of the prisms 210, 110, 120, 130 from 100 are in the ratio of $\frac{1}{2}:1:2:3$, or, more generally, that

$$\frac{\tan\,(100 \wedge hk0)}{\tan\,(100 \wedge 110)} = \frac{k}{h} \quad \text{or} \quad \frac{\tan\,(010 \wedge hk0)}{\tan\,(010 \wedge 110)} = \frac{h}{k}.$$

Also for the clinodomes the tangents of the angles of 012, 011, 021 from 001 are in the ratio of $\frac{1}{2}:1:2$, etc. A similar relation holds for the tangents of the angles of pyramids in the zones mentioned, as 121, 111, 212, etc.

For zones other than those mentioned in the preceding article, as from 100 to a clinodome, or from 001 to a prism, the more general *cotangent formula* given in Art. 47 must be employed. This relation is simplified for certain common cases.

For any zone starting from 001, as the zone 001, 100, or 001, 110, or 001, 210, etc.; if two angles are known, viz., the angles between 001 and those two faces in the given zone which fall (1) in the zone 010, 101, and (2) in the prismatic zone 010, 100; then the angle between 001 and any other face in the given zone can be calculated.

Thus,

$$\begin{array}{llll} & \text{Let } 001 \wedge 101 = PQ & \text{and} & 001 \wedge 100 = PR, \\ \text{or} & \text{“ } 001 \wedge 111 = PQ & \text{“} & 001 \wedge 110 = PR, \\ \text{or} & \text{“ } 001 \wedge 212 = PQ & \text{“} & 001 \wedge 210 = PR, \text{ etc.} \end{array}$$

Then for these, or any similar cases, the angle (PS) between 001 and any face in the given zone (as 201, or 221, or 421, etc., or in general $h0l$, hhl, etc.) is given by the equation

$$\frac{\cot PS - \cot PR}{\cot PQ - \cot PR} = \frac{l}{h}.$$

For the corresponding zones from 001 to $\bar{1}00$, to $\bar{1}10$, to $\bar{2}10$, etc., the expression has the same value; but here

$$\begin{array}{llll} & PQ = 001 \wedge \bar{1}01, & PR = 001 \wedge \bar{1}00, & PS = 001 \wedge \bar{h}0l, \\ \text{or} & 001 \wedge \bar{1}11, \text{ etc.}, & 001 \wedge \bar{1}10, \text{ etc.}, & 001 \wedge \bar{h}hl, \text{ etc.} \end{array}$$

If, however, 100 is the starting-point, and

$$\begin{array}{lll} & 100 \wedge 101 = PQ, & 100 \wedge 001 = PR, \\ \text{or} & 100 \wedge 111 = PQ, & 100 \wedge 011 = PR, \text{ etc.,} \end{array}$$

then the relation becomes

$$\frac{\cot PS - \cot PR}{\cot PQ - \cot PR} = \frac{h}{l}.$$

VI. TRICLINIC SYSTEM.

197. The TRICLINIC SYSTEM includes all the forms which are referred to three unequal axes with all their intersections oblique.

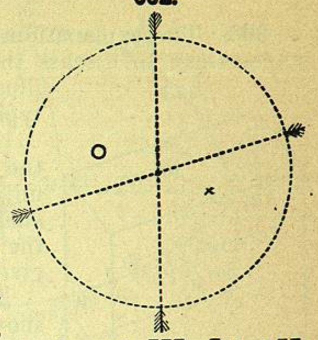
The axes are here designated as in the orthorhombic system, the letters used for the lateral axes $\breve{a}$, $\bar{b}$ (or $\bar{a}$, $\breve{b}$), having a short or long mark over them to indicate which is the shorter and which the longer axis. In the majority of cases, $\breve{a}$ is the brachy-axis and $\bar{b}$ the macro-axis. But this is not invariably true; thus with rhodonite the ratio of $\bar{a} : \breve{b} = 1{\cdot}073 : 1$. The vertical axis is always designated by $\dot{c}$. The angle between the axes b and $\dot{c}$ is called α, that between a and $\dot{c}$ is β, and that between a and b is γ (Fig. 333).

