But the ratio of $k : l$ must be rational and the number derived agrees most closely with 10 : 3.

Again, the angle ($001 \wedge h0l$) may now be calculated from the same triangle and the value 59° 38⅔′ obtained. From this the ratio of h to l is derived since

$$\frac{\tan\,(001 \wedge h0l)}{\tan\,(001 \wedge 101)} = \frac{\tan 59°\ 38\frac{2}{3}'}{\tan 45°\ 43\frac{1}{4}'} = 1{\cdot}665 = \frac{h}{l}.$$

This ratio is nearly equal to 5 : 3, and the two values thus obtained give the symbol 5·10·3. If, however, from the triangle $001 \cdot 0kl \cdot \omega$, the angle at 001 is calculated, the value 26° 42¾′ is obtained, which is also the angle ($010 \wedge hk0$). From this the ratio $h : k$ is deduced, since

$$\frac{\tan\,(010 \wedge 110)}{\tan\,(010 \wedge hk0)} = \frac{\tan 45°\ 12\frac{5}{6}'}{\tan 26°\ 42\frac{3}{4}'} = 2{\cdot}002 = \frac{k}{h}.$$

The value of $\frac{k}{h}$ is hence closely equal to 2; this combined with that first obtained $\left(\frac{k}{l} = \frac{10}{3}\right)$ gives the same symbol 5·10·3.

This symbol being more than usually complex calls for fairly accurate measurements. How accurate the symbol obtained is can best be judged by comparing the measured angles with those calculated from the symbol. For example, in the given case the calculated angles for ω (5·10·3) are $b\omega$ ($010 \wedge 5{\cdot}10{\cdot}3$) = 30° 16′, $\omega\omega'$ ($5{\cdot}10{\cdot}3 \wedge \bar{5}{\cdot}10{\cdot}3$) = 51° 35′. The correctness of the value deduced is further established if it is found that the given face falls into prominent zones.

It will be understood further that the zonal relations, explained on pp. 29, 30, play an important part in all calculations. For example, in Fig. 310, if the symbol of τ were unknown, it could be obtained from a single angle (as $b\tau$), since for this zone $h = l$.

179. Formulas.—Although it is not often necessary to employ formulas in calculations, a few are added here for sake of completeness.

(1) For the distance between the pole of any face P (hkl) and the pinacoids a, b, c, we have in general:

$$\cos^2 \mathrm{P}a = \cos^2 (hkl \wedge 100) = \frac{h^2c^2}{h^2c^2 + k^2a^2c^2 + l^2a^2};$$

$$\cos^2 \mathrm{P}b = \cos^2 (hkl \wedge 010) = \frac{k^2a^2c^2}{h^2c^2 + k^2a^2c^2 + l^2a^2};$$

$$\cos^2 \mathrm{P}c = \cos^2 (hkl \wedge 001) = \frac{l^2a^2}{h^2c^2 + k^2a^2c^2 + l^2a^2}.$$

Here a and c in the formulas are the two axes $\grave{a}$ and $\acute{c}$.

(2) For the distance (PQ) between the poles of any two faces (hkl) and (pqr):

$$\cos \mathrm{PQ} = \frac{hpc^2 + kqa^2c^2 + lra^2}{\sqrt{[h^2c^2 + k^2a^2c^2 + l^2a^2][p^2c^2 + q^2a^2c^2 + r^2a^2]}}.$$

V. MONOCLINIC SYSTEM.

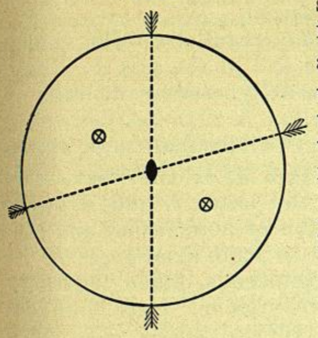
180. The MONOCLINIC SYSTEM includes all the forms which are referred to three unequal axes, having one of their axial inclinations oblique.

The axes are designated as follows: the inclined or clinodiagonal axis is $\grave{a}$; the orthodiagonal axis is $\bar{b}$, the vertical axis is $\acute{c}$. The acute angle between the axes $\grave{a}$ and $\acute{c}$ is represented by the letter β; the angles between $\grave{a}$ and $\bar{b}$ and $\bar{b}$ and $\acute{c}$ are right angles. See Fig. 312. Crystals are usually drawn with the axis $\acute{c}$ vertical and the axis $\grave{a}$ directed to the front and inclined downward.

