

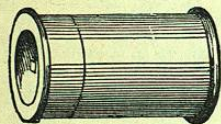
degrees of absorption upon the rays transmitted by vibrations in these two directions. For example, a transparent crystal of zircon looked through in the direction of the vertical axis appears of a pinkish-brown color, while in a lateral direction the color is asparagus-green. This is because the rays (extraordinary) vibrating *parallel* to the axis are absorbed with the exception of those which together give the green color, and those vibrating *laterally* (ordinary) are absorbed except those which together appear pinkish-brown.

Again, all crystals of tourmaline in the direction of the vertical axis are opaque, since the ordinary ray, vibrating normal to the axis, is absorbed, while light-colored varieties, looked through laterally, are transparent, for the extraordinary ray, vibrating parallel to the axis, is not absorbed; the color differs in different varieties. Thus all uniaxial crystals may be *dichroic* or have two distinct axial colors.

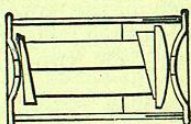
The absorption-colors are most satisfactorily investigated by examining a section cut parallel to the vertical (optic) axis under the microscope provided with a single polarizing nicol. The light that passes through the section is then that corresponding to vibrations coinciding with the known vibration-plane of the nicol, and as the section is rotated 90°, both the two axial colors are observed in succession. References to some important papers on this subject are given on p. 219.

An instrument called a *dichroscope*, contrived by Haidinger, is sometimes used for examining this property of crystals. An oblong rhombohedron of Iceland spar has a glass prism of 18° cemented to each extremity. It is placed in a metallic cylindrical case, as in the figure, having a convex lens at one end, and a square hole at the other. On looking

531.



532.



through it, the square hole appears double; one image belongs to the ordinary and the other to the extraordinary ray. When a pleochroic crystal is examined with it by transmitted light, on revolving it the two squares, at intervals of 90° in the revolution, have different colors, corresponding to the vibration-planes of the ordinary and extraordinary ray in calcite. Since the two images are situated side by side, a very slight difference of color is perceptible.

**366. Circular Polarization.**—The subject of elliptically polarized light and circular polarization has already been briefly alluded to in Art. 323. This phenomenon is most distinctly observed among minerals in the case of crystals belonging to the rhombohedral-trapezohedral group, that is, quartz and cinnabar.

It has been explained that a section of an ordinary uniaxial crystal cut normal to the vertical (optic) axis appears dark in parallel polarized light for every position between crossed nicols. If, however, a similar section of quartz, say 1 mm. in thickness, be examined under these conditions, it appears dark in monochromatic light only, and that not until the analyzer has been rotated so that its vibration-plane makes for sodium light an angle of 24° with that of the polarizer. In other words, this quartz section has rotated the plane of polarization (*i.e.*, the vibration-plane normal to it) some 24°, and here either to the right or to the left, looking in the direction of the light. The *amount* of this rotation increases with the thickness of the section, and as the wavelength of the light diminishes (for red this angle of rotation for a section of 1 mm. is about 19°, for blue 32°). The direction of the rotation is to the right

or left, as defined above—according as the crystal is crystallographically right-handed or left-handed (p. 83).

If the same section of quartz (cut perpendicular to the axis) be viewed between crossed nicols in converging polarized light, it is found that the interference-figure differs from that of an ordinary uniaxial crystal. The central portion of the black cross has disappeared, and instead the space within the inner ring is brilliantly colored.\* Furthermore, when the analyzing nicol is revolved, this color changes from blue to yellow to red, and it is found that in some cases this change is produced by revolving the nicol to the *right*, and in other cases to the *left*; the first is true with right-handed crystals, and the second with left-handed. If sections of a right-handed and left-handed crystal are placed together in the polariscope, the center of the interference-figure is occupied with a four-rayed spiral curve, called, from the discoverer, *Airy's spiral*. Twins of quartz crystals are not uncommon, consisting of the combination of right- and left-handed individuals (according to the Brazil law) which show these spirals of Airy. With cinnabar similar phenomena are observed. Twins of this species also not infrequently show Airy's spirals in the polariscope.