It is to be noted that there is no necessary relation between the values of α, β, and γ, any one may be greater or less than 90°; this is determined by the choice of the fundamental forms.

1. NORMAL GROUP (31). AXINITE TYPE.

198. Symmetry.—The normal group of the triclinic system is characterized by a center of symmetry, the point of intersection of the three axes, but there is no plane and no axis of symmetry.* This symmetry is shown in the accompanying spherical projection (Fig. 332).

332.



199. Forms.—Each form of the group includes two faces, parallel to one another and symmetrical with reference to the center of symmetry. This is true as well of the form with the general symbol (hkl) as of one of the special forms, as, for example, the a-pinacoid (100).

Hence, as shown in the following table, the four prismatic faces 110, $1\bar{1}0$, $\bar{1}\bar{1}0$, $\bar{1}10$ include two forms, namely, 110, $\bar{1}\bar{1}0$, and $\bar{1}10$, $1\bar{1}0$. The same is true of the domes. Further, any eight corresponding pyramidal faces, as, for example, 111, $\bar{1}11$, $\bar{1}\bar{1}1$, $1\bar{1}1$, $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}\bar{1}$, $1\bar{1}\bar{1}$ belong to four distinct forms, namely, **111**, **$\bar{1}\bar{1}\bar{1}$**; **$\bar{1}11$**, **$1\bar{1}\bar{1}$**; $\bar{1}\bar{1}1$, $11\bar{1}$; $1\bar{1}1$, $\bar{1}1\bar{1}$, and similarly in general.

The various types of forms are given in the following table:

	Miller.		Naumann.
Macropinacoid or a-pinacoid	(100)	$\breve{a} : \infty\bar{b} : \infty\dot{c}$	$\infty P\bar{\infty}$ or i-$\bar{i}$, a
Brachypinacoid or b-pinacoid	(010)	$\infty\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P\breve{\infty}$ or i-$\breve{i}$, b
Base or c-pinacoid	(001)	$\infty\breve{a} : \infty\bar{b} : \dot{c}$	OP or O, c
Unit prism	(110)	$\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P'$ or I', m
	($1\bar{1}0$)	$\breve{a} : -\bar{b} : \infty\dot{c}$	$\infty{}'P$ or $'I$, M
Macroprisms $h > k$	($hk0$)	$\breve{a} : n\bar{b} : \infty\dot{c}$	$\infty P'\bar{n}$ or i-$\bar{n}'$
	($h\bar{k}0$)	$\breve{a} : -n\bar{b} : \infty\dot{c}$	$\infty{}'P\bar{n}$ or $'i$-$\bar{n}$
Brachyprisms $h < k$	($hk0$)	$n\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P\breve{n}$ or i-$\breve{n}$
	($h\bar{k}0$)	$n\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P\breve{n}$ or i-$\breve{n}$
Macrodomes	($h0l$)	$\breve{a} : \infty\bar{b} : m\dot{c}$	$m'P'\bar{\infty}$ or $'m$-$\bar{i}'$
	($\bar{h}0l$)	$-\breve{a} : \infty\bar{b} : m\dot{c}$	$m{,}P{,}\infty$ or $,m$-$\bar{i}'$
Brachydomes	($0kl$)	$\infty\breve{a} : \bar{b} : m\dot{c}$	$mP\infty'$ or m-$\breve{i}$
	($0\bar{k}l$)	$\infty\breve{a} : -\bar{b} : m\dot{c}$	$'mP\infty$ or m-$\breve{i}$
Unit pyramids	(hhl)	$a : b : mc$	mP' or m'
	($\bar{h}hl$)	$-a : b : mc$	$m{,}P$ or $,m$
	($\bar{h}\bar{h}l$)	$-a : -b : mc$	$mP{,}$ or $m{,}$
	($h\bar{h}l$)	$a : -b : mc$	$m'P$ or $'m$
Macropyramids† $h > k$	(hkl)	$\breve{a} : n\bar{b} : m\dot{c}$	$mP'\bar{n}$ or m-$\bar{n}'$
	($\bar{h}kl$)	$-\breve{a} : n\bar{b} : m\dot{c}$	$m{,}P\bar{n}$ or $,m$-$\bar{n}$
	($\bar{h}\bar{k}l$)	$-\breve{a} : -n\bar{b} : m\dot{c}$	$mP{,}\bar{n}$ or m-$\bar{n}{,}$
	($h\bar{k}l$)	$\breve{a} : -n\bar{b} : m\dot{c}$	$m{,}P\bar{n}$ or $'m$-$\bar{n}$

