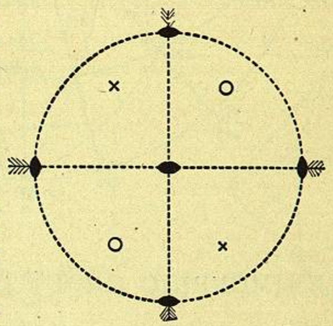


Struvite.

3. SPHENOIDAL GROUP (27). EPSOMITE TYPE.

174. Symmetry and Typical Forms.—The forms of the remaining group of the system, the *orthorhombic-sphenoidal* group, are characterized by three unlike rectangular axes of binary symmetry, but they have no plane and no center of symmetry (Fig. 308). The general form hkl here has four faces only, and the corresponding solid is a rhombic sphenoid, analogous to the sphenoid of the tetragonal system. The complementary plus and minus sphenoids are enantiomorphous. Fig. 309 represents a typical crystal, of epsomite, with the plus sphenoid, z $(1\bar{1}1)$. Other crystals of this species often show both plus and minus complementary forms, but usually unequally developed.

308.



309.

Epsomite.

Mathematical Relations of the Orthorhombic System.

175. Choice of Axes.—As explained in Art. **165**, the three crystallographic axes are fixed as regards direction in all orthorhombic crystals, but any one of them may be made the vertical axis, $\acute{c}$; and of the two lateral axes, which is the longer ($\bar{b}$) and which the shorter ($\breve{a}$) cannot be determined until it is decided which faces to assume as the fundamental, or unit, pyramid, prism, or domes.

The choice is generally so made, in a given case, as to best bring out the relation of the crystals of the species in hand to others allied to them in form or in chemical composition, or in both respects; or, so as to make the cleavage parallel to the fundamental form; or, as suggested by the common habit of the crystals, or other considerations.

176. Axial and Angular Elements.—The *axial elements* are given by the ratio of the lengths of the three axes in terms of the macro-axis, $\bar{b}$, as unity. For example, with barite the axial ratio is

$$\breve{a} : \bar{b} : \acute{c} = 0{\cdot}81520 : 1 : 1{\cdot}31359.$$

The *angular elements* are usually taken as the angles between the three pinacoids and the unit faces in the three zones between them. Thus, again for barite, these elements are

$$100 \wedge 110 = 39^\circ\ 11'\ 13'', \quad 001 \wedge 101 = 58^\circ\ 10'\ 36'', \quad 001 \wedge 011 = 52^\circ\ 43'\ 8''.$$

Two of these angles obviously determine the third angle as well as the axial ratio. The degree of accuracy to be attempted in the statement of the axial ratio depends upon the character of the fundamental measurements from which this ratio has been deduced. There is no good reason for giving the values of $\breve{a}$ and $\acute{c}$ to many decimal places if the probable error of the measurements amounts to many minutes. In the above case the measurements (by Helmhacker) are supposed to be accurate within a few seconds. It is convenient, however, to have the angular elements correct, say, within 10″, so that the calculated angles obtained from them will not vary from those derived direct from the measured angles by more than 30″ to 1′.

177. Calculation of the Axes.—The following simple relations (cf. Art. **46**) connect the axes with the angular elements:

$$\tan(100 \wedge 110) = \breve{a}, \quad \tan(001 \wedge 011) = \acute{c}, \quad \tan(001 \wedge 101) = \frac{\acute{c}}{\breve{a}}.$$

These equations serve to give either the axes from the angular elements, or the angular elements from the axes. It will be noted that the axes are not needed for simple purposes of calculation, but it is still important to have them, for example to use in comparing the morphological relations of allied species.

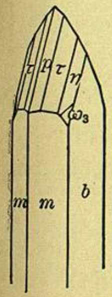
In practice it is easy to pass from the measured angles, assumed as the basis of calculation (or deduced from the observations by the method of least squares), to the angular elements, or from either to any other angles by the application of the tangent principle (Art. **47**) to the pinacoidal zones, and by the solution of the right-angled spherical triangles given on the sphere of projection.