### C. BIAXIAL CRYSTALS.

#### General Optical Relations.

**367. Principal Refractive Indices.**—All crystals of the third or anisometric class, that is, those of the orthorhombic, monoclinic, and triclinic systems, are optically *biaxial*. In the directions of the optic axes there is a single value only for the light velocity, but in other directions † a light-ray is separated into two rays propagated with different velocities; that is, it suffers double refraction.

The study of biaxial crystals shows that there are two directions within them at right angles to each other, corresponding to which, as vibration-axes, the refractive indices have respectively a *minimum* ( $\alpha$ ) and a *maximum* value ( $\gamma$ ) for the given substance. Further, in a third direction at right angles to each of those just named, the refractive index has a certain intermediate value, related to the others by a simple mathematical law. These three rectangular directions, or ether-axes, are properly axes of vibration, and the three corresponding refractive indices determine the rate of this transverse vibration and hence the velocity of the light-ray which corresponds to each of them. The values of the velocities are respectively proportional to  $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ .

The indices  $\alpha, \beta, \gamma$  are called the *principal refractive indices* for the given substance. The mean refractive power is given by their arithmetical mean, *viz.*,  $\frac{\alpha + \beta + \gamma}{3}$ . Further, the difference between the greatest and least index,  $\gamma - \alpha$ , measures the birefringence or *strength* of the double refraction.

**368. Optical Structure of Biaxial Crystals.**—It is found further that the optical structure of a biaxial crystal can be represented by an ellipsoid having as its axes the three lines mentioned in the preceding article which are at right angles to each other and proportional in length to the indices  $\alpha, \beta, \gamma$ . This indeed would be inferred (following Fresnel) from the analogy of uniaxial crystals. The position of the axes named, or, in other words, the symmetry of

\* Very thin sections of quartz, however, show (*e.g.*, with the microscope) the dark cross of an ordinary uniaxial crystal.

† On the distinction between the primary and the secondary optic axes, see Art. 371.

this ellipsoid, is such as to correspond to the general crystallographic symmetry of the crystal. For example, to repeat the statement already made (Art. 348), in the *orthorhombic* system the axes of this ellipsoid coincide in direction with the crystallographic axes. In the *monoclinic* system, one of them coincides with the axis of crystallographic symmetry ( $\bar{b}$ ), the other two lie in the plane of symmetry, that is, in the plane of the crystallographic axes  $\bar{a}$  and  $\bar{c}$ . In the *triclinic* system there is no necessary connection between the position of the ether-axes and the crystallographic axes.

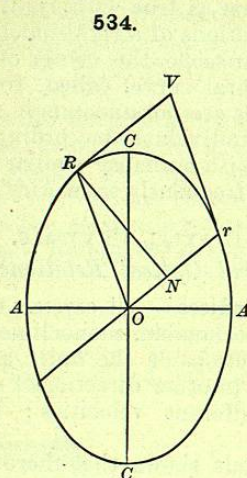
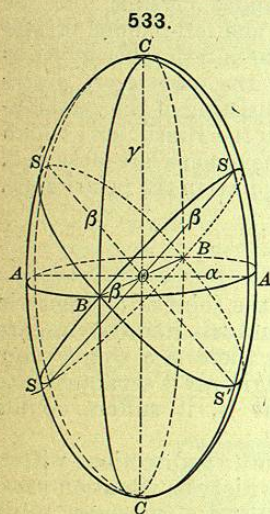
**369. Indicatrix.**—It may be shown, as is done by Fletcher, that the ellipsoid mentioned, whose axes represent in magnitude the three principal refractive indices,  $\alpha$ ,  $\beta$ ,  $\gamma$  (where  $\alpha < \beta < \gamma$ ), not only exhibits the character of the optical symmetry, but also serves to represent the direction, velocity and plane of polarization of a light-ray in any direction whatever, precisely analogous to the spheroid mentioned on p. 196. See Fig. 533 (from Groth), also Figs. 518, 519. That is, for the two rays having any *direction*, as  $Or$  in the plane of the axes  $AA$ ,  $CC$  (Fig. 534), the velocities will be proportional to  $\frac{1}{RN}$

and  $\frac{1}{OB} (= \frac{1}{\beta})$  respectively, and the planes of polarization will be perpendicular to these lines. From the equation of this index-ellipsoid, called by Fletcher the *indicatrix* (see Art. 355), it is possible, as shown by the author named, to deduce by ordinary analytical methods the mathematical expression for the wave-surface, the position of the two sets of optic axes (later explained), etc.