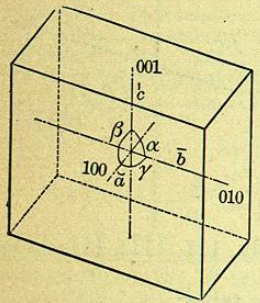
* On the other method of viewing the symmetry here described, see Art. **17**, p. 10.

† In the above table it is assumed that the axial ratio is $\breve{a} : \bar{b} : \dot{c}$. If it were $\bar{a} : \breve{b} : \dot{c}$, the names brachy- and macro- would be exchanged, and also the long and short marks in the Naumann symbols. The use of accents to distinguish prisms, domes, and pyramids according to their position is to be noted.

Brachypyramids $h < k$	(hkl)	$n\breve{a} : \bar{b} : m\dot{c}$	$mP'\breve{n}$ or $m\text{-}\breve{n}'$
	$(\bar{h}kl)$	$-n\breve{a} : \bar{b} : m\dot{c}$	$m{,}P\breve{n}$ or ${,}m\text{-}\breve{n}$
	$(\bar{h}\bar{k}l)$	$- : n\breve{a} : -\bar{b} : m\dot{c}$	$mP{,}\breve{n}$ or $m\text{-}\breve{n}{,}$
	$(h\bar{k}l)$	$n\breve{a} : -\bar{b} : m\dot{c}$	$m'P\breve{n}$ or $'m\text{-}\breve{n}$

200. The explanations given under the two preceding systems make it unnecessary to discuss in detail the various forms individually, except as illustrated in the case of crystals belonging to certain typical triclinic species.

333.



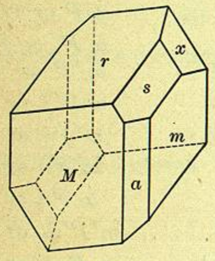
It may be mentioned, however, that Fig. 333 shows the *diametral prism*, which is bounded by three sets of unlike faces, the pinacoids *a*, *b*, and *c*. This is the analogue of the cube of the isometric system, but here the like faces, edges, and solid angles include only a given face, edge and angle, and that opposite to it.

201. Illustrations.—A typical triclinic crystal is shown in Fig. 334 of axinite, already introduced. Here *a* (100) is the macropinacoid; *m* (110) and *M* (1$\bar{1}$0) the two unit prisms; *s* (201) a macrodome, and *x* (111) and *r* (1$\bar{1}$1) two unit pyramids. The axial ratio is as follows:

$$\breve{a} : \bar{b} : \dot{c} = 0{\cdot}49 : 1 : 0{\cdot}48,\ \alpha = 82°\ 54',\ \beta = 91°\ 52',\ \gamma = 131°\ 32'.$$

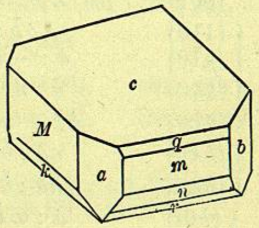
Figs. 335, 336 show two crystals of rhodonite, a species which is allied to pyroxene, and which approximates to it in angle and habit. Here the faces

334.

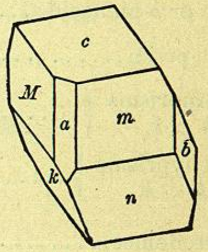


Axinite.

335.



336.