1. NORMAL GROUP (28). GYPSUM TYPE.

181. Symmetry.—In the normal group of the monoclinic system there is one plane of symmetry and one axis of binary symmetry normal to it. The plane of symmetry is always the plane of the axes $\grave{a}$ and $\acute{c}$, and the axis of symmetry coincides with the axis $\bar{b}$, normal to this plane. The position of one axis ($\bar{b}$) and that of the plane of the other two axes ($\grave{a}$ and $\acute{c}$) is thus fixed by the symmetry; but the latter axes may occupy different positions in this plane. Fig. 311 shows the typical spherical projection, projected on the plane of symmetry. Fig. 327 is the projection of an actual crystal of epidote; here, as is usual, the plane of projection is normal to the prismatic zone.

311.



182. Forms.—The various forms* belonging to this group, with their symbols, are given in the following table. As more particularly explained later, an orthodome includes two faces only, and a pyramid four only.

	Miller.		Naumann.
1. Orthopinacoid or a-pinacoid	(100)	$\grave{a} : \infty\bar{b} : \infty\acute{c}$	$\infty P\bar{\infty}$ or i-$\bar{i}$, a
2. Clinopinacoid or b-pinacoid	(010)	$\infty\grave{a} : \bar{b} : \infty\acute{c}$	$\infty P\grave{\infty}$ or i-$\grave{i}$, b
3. Base or c-pinacoid	(001)	$\infty\grave{a} : \infty\bar{b} : \acute{c}$	$0P$ or O, c
4. Unit prism	(110)	$\grave{a} : \bar{b} : \infty\acute{c}$	∞P or I, m
4. Orthoprisms	($hk0$) $h > k$	$\grave{a} : n\bar{b} : \infty\acute{c}$	$\infty P\bar{n}$ or i-$\bar{n}$, as (210) i-$\bar{2}$
4. Clinoprisms	($hk0$) $h < k$	$n\grave{a} : \bar{b} : \infty\acute{c}$	$\infty P\grave{n}$ or i-$\grave{n}$, as (120) i-$\grave{2}$
5. Orthodomes	($h0l$)	$\grave{a} : \infty\bar{b} : m\acute{c}$	$-mP\bar{\infty}$ or $-m$-$\bar{i}$, as (101) -1-$\bar{i}$
	($\bar{h}0l$)	$\grave{a} : \infty\bar{b} : -m\acute{c}$	$mP\infty$ or m-$\bar{i}$, as ($\bar{1}01$) 1-$\bar{i}$
6. Clinodomes	($0kl$)	$\infty\grave{a} : \bar{b} : m\acute{c}$	$mP\grave{\infty}$ or m-$\grave{i}$, as (011) 1-$\grave{i}$
7. Unit pyramids	(hhl)	$\grave{a} : \bar{b} : m\acute{c}$	$-mP$ or $-m$, as (111) -1
	($\bar{h}hl$)	$\grave{a} : \bar{b} : -m\acute{c}$	mP, as ($\bar{1}11$) 1
7. Orthopyramids	(hkl) $h > k$	$\grave{a} : n\bar{b} : m\acute{c}$	$-mP\bar{n}$ or $-m$-$\bar{n}$, as (211)-2-$\bar{2}$
	($\bar{h}kl$) $h > k$	$\grave{a} : n\bar{b} : -m\acute{c}$	$mP\bar{n}$ or m-n, as ($\bar{2}11$) 2-$\bar{2}$
7. Clinopyramids	(hkl) $h < k$	$n\grave{a} : \bar{b} : m\acute{c}$	$-mP\grave{n}$ or $-m$-$\grave{n}$, as (121)-2-$\grave{2}$
	($\bar{h}kl$) $h < k$	$n\grave{a} : \bar{b} : -m\acute{c}$	$mP\grave{n}$ or m-$\grave{n}$, as ($\bar{1}21$) 2-$\grave{2}$

The Naumann symbols given above are analogous to those of the orthorhombic system. The long mark employed is to be understood to be conventional only and as referring to the ortho-axis, $\bar{b}$. It does not imply that this axis is longer than the clino-axis, $\grave{a}$, though this is commonly the case. The inclined mark refers to the inclined axis, $\grave{a}$. With some authors these marks pass through the P, instead of being written over the letter (or number) following.

* On the general use of the terms pinacoid, prisms, domes, pyramids, see pp. 26, 90.

183. Pinacoids.—The pinacoids are the orthopinacoid, clinopinacoid, and the basal plane.

The *orthopinacoid*, (100), includes the two faces parallel to the plane of the ortho-axis, $\bar{b}$ and the vertical axis $\acute{c}$. They have the symbols 100 and $\bar{1}00$. This form is designated by the letter *a*, since it is situated at the extremity of the $\grave{a}$ axis; it is hence conveniently called the *a-pinacoid*.

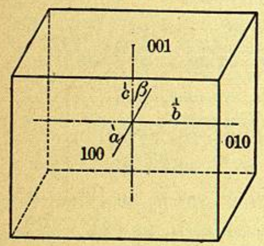
The *clinopinacoid*, (010), includes the two faces parallel to the plane of symmetry; that is, the plane of the clino-axis $\grave{a}$ and to the axis $\acute{c}$. They have the symbols 010 and $0\bar{1}0$. The clinopinacoid is designated by the letter *b*, and is called the *b-pinacoid*.