One important relation appears at once from a first study of this ellipsoid. Obviously for two definite positions of transverse planes passing through the center ( $SS$ ,  $S'S'$ , Fig. 533), these positions depending upon the relative values of  $\alpha$  and  $\gamma$ , the cross-sections will be *circles* each having a radius equal to the index  $\beta$ , intermediate in value between  $\alpha$  and  $\gamma$ ; similarly all sections parallel to these are also circles. Hence, light propagated in a direction normal to these planes, that is, by vibrations lying in them, will suffer no double refraction—and after the analogy of uniaxial crystals these directions are called *optic axes*; they are the *primary optic axes* mentioned in Art. 371.

**370. Wave-surface.**—Following out the analogy of uniaxial crystals, Fresnel deduced the now generally accepted "wave-surface" for biaxial crystals. That it gives correctly the law of the varying refractive indices (that is, of varying light-velocity) in a biaxial crystal has been demonstrated by the agreement between the requirements of the theory and the results of experiment.

The form of the sections of this wave-surface with the three rectangular



axial planes are easily deduced by very elementary considerations, though the full analytical development is most satisfactorily derived from the equation of the indicatrix as shown by Fletcher.

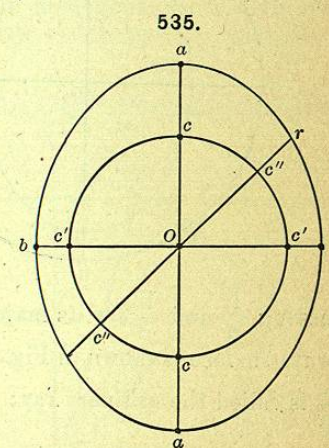
First consider the section of the wave-surface for the transverse plane of the axes  $AB$  (Fig. 533). Light passing in the direction of the axis  $AA$  will be separated into two rays; for one of these the line of transverse vibration will correspond to the axis  $CC$ , and hence its rate, or, in other words, the velocity of the ray itself, will be proportional to  $\frac{1}{\gamma}$ . For the second, the direction of transverse vibration will be that of the axis  $BB$ , and its rate, that is, the velocity of the ray itself, will be proportional to  $\frac{1}{\beta}$ . Let these values be

represented in Fig. 535 by  $Oc'$  ( $= \frac{1}{\gamma}$ ) and  $Ob$  ( $= \frac{1}{\beta}$ ). Again, in the direction of the axis  $BB$ , there will be two rays whose vibration-directions are respectively parallel to the axes  $AA$  and  $CC$ , and their velocities in the direction of  $BB$  proportional to  $\frac{1}{\alpha}$  and  $\frac{1}{\gamma}$  respectively. Let these be represented by  $Oa$  and  $Oc$ . For some other direction in the same plane, there will be two rays, one of whose vibration-directions corresponds to  $CC$ , and its velocity to  $\frac{1}{\gamma}$ , represented in Fig. 533 by  $Oc''$ , while for the other there will be an intermediate vibration-direction and a velocity between  $\frac{1}{\alpha}$  and

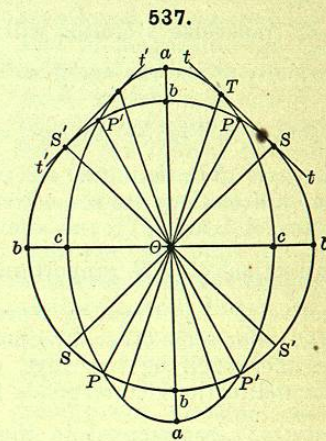
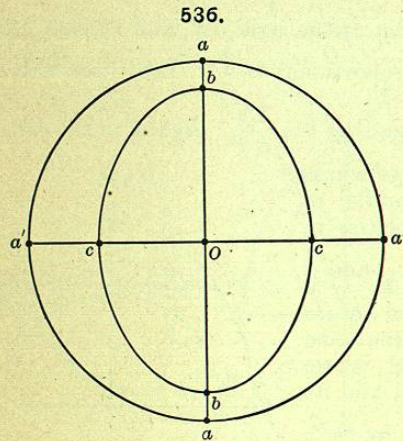
$\frac{1}{\gamma}$ , and it can be shown (after the analogy of uniaxial crystals and as proved by experiment) that this value is given by the line  $Or$  in the ellipse whose major and minor axes ( $Oa$  and  $Ob$ ) are  $\frac{1}{\alpha}$  and  $\frac{1}{\gamma}$ . Hence the circle  $cc'$  represents the section of the wave-surface for the rays in the given plane, whose vibration-direction corresponds to the axis  $CC$ , and the velocity to the constant value  $\frac{1}{\gamma}$ . While for other rays the vibration-directions change from  $AA$  to  $BB$ , and the velocity from  $\frac{1}{\alpha}$  to  $\frac{1}{\beta}$ .