Thus any face hkl (see p. 28) lies in the three pinacoidal zones, 100 and $0kl$, 010 and $h0l$, 001 and $hk0$. For example, the position of the face, 321, is fixed if the positions of two of the poles, 301, 021, 320, are known. These last are given, respectively, by the equations

$$\tan(001 \wedge 301) = 3 \times \tan(001 \wedge 101), \quad \tan(001 \wedge 021) = 2 \times \tan(001 \wedge 011),$$
$$\tan(100 \wedge 320) = \tfrac{2}{3} \times \tan(100 \wedge 110).$$

178. Example.—Fig. 310 represents a crystal of stibnite from Japan, with the faces p (111), τ (343), η (353), etc. On this the following measured angles were taken as fundamental:

310.



$$\eta\eta' \ (353 \wedge \bar{3}53) = 55^\circ\ 1'\ 0'',$$
$$\eta\eta''' \ (353 \wedge 3\bar{5}3) = 99^\circ\ 39'\ 0''.$$

Hence, the angles $353 \wedge 010 = 40^\circ\ 10\tfrac{1}{2}'$ and $353 \wedge 053 = 27^\circ\ 30\tfrac{1}{2}'$ are known without calculation. The right-angled spherical triangle* 010 · 053 · 353 yields the angle $(010 \wedge 053)$ and hence $(001 \wedge 053)$; also the angle at 010, which is equal to $(001 \wedge 101)$. But $\tan(001 \wedge 011) = \tfrac{3}{5} \times \tan(001 \wedge 053)$, and $\tan(001 \wedge 011) = \acute{c}$. Also since $\tan(001 \wedge 101) = \frac{\acute{c}}{\breve{a}}$, the axial ratio is thus known, and two of the angular elements.

The third angular element $(001 \wedge 110)$ can be calculated independently, for the angle at 001 in the triangle 001 · 053 · 353 is equal to $(010 \wedge 350)$ and $\tan(010 \wedge 350) \times \tfrac{5}{3} = (010 \wedge 110)$, the complement of $(100 \wedge 110)$.

Then since $\tan(100 \wedge 110) = \breve{a}$, this can be used to check the value of $\breve{a}$ already obtained. The further use of the tangent principle with the occasional solution of a right-angled triangle will serve to give any desired angle from either the fundamental angles direct, or from the angular elements.

Again, the symbol of any unknown face can be readily calculated if two measured angles of tolerable accuracy are at hand. For example, for the face ω, suppose the measured angles to be

$$b\omega \ (010 \wedge hkl) = 30^\circ\ 15', \quad \omega\omega' \ (hkl \wedge \bar{h}kl) = 51^\circ\ 32'.$$

The solution of the triangle $b \cdot \omega \cdot 0kl$ gives the angle $(010 \wedge 0kl) = 16^\circ\ 25'\ 20''$, and

$$\frac{\tan(001 \wedge 0kl)}{\tan(001 \wedge 011)} = \frac{\tan 73^\circ\ 34\tfrac{2}{3}'}{\tan 45^\circ\ 30\tfrac{1}{2}'} = 3{\cdot}333+, = \frac{k}{l}.$$

* The student in this as in every similar case should draw a spherical projection (not necessarily accurately constructed) to show, if only approximately, the relative position of the faces present.

But the ratio of $k:l$ must be rational and the number derived agrees most closely with 10 : 3.

Again, the angle $(001 \wedge h0l)$ may now be calculated from the same triangle and the value 59° $38\frac{2}{3}'$ obtained. From this the ratio of h to l is derived since

$$\frac{\tan(001 \wedge h0l)}{\tan(001 \wedge 101)} = \frac{\tan 59° 38\frac{2}{3}'}{\tan 45° 43\frac{1}{4}'} = 1{\cdot}665 = \frac{h}{l}.$$

This ratio is nearly equal to 5 : 3, and the two values thus obtained give the symbol 5·10·3. If, however, from the triangle 001·$0kl$·ω, the angle at 001 is calculated, the value 26° $42\frac{3}{4}'$ is obtained, which is also the angle $(010 \wedge hk0)$. From this the ratio $h:k$ is deduced, since

$$\frac{\tan(010 \wedge 110)}{\tan(010 \wedge hk0)} = \frac{\tan 45° 12\frac{5}{6}'}{\tan 26° 42\frac{3}{4}'} = 2{\cdot}002 = \frac{k}{h}.$$

The value of $\frac{k}{h}$ is hence closely equal to 2; this combined with that first obtained $\left(\frac{k}{l} = \frac{10}{3}\right)$ gives the same symbol 5·10·3.