The *base* or *basal pinacoid*, (001), includes the two terminal faces, above and below, parallel to the plane of the lateral axes $\grave{a}$, $\bar{b}$; they have the symbols 001 and $00\bar{1}$. The base is designated by the letter *c*, and is often called the *c-pinacoid*. It is obviously inclined to the orthopinacoid, and the normal angle between the two faces (100 ∧ 001) is the acute axial angle β.

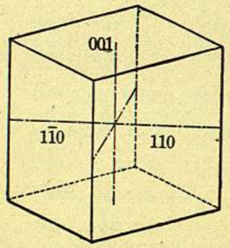
The diametral prism, formed by these three pinacoids, taken together, Fig. 312, is the analogue of the cube in the isometric system. It is bounded by three sets of unlike faces; it has four similar vertical edges; also four lateral similar edges parallel to the axis $\grave{a}$, but the remaining edges, parallel to the axis $\bar{b}$, are only similar two-and-two. Of its eight solid angles there are two sets of four each; the two above in front are similar to those below behind, and the two below in front to those behind above.

184. Prisms.—The prisms are all of one type, the oblique rhombic prism. They include the *unit prism*, (110), designated by the letter *m*, shown in Fig. 313; also the *orthoprisms*, ($hk0$) where $h > k$, lying between *a* (100) and *m* (110), and the *clinoprisms*, ($hk0$) where $h < k$, lying between *m* (110) and *b* (010). The orthoprisms and clinoprisms correspond respectively to the macroprisms and brachyprisms of the orthorhombic system, and the explanation on p. 91 will hence make their relation clear. Common cases of these prisms are shown in the figures given later.

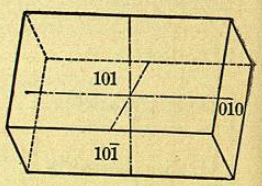
312.



313.



314.



185. Orthodomes.—The four faces parallel to the ortho-axis $\bar{b}$, and meeting the other two axes, fall into two sets of two each, having the general symbols ($h0l$) and ($\bar{h}0l$). These forms are called *orthodomes*, they are strictly hemi-orthodomes. For example, the unit orthodome (101) has the faces 101 and $\bar{1}0\bar{1}$; they would replace the two obtuse edges between *a* (100) and *c* (001) in Fig. 312. The other unit orthodome ($\bar{1}01$) has the faces $\bar{1}01$ and $10\bar{1}$, they would replace the acute edges between *a* and *c*. These two independent forms are shown together, with *b* (010), in Fig. 314.

Similarly the faces 201, $\bar{2}0\bar{1}$ belong to the form (201), and $\bar{2}01$, $20\bar{1}$ to the independent but complementary form ($\bar{2}01$).

In the symbols of Naumann, the hemi-orthodomes between the base and the front orthopinacoid above, and hence corresponding to the obtuse edge between them, are distinguished by the minus sign ($-$ 1-$\bar{\imath}$ = 101, etc.), while those between the base and the orthopinacoid behind are called plus * (+ 1-$\bar{\imath}$ = $\bar{1}01$); the + sign, however, is usually omitted. The two sets of hemi-pyramids (see beyond) are similarly distinguished.

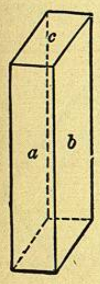
186. Clinodomes.—The *clinodomes* are the forms whose faces are parallel to the inclined axis, $\grave{a}$, while intersecting the other two axes. Their general symbol is hence ($0kl$) and they lie between the base (001) and the clinopinacoid (010). Each form has four faces; thus for the unit clinodome these have the symbols, 011, $0\bar{1}1$, $0\bar{1}\bar{1}$, $01\bar{1}$. The form *n* (021) in Fig. 321 is a clinodome.

187. Pyramids.—The *pyramids* in the monoclinic system are all hemi-pyramids, embracing four faces only in each form, corresponding to the general symbol (hkl). This obviously follows from the symmetry; it is shown, for example, in the fact already stated that the solid angles of the diametral prism (Fig. 312, see above), which are replaced by these pyramids, fall into two sets of four each. Thus any general symbol, as (321), includes the two independent forms (321) and ($\bar{3}21$) with the faces

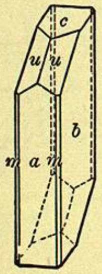
321, $3\bar{2}1$, $32\bar{1}$, $3\bar{2}\bar{1}$, and $\bar{3}21$, $\bar{3}\bar{2}1$, $\bar{3}2\bar{1}$, $\bar{3}\bar{2}\bar{1}$.