The ray propagated by vibrations in the direction of the axis  $CC$ , which has the constant velocity  $\frac{1}{\gamma}$ , that is, the ray whose wave-front in this cross-section is a circle, is called the *ordinary ray*, since on refraction it remains in the plane of incidence. The other ray, whose velocity varies with the direction from  $\frac{1}{\alpha}$  to  $\frac{1}{\beta}$ , is called the *extraordinary ray*.

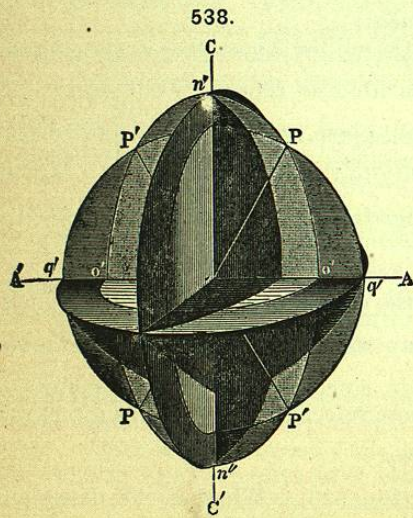
Again, take the plane of the axes  $BC$  (Fig. 533). Whether the direction of the light be that of  $B$  or of  $C$ , or any intermediate line in the same plane, there will be in each position one ray whose vibration-direction is that of the



axis  $A$ , and whose velocity is hence expressed by  $\frac{1}{\alpha}$ ; for it the section of the wave-surface will be a circle. For the other ray, if parallel to  $B$ , the vibration-direction will be that of the axis  $C$ , and its velocity is represented by  $\frac{1}{\gamma}$ . If it is parallel to  $C$ , its vibration-direction is that of  $B$ , and its velocity is given by  $\frac{1}{\beta}$ . As in the other case, intermediate values will be given by the ellipse



having  $\frac{1}{\beta}$  and  $\frac{1}{\gamma}$  for its major and minor axes. The combined section of the wave-surface is shown in Fig. 536. Here also, the ray with the constant velocity  $\frac{1}{\alpha}$  is called the ordinary ray; the other is the extraordinary ray.



For the third plane, that of the axes  $AC$ , one ray will always have as its vibration-direction that of the axis  $B$ , and its velocity will hence be expressed by  $\frac{1}{\beta}$ . For the other ray, if parallel to  $A$ , the vibration-direction is that of  $C$ , and the velocity is expressed by  $\frac{1}{\gamma}$ . If parallel to  $C$ , the vibration-direction is that of  $A$  and the velocity  $\frac{1}{\alpha}$ , and similarly for intermediate positions. The section of the wave-surface constructed from these values is given in Fig. 537. Here the circle (radius =  $\frac{1}{\beta}$ ) cuts the ellipse at the points  $PP, P'P'$ . The complete wave-surface, of which the three axial sections are given in

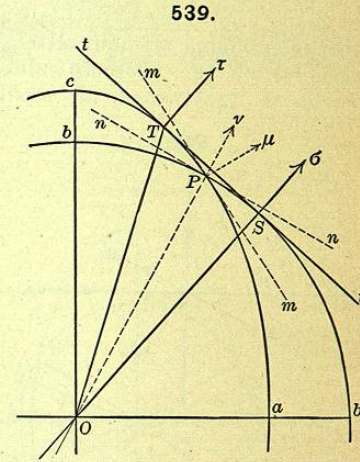
Figs. 535, 536, and 537, can be constructed, but it is not easy to form a

complete knowledge of the form without having a model in hand. Some idea of it may be gathered from Fig. 538.

