CHAPTER IV.

PARABOLIC RIB WITH FIXED ENDS.

57. Values of Ordinates. — Passing next to the parabolic arch, fixed at the ends, we recall, from § 16, that, to locate the equilibrium polygon for a single load at any point, we need *three ordinates*, one at each end, and the third passing through the weight, and that the three conditions by which these must be obtained are, 1st, that the change of span is zero; 2d, that the change of inclination at the abutments is zero; and, 3d, that the abutment deflection is zero. As expressed in the notation used, the three equations of condition are

$$\Sigma\, \mathrm{E\,F}\,.\,\mathrm{D\,E} = 0,$$
$$\Sigma\, \mathrm{E\,F} = 0,$$
$$\Sigma\, \mathrm{E\,F}\,.\,\mathrm{D\,B} = 0.$$

If, in Fig. 12, I N L represents the desired equilibrium polygon for a weight W, attached to the rib A Q B at a point distant T G, $= b$, horizontally from the middle of the span; and if the span A B $= 2c$, the rise of the arch $= k$, A I $= y_1$, G N $= y_0$, and B L $= y_2$, we will prove that

$$y_0 = \tfrac{8}{5} k, \quad (1.)$$
$$y_1 = \tfrac{2}{15} \cdot \frac{c + 5b}{c + b} k = \tfrac{2}{15} \frac{1 + 5n}{1 + n} k, \quad (2.)$$
$$y_2 = \tfrac{2}{15} \cdot \frac{c - 5b}{c - b} k = \tfrac{2}{15} \frac{1 - 5n}{1 - n} k, \quad (3.)$$

when $b = n\,c$.

58. Value of First Equation. — As before, the first condition may be written,

$$\Sigma\, \mathrm{E\,F}\,.\,\mathrm{D\,E} = \Sigma\, (\mathrm{D\,E} - \mathrm{D\,F})\, \mathrm{D\,E} = 0, \text{ or } \Sigma\, \mathrm{D\,E}^2 = \Sigma\, \mathrm{D\,F}\,.\,\mathrm{D\,E}. \quad (1.)$$

If A D $= x$, D E $= \frac{k}{c^2}(2c - x)\, x$, as in § 39. A G $= c + b$; G B $= c - b$. If y_1 or y_2 becomes negative, it is to be laid off below A B, but otherwise above: the figure represents y_2 as negative; and, in the majority of cases, y_1 and y_2 have opposite signs. If a line be drawn horizontally from I, D F, as long as it is on the left of y_0, will be divided into a constant part y_1, and a remainder which varies with the distance from I. Hence we see that

$$\mathrm{D\,F} = y_1 + \frac{y_0 - y_1}{c + b} x.$$

For the right-hand member of (1.), between A and G, we therefore get

$$\int_0^{c+b} \left(y_1 + \frac{y_0 - y_1}{c + b}\, x\right) \frac{k}{c^2}(2cx - x^2)\, dx =$$

$$\frac{k}{c^2} y_1 \int_0^{c+b} (2cx - x^2)\, dx + \frac{k}{c^2} \cdot \frac{y_0 - y_1}{c + b} \int_0^{c+b} (2cx^2 - x^3)\, dx =$$

$$\frac{k}{c^2} y_1 \left[c\,(c + b)^2 - \tfrac{1}{3}(c + b)^3\right] + \frac{k}{c^2}(y_0 - y_1)\left[\tfrac{2}{3} c\,(c + b)^2 - \tfrac{1}{4}(c + b)^3\right]. \quad (2.)$$

For the portion between G and B, if we write $c - b$ for $c + b$, and reckon x from B to the left, we get

$$\mathrm{D\,F} = y_2 + \frac{y_0 - y_2}{c - b} x,$$

the sign of y_2 being contained in the symbol. Then the integration for the right-hand member of (1.), between B and G, or between the limits 0 and $c - b$, will give, when we substitute y_2 for y_1, and $c - b$ for $c + b$,

$$\frac{k}{c^2} y_2 \left[c\,(c - b)^2 - \tfrac{1}{3}(c - b)^3\right] + \frac{k}{c^2}(y_0 - y_2)\left[\tfrac{2}{3} c\,(c - b)^2 - \tfrac{1}{4}(c - b)^3\right]. \quad (3.)$$