The pyramids may be *unit pyramids*, (hhl), orthopyramids, (hkl) when $h > k$, or *clinopyramids*, (hkl) when $h < k$. These correspond respectively to the three prisms already named. They are analogous also to the unit pyramids, macropyramids and brachypyramids of the orthorhombic system, and the explanation given on pp. 91, 92 should serve to make their relations clear. But it must be remembered that each general symbol embraces two forms, (hhl) and ($\bar{h}kl$) with four faces each, as above explained.

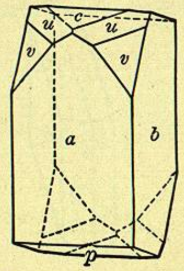
315.



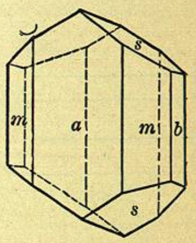
316.



317.



318.



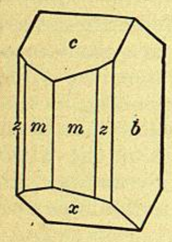
Pyroxene.

188. Illustrations.—Figs. 315–318 of pyroxene ($\grave{a}:\bar{b}:\acute{c} = 1{\cdot}092:1:0{\cdot}589$, $\beta = 74° = ac$) show typical monoclinic forms. Fig. 315 shows the diametral

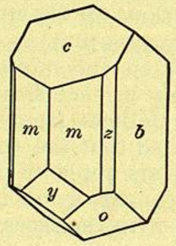
* This choice of signs by Naumann was unfortunate, being contrary to ordinary usage; it is, however, too generally accepted to admit of being reversed. He was led to adopt it because the internal angle of the upper front edge between 001 and 100 is obtuse and hence the cosine (*e.g.* in the general cosine formula for the angle between two faces) is *negative.*

prism. Of the other forms, m is the unit prism (110); p ($\bar{1}01$) is an orthodome; u (111), v (221), s ($\bar{1}11$) are unit pyramids; for other figures see p. 387. Again, Figs. 319–321 represent common crystals of orthoclase ($\grave{a}:\bar{b}:\acute{c} = 0{\cdot}659:1:0{\cdot}555$, $\beta = 64° = ac$). Here z (130) is a clinoprism; x ($\bar{1}01$) and y ($\bar{2}01$) are orthodomes; n (021) is a clinodome; o ($\bar{1}11$) a unit pyramid. Since (Fig. 319) c and x happen to make nearly equal angles with the vertical edge of the prism m, the combination often stimulates an orthorhombic crystal.

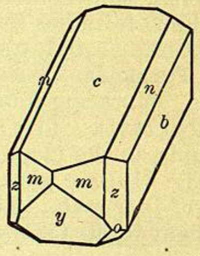
319.



320.



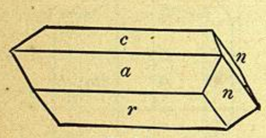
321.



Orthoclase.

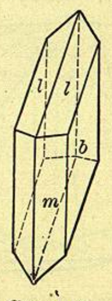
Fig. 322 shows a monoclinic crystal, epidote, prismatic in the direction of the ortho-axis; the forms are r ($\bar{1}01$) and n ($\bar{1}11$). Fig. 323 of gypsum is flattened ∥ b; it shows the unit pyramid l (111) with the unit prism m (110). Fig. 324 of monazite is prismatic in habit by extension of the pyramid v ($\bar{1}11$). It shows also the orthodome w (101); the clinodome e (011); the pyramids r (111), s (121), z ($\bar{3}11$), i ($\bar{2}11$).

322.



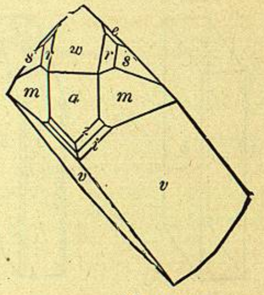
Epidote.

323.



Gypsum.

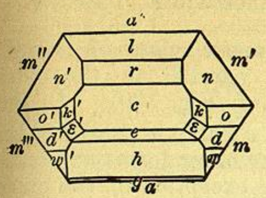
324.



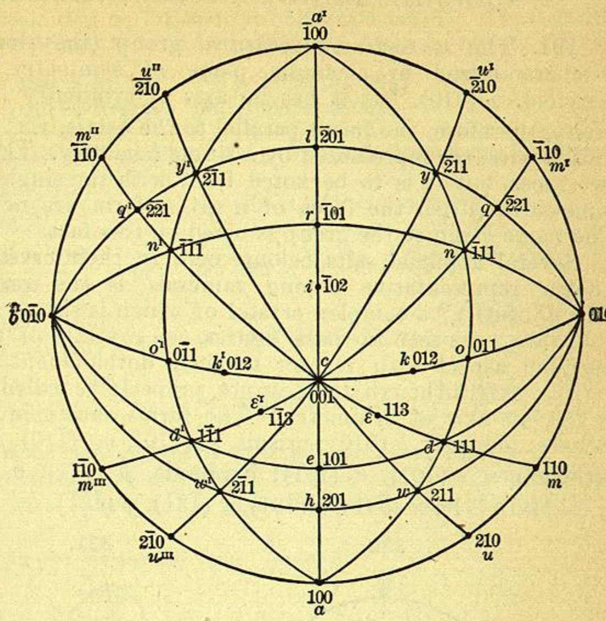
Monazite.