**371. Primary and Secondary Optic Axes.**—It has already been briefly stated (Art. 367) that there are two directions, namely, those normal to the circular cross-sections of the indicatrix ( $SS, S'S'$ , Fig. 533) in which the light is propagated by transverse vibrations of like rate ( $\frac{1}{\beta}$ ). Hence in these directions in a crystal there is no double refraction within the crystal; nor is there when the ray emerges. These two directions bear so close an analogy to the optic axes of a uniaxial crystal that they are also called *optic axes*, and the crystals here considered are hence named *biaxial*. In Fig. 537, these optic axes have the direction  $SS, S'S'$  normal to the tangent planes  $tt, t't'$ , and the direction of the external wave is given by the normal  $S\sigma$  (Fig. 539).

Properly speaking the directions mentioned are those of the *primary optic axes*, for there are also two other somewhat analogous directions,  $PP, P'P'$ , of Fig. 537, called for sake of distinction the *secondary optic axes*. The properties of the latter directions are obvious from the following considerations.

In the section of the wave-surface shown in Fig. 537 (also enlarged, in Fig. 539), corresponding to the axial plane  $AC$ , it is seen that the circle with radius  $Ob (= \frac{1}{\beta})$  intersects the ellipse whose major and minor axes are  $Oa (= \frac{1}{\alpha})$  and  $Oc (= \frac{1}{\gamma})$  in the four points  $P, P, P', P'$ . Corresponding to these directions the velocity of propagation is obviously the same for both rays. Hence within the crystal these rays travel together without double refraction. Since, however, there is no common wave-front for these two rays (for the tangent for one ray is represented by  $mm$  and for the other by  $nn$ , Fig. 539) they do suffer double refraction on emerging; in fact, two external light-waves are formed whose directions are given by the normals  $P\mu$  and  $P\nu$ . These directions,  $PP, P'P'$ , therefore have a relatively minor interest, and whenever, in the pages following, optic axes are spoken of, they are always the *primary optic axes*, that is, those having the directions  $SS, S'S'$  (Fig. 537), or  $OS, OS'$ , Fig. 539.\* In practice, however, as remarked in the next article, the angular variation between the two sets of axes is usually very small, perhaps  $1^\circ$  or less.



**372. Interior and Exterior Conical Refraction.**—The tangent plane to the wave-surface drawn normal to the line  $OS$  through the point  $S$  (Fig. 539) may be shown to meet it in a small circle on whose circumference lie the points  $S$  and  $T$ . This circle is the base of the interior cone of rays  $SOT$ , whose remarkable properties will be briefly hinted at. If a section of a biaxial crystal be cut with its faces normal to  $OS$ , those parallel rays belonging to a cylinder having this circle as its base, incident upon it from without, will be propagated within as the cone  $SOT$ . Conversely, rays from within corresponding in position to the surface of this cone will emerge *parallel* and form a circular cylinder. This phenomenon is called *interior conical refraction*.

\* Fletcher calls the primary axes *binormals*, the secondary axes *biradials*.

On the other hand, if a section be cut with its faces normal to  $OP$ , those rays having the direction of the surface of a cone formed by perpendiculars to  $mm$  and  $nn$  will be propagated within parallel to  $OP$ , and emerging on the other surface form without a similar cone on the other side. This phenomenon is called *exterior conical refraction*.

In the various figures given (535-539) the relations are much exaggerated for the sake of clearness; in practice the relatively small difference between the indices of refraction  $\alpha$  and  $\gamma$  makes this cone of small angular size, rarely over  $2^\circ$ . For example, with sulphur, which has very strong double refraction ( $\gamma - \alpha = 0.29$ ; compare the values given in Art. 359 and Art. 382), the values of  $\alpha, \beta, \gamma$  for yellow sodium light were measured by Schrauf as follows:

$$\alpha = 1.95047, \quad \beta = 2.03832, \quad \gamma = 2.24052.$$

**373. Axes of Elasticity.**—As intimated in Art. 368, Fresnel appears to have deduced the wave-surface of biaxial crystals, as it is here called following him, by a generalization from that accepted for the more simple uniaxial crystals. The explanation of the observed phenomena, attempted by him, was based upon the assumption that the varying velocity of light shown by the varying values of the refractive indices depended upon the variable *elasticity* of the ether within the crystal. Since, as stated on p. 160, it seems better not to insist upon this hypothesis and since, further, it is possible to describe all the phenomena without attempting to explain the properties of the ether upon which the ultimate values of the pulses depend which manifest themselves as light-waves, all mention of elasticity has been thus far avoided.