The left-hand member of (1.) was shown to be, in § 39, (2.),

$$\int_0^{2c} \frac{k^2}{c^4}(2cx - x^2)^2\, dx = \tfrac{16}{15} k^2 c. \quad (4.)$$

The two portions, (2.) and (3.), of the right-hand member, being added together, when the coefficients of y_0, y_1, and y_2 are reduced, will be equated with (4.), the left-hand member of (1.), producing

$$\frac{k}{6c^2}\left\{y_0(5c^3 - cb^2) + \tfrac{1}{2}y_1(c+b)^2(3c-b) + \tfrac{1}{2}y_2(c-b)^2(3c+b)\right\} = \tfrac{16}{15}k^2 c,$$

or

$$2c(5c^2 - b^2)y_0 + (c+b)^2(3c-b)y_1 + (c-b)^2(3c+b)y_2 = \tfrac{64}{5}kc^3. \quad (5.)$$

59. **Values of Second and Third Equations.** — It is not necessary to integrate in order to obtain equations from the other two conditions, although they may be derived quite simply in that way. The second condition may be written,

$$\Sigma\, \text{EF} = \Sigma(\text{DE} - \text{DF}) = 0, \quad \text{or} \quad \Sigma\, \text{DE} = \Sigma\, \text{DF}.$$

The first member is the summation of all the ordinates to the arch, or the included area between the rib and the line A B. The area of a parabolic segment being equal to two-thirds of the rectangle of the same base and altitude, the area will be $\frac{2}{3} \cdot 2c \cdot k$, or $\frac{4}{3}ck$. The second member will be the summation of all the ordinates to the two inclined lines, or the area of the two trapezoids, giving

$$\tfrac{1}{2}(y_0 + y_1)(c+b) + \tfrac{1}{2}(y_0 + y_2)(c-b), \quad \text{or} \quad cy_0 + \tfrac{1}{2}(c+b)y_1 + \tfrac{1}{2}(c-b)y_2.$$

Equating the two values, we obtain the second equation,

$$2cy_0 + (c+b)y_1 + (c-b)y_2 = \tfrac{8}{3}ck \quad (1.)$$

The condition that Σ E F . D B = 0, or that Σ (D E − D F) D B = 0, gives

$$\Sigma\, \text{DE} \cdot \text{DB} = \Sigma\, \text{DF} \cdot \text{DB},$$

and this condition is satisfied by the equivalent step of multiplying each area, just obtained, by the horizontal distance of its centre of gravity from one abutment, the right one for example, and equating the products. The left-hand member will then plainly be $\frac{4}{3}ck \cdot c$, or $\frac{4}{3}c^2k$. As the second expression above for the area of the trapezoids has three terms which correspond to the three triangles formed by drawing lines from N to A and B, we may multiply each triangle by the distance of its centre of gravity from B, obtaining

$$cy_0(c - \tfrac{1}{3}b) + \tfrac{1}{2}(c+b)y_1[c - b + \tfrac{2}{3}(c+b)] + \tfrac{1}{2}(c-b)y_2\tfrac{1}{3}(c-b),$$

or,

$$\tfrac{1}{3}cy_0(3c-b) + \tfrac{1}{6}(c+b)y_1(5c-b) + \tfrac{1}{6}y_2(c-b)^2.$$

Equating the two members, and clearing of fractions, we find that

$$2c(3c-b)y_0 + (c+b)(5c-b)y_1 + (c-b)^2y_2 = 8c^2k. \quad (2.)$$

60. **Solution of Equations.** — Equations (5.), § 58, and (1.) and (2.), § 59, contain the three unknown quantities. The eliminations may be performed as follows: —

Multiply (1.) by $c - b$, obtaining

$$2c(c-b)y_0 + (c+b)(c-b)y_1 + (c-b)^2y_2 = (c^2 - bc)\tfrac{8}{3}k.$$

Subtract from (2.)

$$4c^2y_0 + 4c(c+b)y_1 = (2c^2 + bc)\tfrac{8}{3}k. \quad (a.)$$

Multiply (2.) by $3c + b$,

$$2c(9c^2 - b^2)y_0 + (c+b)(15c^2 + 2cb - b^2)y_1 + (c-b)^2(3c+b)y_2 = (3c^3 + bc^2)8k.$$