189. Projections.—Fig. 325 shows a projection of a crystal of epidote (cf. Fig. 853, p. 438) on a plane normal to the prismatic zone, and Fig. 326 one of a similar crystal on a plane parallel to b (010); both should be carefully studied, as also the spherical projection of the same species, Fig. 327. The symbols of the prominent faces are given in Fig. 327.

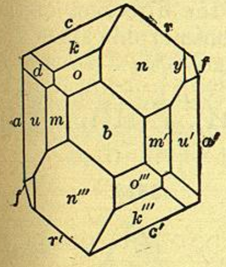
325.



327.



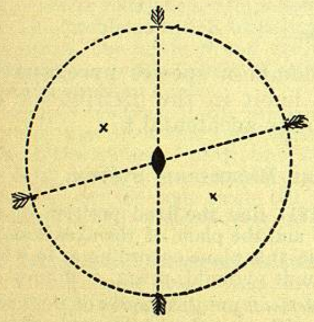
326.



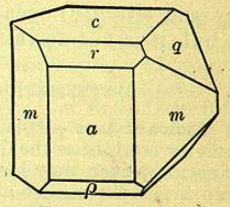
2. HEMIMORPHIC GROUP (29).

190. The *monoclinic-hemimorphic* group is characterized by a single axis of binary symmetry, the crystallographic axis $\grave{b}$, but it has no plane of symmetry. It is illustrated by the spherical projection (Fig. 328); also by Fig. 329, a common form of tartaric acid; sugar crystals also belong here.

328.



329.



Tartaric Acid.
Forms: r (101), ρ (10$\bar{1}$), q (011).

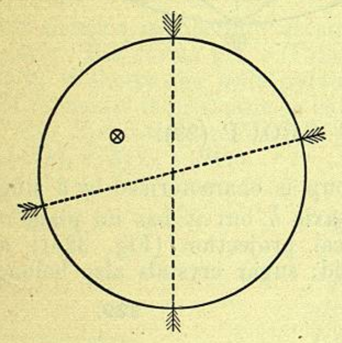
The hemimorphic character is distinctly shown in the distribution of the clinodomes and pyramids; corresponding to this the artificial salts belonging here often exhibit marked pyro-electrical phenomena.

3. CLINOHEDRAL GROUP (30). CLINOHEDRITE TYPE.

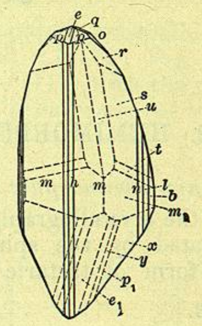
191. The *monoclinic-clinohedral* group (the domatic class of Groth) is characterized by a single plane of symmetry, parallel to the clinopinacoid, b (010), but it has no axis of symmetry (cf. Fig. 330). In this group, therefore, the forms parallel to the $\bar{b}$-axis, viz., c (001), a (100) and the orthodomes, are represented by a single face only. The other forms have each two faces, but it is to be noted that, with the single exception of the clinopinacoid b (010), the faces of a given form are never parallel to another. The name given to the group is based on this fact.

Several artificial salts belong here in their crystallization, but the only known representative among minerals is the rare silicate, clinohedrite ($H_2CaZnSiO_5$),* a complex crystal of which is shown in two positions in Figs. 331, 331*a*. As seen in these figures, the crystals of the group have a hemimorphic aspect with respect to their development in the direction of the vertical axis, although they cannot properly be called hemimorphic since this is not an axis of symmetry. The forms shown in Figs. 331, 331*a* are as follows: pinacoid, b (010); prisms, m (110), m_1 ($\bar{1}10$), h (320), n (120), l (130); orthodomes, e (101), e_1 ($\bar{1}0\bar{1}$); pyramids, p (111), p_1 ($\bar{1}1\bar{1}$), q ($\bar{1}11$); r ($\bar{3}31$), s, ($\bar{5}51$), t ($\bar{7}71$), u ($\bar{5}31$), o ($\bar{1}31$), x ($\bar{1}3\bar{1}$), y ($\bar{1}2\bar{1}$).

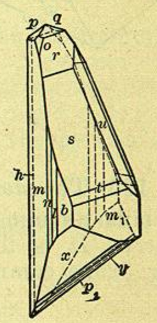
330.



331.



331*a*.



Clinohedrite.