These "axes of elasticity" are of great convenience in describing the optical properties of crystals, and it is hence necessary to make frequent use of them. They are uniformly represented by the letters  $a, b, c$ , where  $a > b > c$  as shown in Fig. 540, and where

$$\text{further it is true that } a : b : c = \frac{1}{\alpha} : \frac{1}{\beta} : \frac{1}{\gamma}$$

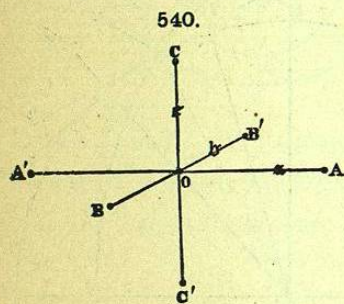
$\alpha, \beta, \gamma$  being the three principal refractive indices ( $\alpha < \beta < \gamma$ ). The three axes as given in Fig. 540 then have the direction of the three ellipsoidal axes (Fig. 533); the maximum value of the elasticity,  $a$ , corresponds to the

minimum value of the observed refractive index  $\alpha$ , and this in turn corresponds to the maximum velocity of a ray propagated perpendicular to the plane of the axes  $a$  and  $b$  by vibrations having a direction parallel to this axis  $a$ ; similarly for the axes  $b$  and  $c$ .

Whenever in this work the axes  $a, b, c$ , which may be simply called the *ether-axes*, are spoken of in describing the optical characters of crystals, it is to be understood that they have the directions indicated, corresponding respectively, as just explained, to the ellipsoidal axes; moreover, their relative magnitude is expressed as follows:  $a > b > c$ .

**374. Bisectrices, or Mean-lines.**—As shown in Art. 371, the optic axes always lie in the plane of the axes  $\alpha, \gamma$  of the indicatrix (that is, of the ether-axes  $a$  and  $c$ ); this is called the optic axial plane (or briefly, *ax. pl.*). The value of the optic axial angle is known when the values of the refractive indices,  $\alpha, \beta, \gamma$ , are given, as stated in the next article. That axis (cf. Fig. 539, also Figs. 541, 542) which bisects the acute angle of the optic axes is called the *acute bisectrix*, or *first mean-line*, and that bisecting the obtuse angle is the *obtuse bisectrix*, or *second mean-line*.

The acute bisectrix is often represented by  $Bx_a$ , the obtuse bisectrix by  $Bx_o$ .

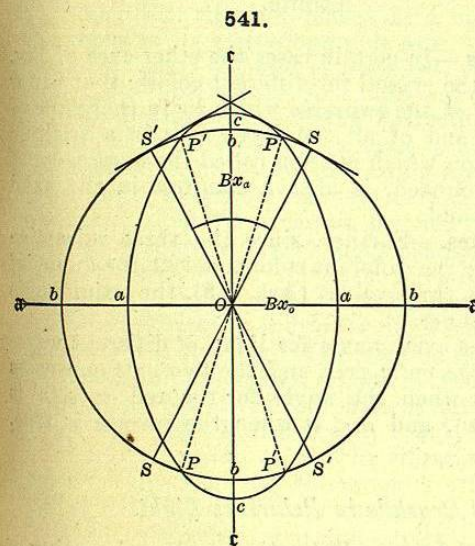


If the word *bisectrix* is used alone without special qualification it is always to be understood as referring to the acute bisectrix.