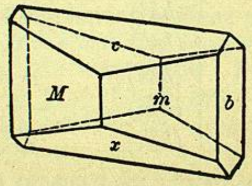


Rhodonite.

are: Pinacoids *a* (100), *b* (010), *c* (001); prisms *m* (110), *M* (1$\bar{1}$0); pyramids *q* (221), *k* ($\bar{2}$21), *n* ($\bar{2}\bar{2}$1), *r* ($\bar{1}\bar{1}$1).

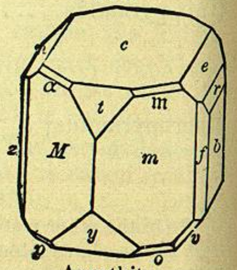
Further illustrations are given by Fig. 337 of albite and Fig. 338 of anorthite. The symbols of the faces, besides the pinacoids and the unit prisms, are as follows: Fig. 337, *x* ($\bar{1}$01); Fig. 338, prisms *f* (130), *z* (1$\bar{3}$0); domes *t* (207), *y* ($\bar{2}$01), *e* (021), *r* (061), *n* (0$\bar{2}$1); pyramids m (111), *α* (1$\bar{1}$1), *o* ($\bar{1}\bar{1}$1), *p* ($\bar{2}$11). In Fig. 338 of anorthite the similarity of the crystal to one of orthoclase is evident on slight examination (cf. Figs. 319, 320), and careful study with the measurement of angles shows that the correspondence is very

337.



Albite.

338.

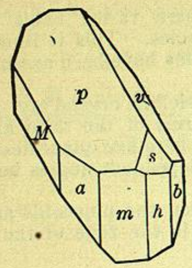


Anorthite.

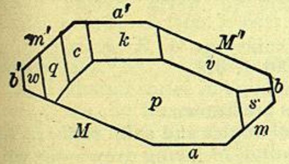
close. Hence in this case the choice of the fundamental planes is readily made.

Fig. 339 represents a crystal (artificial) of blue vitriol, the mineral chalcanthite; Fig. 339*a* gives a projection on a zone normal to the prisms, and Fig. 340 a spherical projection. The last figure also shows the symbols of the different faces.

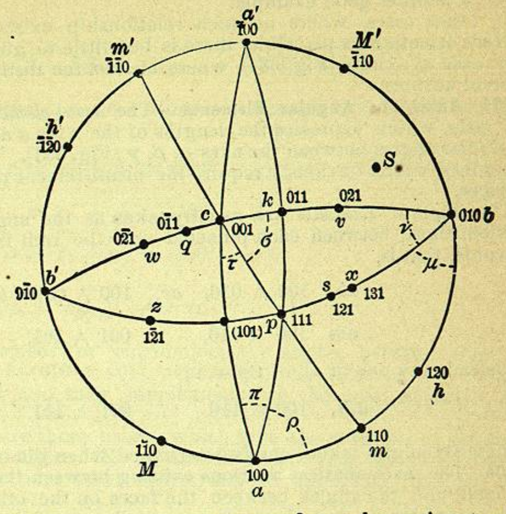
339.



339*a*.



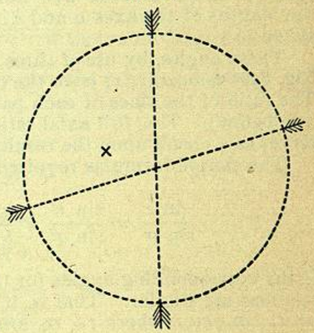
340.



2. ASYMMETRIC GROUP (32).

202. Besides the normal group of the triclinic system there is another possible group, possessing symmetry neither with respect to a plane, axis nor center; in it a given form has *one face* only. This group, the *asymmetric* class of Groth, finds examples among a number of artificial salts. One of these is calcium thiosulphate ($CaS_2O_3 . 6H_2O$); as yet no mineral species is known to be included here. This is the most general of all the thirty-two types of forms classified according to their symmetry and comes first, therefore, if the groups are arranged in order according to the degree of symmetry characterizing them. This group is one of those whose crystals may show circular polarization. This is true of eleven of the groups which have been described in the preceding pages.