This symbol being more than usually complex calls for fairly accurate measurements. How accurate the symbol obtained is can best be judged by comparing the measured angles with those calculated from the symbol. For example, in the given case the calculated angles for ω (5 10·3) are $b\omega$ $(010 \wedge 5{\cdot}10{\cdot}3) = 30° 16'$, $\omega\omega'$ $(5{\cdot}10{\cdot}3 \wedge \bar{5}{\cdot}10{\cdot}3) = 51° 35'$. The correctness of the value deduced is further established if it is found that the given face falls into prominent zones.

It will be understood further that the zonal relations, explained on pp. 29, 30, play an important part in all calculations. For example, in Fig. 310, if the symbol of τ were unknown, it could be obtained from a single angle (as $b\tau$), since for this zone $h = l$.

179. Formulas.—Although it is not often necessary to employ formulas in calculations, a few are added here for sake of completeness.

(1) For the distance between the pole of any face P (hkl) and the pinacoids ***a, b, c,*** we have in general:

$$\cos^2 \mathrm{P}a = \cos^2(hkl \wedge 100) = \frac{h^2c^2}{h^2c^2 + k^2a^2c^2 + l^2a^2};$$

$$\cos^2 \mathrm{P}b = \cos^2(hkl \wedge 010) = \frac{k^2a^2c^2}{h^2c^2 + k^2a^2c^2 + l^2a^2};$$

$$\cos^2 \mathrm{P}c = \cos^2(hkl \wedge 001) = \frac{l^2a^2}{h^2c^2 + k^2a^2c^2 + l^2a^2}.$$

Here a and c in the formulas are the two axes $\grave{a}$ and $\acute{c}$.

(2) For the distance (PQ) between the poles of any two faces (hkl) and (pqr):

$$\cos \mathrm{PQ} = \frac{hpc^2 + kqa^2c^2 + lra^2}{\sqrt{[h^2c^2 + k^2a^2c^2 + l^2a^2][p^2c^2 + q^2a^2c^2 + r^2a^2]}}.$$

V. MONOCLINIC SYSTEM.

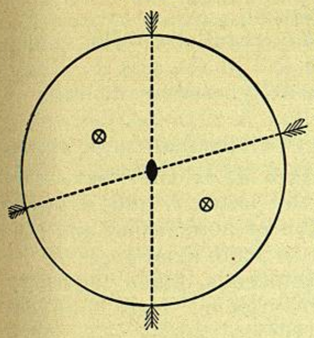
180. The Monoclinic System includes all the forms which are referred to three unequal axes, having one of their axial inclinations oblique.

The axes are designated as follows: the inclined or clinodiagonal axis is $\grave{a}$; the orthodiagonal axis is $\bar{b}$, the vertical axis is $\acute{c}$. The acute angle between the axes $\grave{a}$ and $\acute{c}$ is represented by the letter β; the angles between $\grave{a}$ and $\bar{b}$ and $\bar{b}$ and $\acute{c}$ are right angles. See Fig. 312. Crystals are usually drawn with the axis $\acute{c}$ vertical and the axis $\grave{a}$ directed to the front and inclined downward.

1. NORMAL GROUP (28). GYPSUM TYPE.

181. Symmetry.—In the normal group of the monoclinic system there is one plane of symmetry and one axis of binary symmetry normal to it. The plane of symmetry is always the plane of the axes $\grave{a}$ and $\acute{c}$, and the axis of symmetry coincides with the axis $\bar{b}$, normal to this plane. The position of one axis ($\bar{b}$) and that of the plane of the other two axes ($\grave{a}$ and $\acute{c}$) is thus fixed by the symmetry; but the latter axes may occupy different positions in this plane. Fig. 311 shows the typical spherical projection, projected on the plane of symmetry. Fig. 327 is the projection of an actual crystal of epidote; here, as is usual, the plane of projection is normal to the prismatic zone.