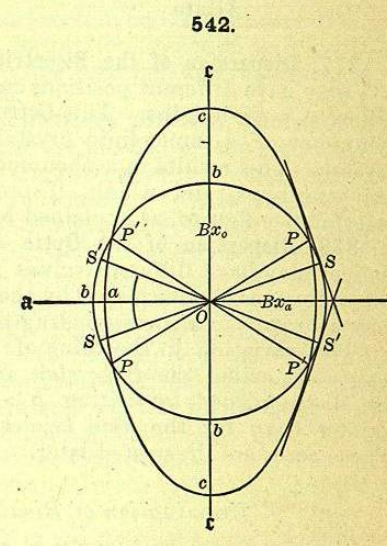
**375. Relation of the Axial Angle to the Refractive Indices.**—If in a given case the values of  $\alpha, \beta$ , and  $\gamma$  are known, the value of the interior optic axial angle ( $2V$ ) can be calculated from them by the following formulas:

$$\cos^2 V = \frac{\frac{1}{\beta^2} - \frac{1}{\gamma^2}}{\frac{1}{\alpha^2} - \frac{1}{\gamma^2}} \quad \text{or} \quad \tan^2 V = \frac{\frac{1}{\alpha^2} - \frac{1}{\beta^2}}{\frac{1}{\beta^2} - \frac{1}{\gamma^2}}$$

**376. Positive and Negative Crystals.**—Biaxial crystals are distinguished as *optically positive (+)* or *negative (-)* after a manner analogous to the usage with uniaxial crystals. Referring to Fig. 533 of the ellipsoid, and also to Fig. 539, it will be obvious at once that for certain relative values of the indices,  $\alpha, \beta, \gamma$ , the interior optic axial angle must be  $90^\circ$ . In other words, in this



Positive Crystal,  $Bx_a = c$ .



Negative Crystal,  $Bx_o = a$ .

case the planes of the optic axes will be equally inclined to the two planes of the ether-axes. Such a case, however, is rare in practice, and when it occurs it is true for light of a certain color\* (wave-length) only, and not for others.

\* For danburite  $2V = 89^\circ 14'$  for green (thallium) and  $90^\circ 14'$  for blue ( $\text{CuSO}_4$ ).

Roughly expressed,\* the optic axes will lie nearer to  $c$  than to  $a$ —that is,  $c$  will be the bisectrix—when the value of the intermediate index,  $\beta$ , is nearer that of  $\alpha$  than to that of  $\gamma$ . Such a crystal, for which  $Bx_a = c$ , is called *optically positive*. It is obvious (cf. Fig. 541) that in this case, as the angle diminishes and becomes nearly equal to zero, the form of the ellipsoid then approaches that of the prolate spheroid of the positive uniaxial crystal as its limit (Fig. 521, p. 195); this shows the appropriateness of the + sign here used.

On the other hand, the optic axes will lie nearer to  $a$  than to  $c$ —that is,  $a$  will be the bisectrix—if the value of the mean index  $\beta$  is nearer† to that of  $\gamma$  than to that of  $\alpha$ . Such a crystal, for which  $Bx_a = a$ , is called *optically negative*. It is seen that in this case (Fig. 542) the smaller the angle the more the ellipsoid approaches the oblate spheroid of the negative uniaxial crystal (Fig. 520, p. 195).

The following are a few examples of positive and negative biaxial crystals:

Positive (+).	Negative (-).
Sulphur.	Aragonite.
Enstatite.	Hypersthene.
Topaz.	Muscovite.
Barite.	Orthoclase.
Chrysolite.	Epidote.
Albite.	Axinite.

**377. Dispersion of the Bisectrices.**—In certain cases the ether-axes of Fig. 533 may have different positions in the crystal for different colors; that is, for different wave-lengths. This is true of the two axes which lie in the plane of symmetry of a monoclinic crystal, and of all the three axes in a triclinic crystal. This results in a phenomenon which is often called the *dispersion of the bisectrices*, and which, if pronounced, is always manifest in the axial interference-figures, as explained beyond.

**378. Dispersion of the Optic Axes.**—Further, since the three refractive indices may have different values for the different colors, and as the angle of the optic axes is determined by these three values (Art. 375), the axial angle may also vary in a corresponding manner.

This variation in the value of the axial angle for light of different wave-lengths is called the *dispersion of the optic axes*, and the two extreme cases are distinguished by writing  $\rho > \nu$  when the angle for the red rays ( $\rho$ ) is greater than for the blue (violet,  $\nu$ ), and  $\rho < \nu$  when the reverse is true. These cases are illustrated later.