Subtract (5.), and divide the remainder by $2c$,

$$4c^2y_0 + 6c(c+b)y_1 = (\tfrac{7}{5}c^2 + bc)4k. \quad (b.)$$

Subtract (*a.*),

$$2c(c+b)y_1 = (\tfrac{4}{15}c^2 + \tfrac{4}{3}bc)k, \quad \text{or} \quad y_1 = \tfrac{2}{15} \cdot \frac{c+5b}{c+b}k.$$

Substituting this value in (*a.*) or (*b.*), we get

$$y_0 = \tfrac{6}{5}k,$$

and by analogy, or by substitution,

$$y_2 = \tfrac{2}{15} \cdot \frac{c-5b}{c-b}k.$$

61. **Remarks.** — The similarity between y_1 and y_2 is to be expected; for, when a load is moved from one side of the centre to an equal distance on the other, y_1 and y_2 change places. Therefore it must be remembered that y_2 is the value of the ordinate at that springing which is *nearer* to the weight. If

the load is in the middle, $b = 0$, and $y_1 = y_2$. It is worthy of notice that y_0 is a constant quantity for all positions of the weight. These ordinates can be easily computed for a weight at different points, and it will be seen that a value of b greater than $\frac{1}{5}c$ will make y_2 negative, or to be plotted below the springing line. The original reasoning showed, and the above equations will prove, that the third condition may be taken about the other abutment, and will still give the same values for the ordinates.

62. **Computation of Ordinates** y_1 **and** y_2. — If we propose to work out data for use with this type of arch also, we must first calculate the values of y_1 and y_2 for all points. Let a rib be divided into ten parts, equal horizontally as before; then, if $b = n\,c$, the results of the following table will be obtained. It

VALUES OF y_1 AND y_2.

$n = \frac{b}{c} =$	0	.2	.4	.6	.8	
$\frac{1+5n}{1+n} =$	$\frac{1}{1}$	$\frac{2.0}{1.2}$	$\frac{3.0}{1.4}$	$\frac{4.0}{1.6}$	$\frac{5.0}{1.8}$	
$\frac{2}{15} \cdot \frac{1+5n}{1+n} =$	0.1333	0.2222	0.2857	0.3333	0.3704	$k = y_1$.
$\frac{1-5n}{1-n} =$	$\frac{1}{1}$	$\frac{0}{0.8}$	$\frac{-1.0}{0.6}$	$\frac{-2.0}{0.4}$	$\frac{-3.0}{0.2}$	
$\frac{2}{15} \cdot \frac{1-5n}{1-n} =$	0.1333	0	—0.2222	—0.6667	—2.0	$k = y_2$.

is so similar to previous ones as to call for no explanation. Only remember that y_1 and y_2 change places for loads on the left of the crown. The equilibrium polygons for one half of the arch are shown in Fig. 12.

63. **Formulæ for** H, P_1 **and** P_2. — To obtain the value of H for a particular position of the load, we lay off y_1, y_0, and y_2 at A, G, and B, draw I N and N L, complete the stress diagram below, and draw 0–3 for H. The vertical components of the abutment reactions will be 2–3 and 3–1. If we draw the horizontal dotted lines from I and L, we shall have similar triangles to those in the stress diagram, and may write

$$y_0 - y_1 : c + b = (2\text{–}3) : \text{H, or}$$

$$P_1 = (2\text{–}3) = \text{H}\frac{y_0 - y_1}{c + b} = \tfrac{8}{15} \cdot \frac{2 + n}{(1 + n)^2}\frac{k}{c} \cdot \text{H},$$

$$y_0 + (-y_2) : c - b = (3\text{–}1) : \text{H, or}$$

$$P_2 = \text{W} - (2\text{–}3) = \text{H}\frac{y_0 - y_2}{c - b} = \tfrac{8}{15} \cdot \frac{2 - n}{(1 - n)^2}\frac{k}{c} \cdot \text{H}.$$

Substitute the value of (2–3) from the first equation, transpose, and obtain

$$\text{H} = \frac{\text{W}}{\frac{y_0 - y_1}{c + b} + \frac{y_0 - y_2}{c - b}} = \tfrac{15}{32} \cdot \frac{(c^2 - b^2)^2}{c^3 k} \cdot \text{W} = \tfrac{15}{32}(1 - n^2)^2 \frac{c}{k}\text{W}.$$

64. **Computation of Values.** — The amount of H for a load at any one point will then be found in the several columns of the table below. The first three values will be seen to be

VALUES OF H, P_1, AND P_2.