311.



182. Forms.—The various forms* belonging to this group, with their symbols, are given in the following table. As more particularly explained later, an orthodome includes two faces only, and a pyramid four only.

	Miller.		Naumann.
1. Orthopinacoid or a-pinacoid	......(100)	$\grave{a}:\infty\bar{b}:\infty\acute{c}$	$\infty P\bar{\infty}$ or $i\text{-}\bar{i}$, a
2. Clinopinacoid or b-pinacoid	......(010)	$\infty\grave{a}:\bar{b}:\infty\acute{c}$	$\infty P\grave{\infty}$ or $i\text{-}\grave{i}$, b
3. Base or c-pinacoid	......(001)	$\infty\grave{a}:\infty\bar{b}:\acute{c}$	$0P$ or O, c
4. Unit prism	..........(110)	$\grave{a}:\bar{b}:\infty\acute{c}$	∞P or I, m
4. Orthoprisms	....($hk0$) $h>k$	$\grave{a}:n\bar{b}:\infty\acute{c}$	$\infty P\bar{n}$ or $i\text{-}\bar{n}$, as (210) $i\text{-}\bar{2}$
4. Clinoprisms	....($hk0$) $h<k$	$n\grave{a}:\bar{b}:\infty\acute{c}$	$\infty P\grave{n}$ or $i\text{-}\grave{n}$, as (120) $i\text{-}\grave{2}$
5. Orthodomes	($h0l$)	$\grave{a}:\infty\bar{b}:m\acute{c}$	$-mP\bar{\infty}$ or $-m\text{-}\bar{i}$, as (101) $-1\text{-}\bar{i}$
	($\bar{h}0l$)	$\grave{a}:\infty\bar{b}:-m\acute{c}$	$mP\infty$ or $m\text{-}\bar{i}$, as ($\bar{1}01$) $1\text{-}\bar{i}$
6. Clinodomes..........	...($0kl$)	$\infty\grave{a}:\bar{b}:m\acute{c}$	$mP\grave{\infty}$ or $m\text{-}\grave{i}$, as (011) $1\text{-}\grave{i}$
7. Unit pyramids......	(hhl)	$\grave{a}:\bar{b}:m\acute{c}$	$-mP$ or $-m$, as (111) -1
	($\bar{h}hl$)	$\grave{a}:\bar{b}:-m\acute{c}$	mP, as ($\bar{1}11$) 1
7. Orthopyramids.	(hkl) $h>k$	$\grave{a}:n\bar{b}:m\acute{c}$	$-mP\bar{n}$ or $-m\text{-}\bar{n}$, as (211)-2-$\bar{2}$
	($\bar{h}kl$) $h>k$	$\grave{a}:n\bar{b}:-m\acute{c}$	$mP\bar{n}$ or $m\text{-}n$, as ($\bar{2}11$) 2-$\bar{2}$
7. Clinopyramids.	(hkl) $h<k$	$n\grave{a}:\bar{b}:m\acute{c}$	$-mP\grave{n}$ or $-m\text{-}\grave{n}$, as (121)-2-$\grave{2}$
	($\bar{h}kl$) $h<k$	$n\grave{a}:\bar{b}:-m\acute{c}$	$mP\grave{n}$ or $m\text{-}\grave{n}$, as ($\bar{1}21$) 2-$\grave{2}$

The Naumann symbols given above are analogous to those of the orthorhombic system. The long mark employed is to be understood to be conventional only and as referring to the ortho-axis, $\bar{b}$. It does not imply that this axis is longer than the clino-axis, $\grave{a}$, though this is commonly the case. The inclined mark refers to the inclined axis, $\grave{a}$. With some authors these marks pass through the P, instead of being written over the letter (or number) following.

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* On the general use of the terms pinacoid, prisms, domes, pyramids, see pp. 26, 90.