#### Examination of Biaxial Crystals in Polarized Light.

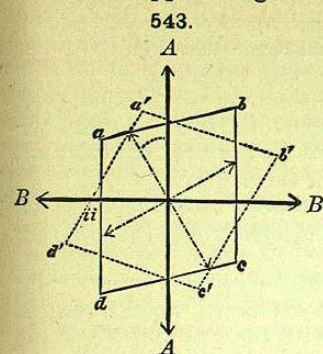
**379. Sections in Parallel Polarized Light. Extinction-angle.**—A section of a biaxial crystal appears dark between crossed nicols when its vibration-planes coincide with the vibration-planes of the nicols. In any other position of the

\*  $\tan V = 45^\circ$  and  $2V = 90$  for a value of  $\beta$  given by the equation

$$\frac{1}{\alpha^2} - \frac{1}{\beta^2} = \frac{1}{\beta^2} - \frac{1}{\gamma^2}.$$

† To compare Figs. 542 and 520, the horizontal axis of the former should be placed in a vertical position; that is, the axis  $AA$  of Fig. 520 corresponds to  $c$  of Fig. 542.

section it appears light and may show the usual bright interference-color.



Hence the use of the polariscope, or polarization-microscope, by the method of extinction, gives a quick means of determining the position of these vibration-planes in a given case.

For example, in Fig. 543, repeated from p. 177, let the two larger rectangular arrows represent the vibration-directions for the two nicols, and between the two prisms suppose a section of a biaxial crystal,  $abcd$ , to be placed so that one edge of a known crystallographic plane coincides with the direction of one of these lines. The field of the microscope, dark before, since the prisms were crossed, is no longer so, and becomes dark again, as explained, only when the crystal is revolved so that *its* vibration-directions (the smaller dotted arrows) coincide with those of the nicols, as is indicated by the maximum extinction of the light. The crystal has then the position  $a'b'c'd'$ . The angle (indicated in the figure) which it has been necessary to revolve the plate to obtain the effect described, is the angle which one of the vibration-directions in the given plate makes with the given crystallographic edge  $ad$ ; it is often called the *extinction-angle*.

When the vibration-planes of a crystal-section coincide in direction with the planes of its crystallographic axes, the extinction is said to be *parallel*; if not, it is called *oblique* or *inclined*. On the practical determination of the extinction-directions see Arts. 380 and 389.

**380. Determination of the Extinction-directions with the Microscope.**—In the use of the microscope, in cases to which the method of the stausoscope is not applicable, instead of depending upon the somewhat uncertain estimate of the point of maximum light-extinction, a convenient way is to employ a plate of quartz, which for a certain position of the analyzer gives the field a tint of color (a purplish pink), to slight changes in which the eye is very sensitive. When the section is revolved on the stage till it has precisely the same tint as the surrounding field, its vibration-planes are those of the cross-hairs in the eyepiece (supposing the lower nicol has the normal position), and from the graduation of the stage their position in the section can be at once determined.

Instead of the quartz, a plate of selenite (Art. 364) of such a thickness as to give the red of the first order is often employed, the nicols being crossed. When this plate is inserted as usual, the crystal-section can have the same color only on condition that its vibration-directions coincide with those of the nicols; hence their crystallographic orientation in the section is readily determined and with considerable accuracy.

A still more delicate method involves the use of the *Bertrand ocular*. This has four sectors of quartz cut  $\perp c$ : two of these, diagonally opposite, are from a right handed, the other pair from a left-handed crystal. When the diameters in which the sectors meet coincide with the vibration-directions of the two-crossed nicols, the two pairs of quadrants have precisely the same color; any change of position, however, of the upper nicol causes them to assume tints complementary to each other.

Assuming now the nicols to be crossed and in the normal position, if a section of a doubly refracting mineral be introduced on the stage of the microscope, the quadrants in general take unlike tints and are brought to the same color as before only when by the revolution of the stage the vibration-directions of the section are made to correspond precisely to those of the crossed nicols, that is, of the diameters of the ocular. The adjustment can be made in this case with great accuracy.

**381. Relation of the Vibration-directions to the Ether-axes.**—In the most