$n =$	0	.2	.4	.6	.8	
$1 - n^2 =$	1	.96	.84	.64	.36	
$(1 - n^2)^2 =$	1	.9216	.7056	.4096	.1296	
H $=$	.4687	.4320	.3308	.1920	.0607	$\frac{c}{k}$W.
$\text{H}\frac{y_0 - y_1}{(1 + n)c} =$	0.5	0.352	0.216	0.104	0.028	W } = P.
$\text{H}\frac{y_0 - y_2}{(1 - n)c} =$	0.5	0.648	0.784	0.896	0.972	W } = P.

greater, and the last two to be smaller, than the corresponding H's in § 41. It will next be necessary to find the vertical components of the reactions by multiplying H by the quantities noted in the last section: the results will be found in the last two lines. The larger value of P occurs at the nearer abutment. It will be noted that these quantities differ in amount from the two supporting forces of a single-span beam or truss.

If the H's for an equal load at each of the nine points of division are added together, we find that, for loads at all points,

$H = 2.4997 \frac{c}{k} W$, which agrees more closely with the amount for a truss or bowstring girder than did the value for a rib with hinged ends, § 42. It is due to the fact that the equilibrium polygon for a single weight crosses the rib oftener in the present case than in that of a rib with hinged ends; so that, when several loads are combined, the polygon will deviate from the parabola (the form of the rib, and the true equilibrium curve for a uniform distributed load) very little.

65. **Computation of Bending Moments.** — If, in place of scaling, we desire to compute the values of M in this case also, we may use the former equation, § 43,

$$M = H (y - z).$$

The values of the ordinates, z, to the parabola will be the same as before. If x denotes the distance from A to the foot of the ordinate y, and x' = the distance from B to the foot of the same ordinate, in which case $x' = 2c - x$, we shall have

$$y = y_1 + \frac{y_0 - y_1}{c + b} x, \text{ on the left of the weight, and}$$

$$y = y_2 + \frac{y_0 - y_2}{c - b} x', \text{ on the right of the weight,}$$

the sign of y_2 being contained in the symbol.

Let us proceed to find the values of M, at both abutments and the nine other points, for a weight on the third point of division from the middle, towards the right. As above,

$$H = 0.192 \frac{c}{k} W; \quad \frac{y_0 - y_1}{c + b} = 0.5417 \frac{k}{c}; \quad \frac{y_0 - y_2}{c - b} = 4.6667 \frac{k}{c};$$

$$z = .36\,k,\ .64\,k,\ .84\,k,\ .96\,k,\ k,\ .96\,k, \text{ \&c., § 43.}$$

VALUES OF M.

								W				
$x =$	0 c	0.2 c	0.4 c	0.6 c	0.8	1.0	1.2	1.4	1.6	0.2 c	0 c	$= x'$
$\times .5417 \frac{k}{c}$	0	.1083	.2166	.3250	.4334	.5417	.6500	.7584	.8667	.9333	0	$\times 4.667 \frac{k}{c}$
$+ y_1$	.3333	.4416	.5500	.6583	.7667	.8750	.9833	1.0917	1.2000	.2667	−.6667	$+ y_2$
$z =$	0	.36	.64	.84	.96	1.00	.96	.84	.64	.36	0	k
$y - z =$	+.3333	+.0816	−.0900	−.1817	−.1933	−.1250	+.0233	+.2517	+.5600	−.0933	−.6667	k
Multiply by $H = 0.192 \frac{c}{k} W$.												
$M =$	+.0640	+.0157	−.0173	−.0349	−.0371	−.0240	+.0045	+.0483	+.1075	−.0179	−.1280	c W

W is placed over the number of the point to which it is attached, and a double line is drawn on one side of W to denote the end of each series, running from the two ends of the table. The dividing line might just as well have been drawn on the left of W, if preferred. More frequent values of any of the preceding quantities may be obtained by interpolation, as explained before.

66. **Table of Bending Moments.** — A table of values of M has been prepared for this case of an arch with fixed ends, the span being divided into ten equal parts, and is here presented, p. 71. A table for twenty divisions may be found in "Engineering News," vol. iv., p. 178. At any one point, for a uniform load at all of the points of division, M reduces nearly to zero, as before. The greatest possible positive M, as well as the greatest possible negative M, for any combination of weights, occurs at each abutment; positive maximum when the span is loaded from the other abutment to and beyond the centre one point; negative when the other portion only of the span is covered. The load on the first point from the middle produces no M at the nearer abutment. There is another maximum at the third or seventh point, with loads nearly the reverse of the ones mentioned above. An inspection of the table will show these facts.