341.



Mathematical Relations of the Triclinic System.

203. Choice of Axes.—It is obvious, from what has been said as to the symmetry of this system, that *any* three faces of a triclinic crystal may be chosen as the pinacoids, or the faces which fix the position of the axial planes and the directions of the axes; moreover, there is a like liberty in the choice of the unit prisms, domes or pyramids which further fix the lengths of the axes.

When the crystal in hand is allied in form or composition to other species whether of the same or different systems, this fact simplifies the problem and makes the choice of the fundamental forms easy. This is well illustrated, as already noted, by the triclinic feldspars (e.g., albite and anorthite, Figs. 337, 338) which are near in angle to the allied monoclinic species orthoclase. Rhodonite (Figs. 335, 336), the triclinic member of the pyroxene group, is another good example.

In other cases, where no such relationship exists, and where varied habit makes different orientations plausible, there is but little to guide the choice. This is illustrated in the case of axinite (Fig. 334), where at least ten distinct positions have been assumed by different authors.

204. Axial and Angular Elements.—The *axial elements* of a triclinic crystal are: (1) the axial ratio, which expresses the lengths of the axes a and $\dot{c}$ in terms of the third axis, b; and (2) the angles between the axes α, β, γ (Fig. 333). There are here five quantities to be determined which obviously require the measurement of five independent angles between the faces.

The *angular elements* are usually taken as the angles between the pinacoids and, in addition, those between each pinacoid and the unit face lying in the zone of the other pinacoids; that is,

	ab,	$100 \wedge 010$,	*ac*,	$100 \wedge 001$,	*bc*,	$010 \wedge 001$;
also	*am*	$100 \wedge 110$,		$001 \wedge 101$,		$001 \wedge 011$

or, instead, any one or all of these,

aM,	$100 \wedge 1\bar{1}0$,	$001 \wedge \bar{1}01$,	$001 \wedge 0\bar{1}1$.

Of these six angles taken, one is determined when the others are known.

205. The mathematical relations existing between the axial angles and axial ratio, on the one hand, and the angles between the faces on the other, admit of being drawn out with great completeness, but they are necessarily complex and in general have little practical value. In fact, most of the problems likely to arise can be solved by means of the triangles of the spherical projection, together with the cotangent formula connecting four planes in the same zone (Art. **47**, p. 31); this will often be laborious and may require some ingenuity, but in general involves no serious difficulty. In connection with the use of the cotangent formula, it is to be noted that in certain commonly occurring cases its form is much simplified; some of these have already been explained under the monoclinic system (Art. **196**). The formulas given there are of course equally applicable here.

206. The first problem may be to find the axial elements from measured angles. Since these elements include five unknown quantities, viz., the three axial angles α, β, γ and the lengths of the axes a and $\dot{c}$ in terms of b, five measured angles are required, as already stated.

These angles, by use of three or more spherical triangles, will serve to give the angles (see **Fig. 340**) $\pi, \rho, \mu, \nu, \tau, \sigma$ (or the corresponding angles π', ρ', etc., in the adjacent quadrants). The ratio of the sines of each pair of these angles fixes the ratios of the corresponding axes (see below). The full axial ratio may be obtained from any two pairs and the third ratio serves as a check upon the results given by the other two.

The simple formulas required are:

$$\frac{\sin\tau}{\sin\sigma} = \frac{\sin\tau'}{\sin\sigma'} = \frac{a}{b}, \quad \frac{\sin\nu}{\sin\mu} = \frac{\sin\nu'}{\sin\mu'} = \frac{\dot{c}}{a}, \quad \frac{\sin\pi}{\sin\rho} = \frac{\sin\pi'}{\sin\rho'} = \frac{\dot{c}}{b}.$$

If the corresponding angles for the general case are given (not those of the unit zones), the relations are similar. That is, if for the face *hkl* the corresponding angles be represented by τ_0, σ_0, etc., where τ_0, σ_0 are the angles between the zone circles 100, 001 and 100, 010 respectively and the zone circle 001, $hk0$ (and similarly for τ_0', σ_0' in the adjacent quadrant, also similarly ν_0, μ_0, etc.), these relations may be expressed in the general form

$$\frac{\sin\tau_0}{\sin\sigma_0} = \frac{\sin\tau_0'}{\sin\sigma_0'} = \frac{a}{\frac{h}{k}b} = \frac{k}{h}\cdot\frac{a}{b},$$

and similarly for $\dfrac{\sin\nu_0}{\sin\mu_0}$, etc.