It is to be noted that crystals of the common species pyroxene (also of ægirite and titanite) occasionally show this habit in the distribution of their faces, but it is not certain that this may not be accidental.†

Mathematical Relations of the Monoclinic System.

192. Choice of Axes.—It is repeated here (Art. **181**), that the fixed position of the plane of symmetry establishes the direction of the axis $\bar{b}$ and the plane of the axes $\grave{a}$ and $\acute{c}$. The latter axes, however, may have varying positions in this plane according as to which faces are taken as the pinacoids a and c, and which the unit pyramid, prism, or domes.

193. Axial and Angular Elements.—The *axial elements* are the *lengths* of the axes $\grave{a}$ and $\acute{c}$ in terms of the unit axis $\bar{b}$, that is, the axial ratio, with also the acute angle of inclination of the axes $\grave{a}$ and $\acute{c}$, called β. Thus for orthoclase the axial elements are:

$$\grave{a} : \bar{b} : \acute{c} = 0{\cdot}6585 : 1 : 0{\cdot}5554 \quad \beta = 63° \, 56\tfrac{3}{4}'.$$

* Penfield and Foote, Am. J. Sc., **5**, 289, 1898.

† See G. H. Williams, Am. J. Sc., **34**, 275, 1887, **38**, 115, 1889.

The angular elements are usually taken as the angle (100 ∧ 001) which is equal to the angle β; also the angles between the three pinacoids 100, 010, 001, respectively, and the unit prism 110, the unit orthodome (101 or $\bar{1}01$) and the unit clinodome 011. Thus again for orthoclase, the angular elements are:

$$001 \wedge 100 = 63° \, 56\tfrac{3}{4}', \quad 100 \wedge 110 = 30° \, 36\tfrac{1}{2}'.$$
$$001 \wedge \bar{1}01 = 50° \, 16\tfrac{1}{2}', \quad 001 \wedge 011 = 26° \, 31'.$$

194. The mathematical relations connecting axial and angular elements are as follows:

$$\grave{a} = \frac{\tan(100 \wedge 110)}{\sin\beta} \quad \text{or} \quad \tan(100 \wedge 110) = \grave{a} \, . \sin\beta; \qquad (1)$$

$$\acute{c} = \frac{\tan(001 \wedge 011)}{\sin\beta} \quad \text{or} \quad \tan(001 \wedge 011) = \acute{c} \, . \sin\beta; \qquad (2)$$

$$\left.\begin{aligned} \acute{c} &= \frac{\grave{a} \, . \tan(001 \wedge 101)}{\sin\beta - \cos\beta \, . \tan(001 \wedge 101)} \quad \text{or} \quad \tan(001 \wedge 101) = \frac{\acute{c}\sin\beta}{\grave{a} + \acute{c} \, . \cos\beta}, \\ \acute{c} &= \frac{\grave{a} \, . \tan(001 \wedge \bar{1}01)}{\sin\beta + \cos\beta \, . \tan(001 \wedge \bar{1}01)} \quad \text{or} \quad \tan(001 \wedge \bar{1}01) = \frac{\acute{c}\sin\beta}{\grave{a} - \acute{c} \, . \cos\beta}. \end{aligned}\right\} \qquad (3)$$

These relations may be made more general by writing in the several cases—

in (1) $hk0$ for 110 and $\frac{k}{h}\grave{a}$ for $\grave{a}$; in (2) $0kl$ for 011 and $\frac{k}{l}\acute{c}$ for $\acute{c}$;

in (3) $h0l$ for 100 and $\frac{h}{l}\acute{c}$ for $\acute{c}$.

Also

$$\frac{\acute{c}}{\grave{a}} = \frac{\sin(001 \wedge 101)}{\sin(100 \wedge 101)} = \frac{\sin(001 \wedge \bar{1}01)}{\sin(100 \wedge \bar{1}01)},$$

and more generally

$$\frac{h}{\grave{a}} \cdot \frac{\acute{c}}{l} = \frac{\sin(001 \wedge h0l)}{\sin(100 \wedge h0l)} = \frac{\sin(001 \wedge \bar{h}0l)}{\sin(100 \wedge \bar{h}0l)}.$$

Note also that

$$\tan\phi = \grave{a} \quad \text{and} \quad \tan\zeta = \acute{c};$$

where ϕ is the angle (Fig. 327) between the zone-circles (001, 100) and (001, 110); also ζ the angle between (100, 001) and (100, 011).

All the above relations are important and should be thoroughly understood.

195. The problems which usually arise have as their object either the deducing of the axial elements, the angle β and the values of $\grave{a}$ and $\acute{c}$ in terms of $\bar{b}$ (= 1), from three measured angles, or the finding of any required interfacial angles from these elements or from the fundamental angles.

The simple relations of the preceding article connect the angular and axial elements, and beyond this all ordinary problems can be solved* either by the solution of spherical triangles on the sphere of projection, or by the aid of the cotangent (and tangent) relation.