67. **Example.** — As soon as H, P, y_1, and y_2 have been obtained for all points, it is easy to draw an equilibrium polygon for any desired arrangement of load. Let us suppose that one must be constructed for weights of 2 tons, 6 tons, 3 tons, and 1 ton, on the 2d, 4th, 5th, and 8th points respectively, from the left abutment, of an arch of 100 feet span and 20 feet rise, Fig. 13, divided into ten equal parts along the span, as previously described. We will proceed as follows: —

The vertical components of the reactions cannot be computed for the load in the gross, as for a beam on two supports, but must be summed up from the values lately given. Referring to those data, we get

	P_1.		H.	
2d joint,	$0.896 \times 2 = 1.792$	tons.	$0.192 \times 2 = 0.384 \frac{c}{k}$	tons.
4th "	$0.648 \times 6 = 3.888$	"	$0.432 \times 6 = 2.592$	"
5th "	$0.5 \times 3 = 1.500$	"	$0.469 \times 3 = 1.407$	"
8th "	$0.104 \times 1 = 0.104$	"	$0.192 \times 1 = 0.192$	"
	$P_1 = 7.284$	"	$H = 4.575$	"

$P_2 = 12 - 7.284 = 4.716$ tons. $H = 4.575 \times 2.5 = 11.44$ tons.

Since $H\,y_1 =$ moment at the springing A, Fig. 13; since each of these loads has a separate H and a definite y_1; and since the H's for the different loads all conspire to produce the total thrust, —we must calculate the arm with which the latter acts at one or both springings, that is, the ordinate y_1' or y_2' of the point whence the equilibrium polygon must start. We satisfy the equation

$$y_1' \cdot \Sigma H = \Sigma H \cdot y_1, \text{ or } y_1' = \frac{\Sigma H \cdot y_1}{\Sigma H},$$

which simply requires that the resultant moment shall be equal to the algebraic sum of the original moments. We therefore multiply each H for a given weight by its y_1, and divide the sum of the products by the total H. The calculation having been made, as here set down, we find that y_1' is equal to —.02 feet, a comparatively insignificant amount. It is well to compute y_2' also, as a check on the accuracy of the subsequent drawing, and it will be found to be +3.34 feet.

y_1.	H.	M.	
$-.667 \times$	$0.384 =$	$-0.256\,c$	tons.
$0 \times$	$2.592 =$	0	
$+.133 \times$	$1.407 =$	$+0.188$	"
$+.333 \times$	$0.192 =$	$+0.064$	"
	4.575)	-0.004	"
		$-0.0009\,k$.	
		20	
	$y_1' =$	-0.018 feet.	

While we may seem to have carried out this example in too much detail, we are aware that inattention to apparently trivial points will sometimes cause trouble, and we have therefore given most of the work at full length. Now lay off the weights in order on the load line, plot P_1 and P_2, lay off H on the proper side, draw the usual radiating lines to the extremity of H, start below A, a distance $-y_1'$, and draw the equilibrium polygon with sides parallel to the inclined lines of the stress diagram, checking the polygon by the fact that it strikes the extremity of the calculated ordinate y_2'. Fig. 13 illustrates this example. The diagram for vertical shear is also shown below, and needs no explanation, as the construction is similar to previous cases. The dotted lines in the stress diagram determine the value of Y_1. It is quite noticeable in this figure, how the shear changes sign wherever the bending moment becomes a maximum.

68. **Table of Shear.**— To find the numerical value of the vertical shear, from which we may derive the normal components resisted by the braces of an arch with fixed ends, we proceed as we did in the case of an arch with hinged ends. The values of P_1, the vertical component of the abutment reaction at the left, have been found. We then need only calculate the value of $Y_1 = 2\frac{k}{c}H$, and form a table, as was done in § 51. It is not necessary to repeat the operations here. A table of shears for an arch with fixed ends, and for ten divisions, has been prepared, and is appended, p. 70. The same remarks apply to it as to the previous similar table for the parabolic arch with hinged ends. For a table for twenty divisions, see "Engineering News," vol. iv., p. 193.