Thus for the face 321 the formulas become

$$\frac{\sin\tau_0}{\sin\sigma_0} = \frac{a}{\frac{3}{2}b} = \frac{2a}{3b}, \quad \frac{\sin\nu_0}{\sin\mu_0} = \frac{3\dot{c}}{a}, \quad \frac{\sin\pi_0}{\sin\rho_0} = \frac{2\dot{c}}{b}.$$

It is also to be noted that

$$\alpha = 180° - A, \quad \beta = 180° - B, \quad \gamma = 180° - C,$$

where A, B, C are the angles in the pinacoidal spherical triangle 100·010·001 at these poles respectively. That is,

$$A = \pi + \rho = \pi_0 + \rho_0 = (180° - \alpha);$$
$$B = \nu + \mu = \nu_0 + \mu_0 = (180° - \beta);$$
$$C = \tau + \sigma = \tau_0 + \sigma_0 = (180° - \gamma).$$

Also

$$180° - A = \pi' + \rho' = \pi_0' + \rho_0' = \alpha.$$

Hence, having given, by measurement or calculation, the angles between the faces *ab* ($100 \wedge 010$), *ac* ($100 \wedge 001$) and *bc* ($010 \wedge 001$), which are the sides of this triangle, the angles A, B, C are calculated and their supplements are the axial angles α, β, γ respectively.

Still another series of equations are those below, which give the relations of the angles μ, ν, ρ, etc., to the axes and axial angles. By means of them, with the sine formulas given above, the angular elements (and other angles) can be calculated from the axial elements.

$$\tan\mu = \frac{a\sin\beta}{\dot{c} + a\cos\beta}; \quad \tan\nu = \frac{\dot{c}\sin\beta}{a + \dot{c}\cos\beta}.$$

$$\tan\rho = \frac{b\sin\alpha}{\dot{c} + b\cos\alpha}; \quad \tan\pi = \frac{\dot{c}\sin\alpha}{b + \dot{c}\cos\alpha}.$$

$$\tan\tau = \frac{a\sin\gamma}{b + a\cos\gamma}; \quad \tan\sigma = \frac{b\sin\gamma}{a + b\cos\gamma}.$$

These equations apply when $\mu + \nu$, etc., is less than 90°; if their sum is greater than 90° the sign in the denominator is negative.

207. The following equations are also often useful; they give the relations between the angles α, β, γ, and the angles μ, ν, etc., already defined.

$$\tan\alpha = \frac{2\sin\rho\sin\rho'}{\sin(\rho - \rho')} = \frac{2\sin\pi\sin\pi'}{\sin(\pi - \pi')}.$$

$$\tan\beta = \frac{2\sin\mu\sin\mu'}{\sin(\mu - \mu')} = \frac{2\sin\nu\sin\nu'}{\sin(\nu - \nu')}.$$

$$\tan\gamma = \frac{2\sin\tau\sin\tau'}{\sin(\tau - \tau')} = \frac{2\sin\sigma\sin\sigma'}{\sin(\sigma - \sigma')}.$$

Also,

$$\alpha + \pi + \rho = \beta + \mu + \nu = \gamma + \tau + \sigma = 180°.$$

The calculation, from the angular elements or from the assumed fundamental measured angles, either (1) of the angular position of any face whose symbol is given, or (2) of the symbol of an unknown face for which measured angles are at hand, requires no further explanation. The cotangent formula is all that is needed in a single zone, and the solution of spherical triangles on the projection (with the use of the sine formulas) will suffice in addition in all ordinary cases.