It is to be noted, in the first place, that all great circles on the sphere of projection (Fig. 327) from 010 cut the zone circle 100, 001, $\bar{1}00$ at right angles, but those from 100 cut the zone circles 010, 001, $0\bar{1}0$ obliquely, as also those from 001 cutting the zone circle 100, 010, $\bar{1}00$.

196. Tangent and Cotangent Relations.—The simple *tangent relation* holds good for all zones from 010 to any pole on the zone circle 100, 001, $\bar{1}00$; in other words, for the prisms, clinodomes, and zones of pyramids in which the ratio of $h : l$ is constant (from 001 to $h0l$ or to $\bar{h}0l$). Thus it is still true, as in the orthorhombic system, that the tangents of the angles

* The general formulas, from which it is possible to calculate directly the angles between any face and the pinacoids, or the angle between any two faces whatever, are so complex as to be of little value.

of the prisms 210, 110, 120, 130 from 100 are in the ratio of $\frac{1}{2} : 1 : 2 : 3$, or, more generally, that

$$\frac{\tan(100 \wedge hk0)}{\tan(100 \wedge 110)} = \frac{k}{h} \quad \text{or} \quad \frac{\tan(010 \wedge hk0)}{\tan(010 \wedge 110)} = \frac{h}{k}.$$

Also for the clinodomes the tangents of the angles of 012, 011, 021 from 001 are in the ratio of $\frac{1}{2} : 1 : 2$, etc. A similar relation holds for the tangents of the angles of pyramids in the zones mentioned, as 121, 111, 212, etc.

For zones other than those mentioned in the preceding article, as from 100 to a clinodome, or from 001 to a prism, the more general *cotangent formula* given in Art. 47 must be employed. This relation is simplified for certain common cases.

For any zone starting from 001, as the zone 001, 100, or 001, 110, or 001, 210, etc.; if two angles are known, viz., the angles between 001 and those two faces in the given zone which fall (1) in the zone 010, 101, and (2) in the prismatic zone 010, 100; then the angle between 001 and any other face in the given zone can be calculated.

Thus,

Let $001 \wedge 101 = PQ$ and $001 \wedge 100 = PR$,
or " $001 \wedge 111 = PQ$ " $001 \wedge 110 = PR$,
or " $001 \wedge 212 = PQ$ " $001 \wedge 210 = PR$, etc.

Then for these, or any similar cases, the angle (PS) between 001 and any face in the given zone (as 201, or 221, or 421, etc., or in general $h0l$, hhl, etc.) is given by the equation

$$\frac{\cot PS - \cot PR}{\cot PQ - \cot PR} = \frac{l}{h}.$$

For the corresponding zones from 001 to $\bar{1}00$, to $\bar{1}10$, to $\bar{2}10$, etc., the expression has the same value; but here

$PQ = 001 \wedge \bar{1}01$, $PR = 001 \wedge \bar{1}00$, $PS = 001 \wedge \bar{h}0l$,
or $001 \wedge \bar{1}11$, etc., $001 \wedge \bar{1}10$, etc., $001 \wedge \bar{h}hl$, etc.

If, however, 100 is the starting-point, and

$100 \wedge 101 = PQ$, $100 \wedge 001 = PR$,
or $100 \wedge 111 = PQ$, $100 \wedge 011 = PR$, etc.,

then the relation becomes

$$\frac{\cot PS - \cot PR}{\cot PQ - \cot PR} = \frac{h}{l}.$$

VI. TRICLINIC SYSTEM.

197. The TRICLINIC SYSTEM includes all the forms which are referred to three unequal axes with all their intersections oblique.

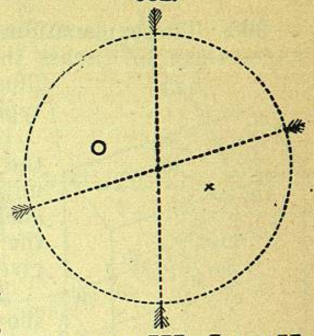
The axes are here designated as in the orthorhombic system, the letters used for the lateral axes $\breve{a}$, $\bar{b}$ (or $\bar{a}$, $\breve{b}$), having a short or long mark over them to indicate which is the shorter and which the longer axis. In the majority of cases, $\breve{a}$ is the brachy-axis and $\bar{b}$ the macro-axis. But this is not invariably true; thus with rhodonite the ratio of $\bar{a} : \breve{b} = 1 \cdot 073 : 1$. The vertical axis is always designated by $\dot{c}$. The angle between the axes b and $\dot{c}$ is called α, that between a and $\dot{c}$ is β, and that between a and b is γ (Fig. 333).

It is to be noted that there is no necessary relation between the values of α, β, and γ, any one may be greater or less than 90°; this is determined by the choice of the fundamental forms.