69. **Extent of Load to produce Maximum M and F.**— A diagram is also presented, Fig. 15, showing, by the full lines, the loads required to produce the maximum +M, from live load, at the point whose number is attached to the line, and by the remaining blank portion the load required for maximum —M at the same point. The broken lines and the blank portion in each *space* represent the way of distributing the load for maximum +F and —F respectively. It is still more apparent from this figure than from Fig. 11, that any investigation which considers the rolling load as continuous from one

PARABOLIC RIB, FIXED AT ENDS.

§ 66. $M = mcW$. Values of m.

	0	1	2	3	4	5	6	7	8	9	10	H
W on 9	+.022	+.006	−.005	−.012	−.013	−.010	−.002	+.011	+.028	+.051	−.121	$.061 \frac{c}{k} W.$
" 8	+.064	+.016	−.017	−.035	−.037	−.024	+.004	+.048	+.107	−.018	−.128	.192 "
" 7	+.095	+.019	−.031	−.054	−.050	−.020	+.036	+.119	+.028	−.036	−.073	.331 "
" 6	+.096	+.011	−.040	−.056	−.037	+.016	+.104	+.026	−.017	−.026	0	.432 "
" 5	+.062	−.006	−.037	−.031	+.012	+.094	+.012	−.031	−.037	−.006	+.062	.469 "
" 4	0	−.026	−.017	+.026	+.104	+.016	−.037	−.056	−.040	+.011	+.096	.432 "
" 3	−.073	−.036	+.028	+.119	+.036	−.020	−.050	−.054	−.031	+.019	+.095	.331 "
" 2	−.128	−.018	+.107	+.048	+.004	−.024	−.037	−.035	−.017	+.016	+.064	.192 "
" 1	−.121	+.051	+.028	+.011	−.002	−.010	−.013	−.012	−.005	+.006	+.022	.061 "

§ 68. $V = nW$. Values of n.

	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10
W on 9	−.081	−.057	−.033	−.008	+.016	+.040	+.064	+.089	+.113	−.863
" 8	−.242	−.165	−.088	−.011	+.066	+.142	+.219	+.296	−.627	−.550
" 7	−.379	−.247	−.115	+.017	+.150	+.282	+.414	−.453	−.321	−.189
" 6	−.426	−.253	−.080	+.093	+.266	+.438	−.389	−.216	−.043	+.130
" 5	−.343	−.156	+.031	+.219	+.406	−.406	−.219	−.031	+.156	+.343
" 4	−.130	+.043	+.216	+.389	−.438	−.266	−.093	+.080	+.253	+.426
" 3	+.189	+.321	+.453	−.414	−.282	−.150	−.017	+.115	+.247	+.379
" 2	+.550	+.627	−.296	−.219	−.142	−.066	+.011	+.088	+.165	+.242
" 1	+.863	−.113	−.089	−.064	−.040	−.016	+.008	+.033	+.057	+.081

abutment over a portion of the span will not determine actual maximum stresses. See § 54.

70. **Comparison of Ribs; Fixed and Hinged at Abutments.** — A comparison of Fig. 15 with Fig. 11 will be instructive, as showing the different loading, when hinges are omitted, to produce maximum bending moments and shears. There are four points near the ends of the rib with fixed ends, which require that loads should be on both ends of the span at once, to produce the maximum +M at those points; and five points at the middle which have the maximum —M under similar circumstances. In some structures such conditions can be realized. If we foot up the plus and minus values of the columns in the tables for M and V, we shall readily see that, with the exception of the springing points, all the points in the arch with fixed ends have *less* maximum bending moments of either kind, for a load W at each loaded point, than in the case of the arch with hinged ends, and, in most cases, the values are materially less. A similar comparison of maximum shears will show that the arch with fixed ends has to carry more shear over its web or bracing for all the divisions of the first and last quarters of the span, and less for the middle half of the span, than an arch with hinged ends. These considerations alone would indicate the superiority of the arch with fixed ends over the other type, as requiring less material in the flanges or chords, and throwing the heavier bracing towards the abutments; the value of the direct thrust, however, as indicated by the previously computed amounts of H, varies according to the amount of load, and conspires with the compression from bending moment, so that the sections of the two chords must be designed for the maximum compression and tension at all points; the effect of rise or fall of temperature will be shown to be greater on the rib with fixed ends, reqnring a greater increase of section to provide for it.