1. NORMAL GROUP (31). AXINITE TYPE.

198. Symmetry.—The normal group of the triclinic system is characterized by a center of symmetry, the point of intersection of the three axes, but there is no plane and no axis of symmetry.* This symmetry is shown in the accompanying spherical projection (Fig. 332).

332.



199. Forms.—Each form of the group includes two faces, parallel to one another and symmetrical with reference to the center of symmetry. This is true as well of the form with the general symbol (hkl) as of one of the special forms, as, for example, the a-pinacoid (100).

Hence, as shown in the following table, the four prismatic faces 110, $\bar{1}10$, $\bar{1}\bar{1}0$, $1\bar{1}0$ include two forms, namely, 110, $\bar{1}\bar{1}0$, and $\bar{1}10$, $1\bar{1}0$. The same is true of the domes. Further, any eight corresponding pyramidal faces, as, for example, 111, $\bar{1}11$, $\bar{1}\bar{1}1$, $1\bar{1}1$, $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}\bar{1}$, $1\bar{1}\bar{1}$ belong to four distinct forms, namely, **111, $\bar{1}\bar{1}\bar{1}$; $\bar{1}11$, $1\bar{1}\bar{1}$;** $\bar{1}\bar{1}1$, $11\bar{1}$; $1\bar{1}1$, $\bar{1}1\bar{1}$, and similarly in general.

The various types of forms are given in the following table:

	Miller.		Naumann.
Macropinacoid or a-pinacoid	(100)	$\breve{a} : \infty\bar{b} : \infty\dot{c}$	$\infty P\bar{\infty}$ or i-$\bar{i}$, a
Brachypinacoid or b-pinacoid	(010)	$\infty\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P\breve{\infty}$ or i-$\breve{i}$, b
Base or c-pinacoid	(001)	$\infty\breve{a} : \infty\bar{b} : \dot{c}$	OP or O, c
Unit prism	(110)	$\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P'$ or I', m
	$(1\bar{1}0)$	$\breve{a} : -\bar{b} : \infty\dot{c}$	$\infty{}'P$ or $'I$, M
Macroprisms $h > k$	$(hk0)$	$\breve{a} : n\bar{b} : \infty\dot{c}$	$\infty P'\bar{n}$ or i-$\bar{n}'$
	$(h\bar{k}0)$	$\breve{a} : -n\bar{b} : \infty\dot{c}$	$\infty{}'P\bar{n}$ or $'i$-$\bar{n}$
Brachyprisms $h < k$	$(hk0)$	$n\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P\breve{n}$ or i-$\breve{n}$
	$(h\bar{k}0)$	$n\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P\breve{n}$ or i-$\breve{n}$
Macrodomes	$(h0l)$	$\breve{a} : \infty\bar{b} : m\dot{c}$	$m'P'\bar{\infty}$ or $'m$-$\bar{i}'$
	$(\bar{h}0l)$	$-\breve{a} : \infty\bar{b} : m\dot{c}$	$m_{,}P_{,}\infty$ or $_{,}m$-$\bar{i}'$
Brachydomes	$(0kl)$	$\infty\breve{a} : \bar{b} : m\dot{c}$	$mP\infty'$ or m-$\breve{i}$
	$(0\bar{k}l)$	$\infty\breve{a} : -\bar{b} : m\dot{c}$	$'mP\infty$ or m-$\breve{i}$
Unit pyramids	(hhl)	$a : b : mc$	mP' or m'
	$(\bar{h}hl)$	$-a : b : mc$	$m_{,}P$ or $_{,}m$
	$(\bar{h}\bar{h}l)$	$-a : -b : mc$	$mP_{,}$ or $m_{,}$
	$(h\bar{h}l)$	$a : -b : mc$	$m'P$ or $'m$
Macropyramids† $h > k$	(hkl)	$\breve{a} : n\bar{b} : m\dot{c}$	$mP'\bar{n}$ or m-$\bar{n}'$
	$(\bar{h}kl)$	$-\breve{a} : n\bar{b} : m\dot{c}$	$m_{,}P\bar{n}$ or $_{,}m$-$\bar{n}$
	$(\bar{h}\bar{k}l)$	$-\breve{a} : -n\bar{b} : m\dot{c}$	$mP_{,}\bar{n}$ or m-$\bar{n}_{,}$
	$(h\bar{k}l)$	$\breve{a} : -n\bar{b} : m\dot{c}$	$m_{,}P\bar{n}$ or $'m$-$\bar{n}$

* On the other method of viewing the symmetry here described, see Art. **17**, p. 10.

† In the above table it is assumed that the axial ratio is $\breve{a} : \bar{b} : \dot{c}$. If it were $\bar{a} : \breve{b} : \dot{c}$, the names brachy- and macro- would be exchanged, and also the long and short marks in the Naumann symbols. The use of accents to distinguish prisms, domes, and pyramids according to their position is to be noted.