# CHAPTER X.

## STIFFENED SUSPENSION-BRIDGES.

160. **Necessity for Stiffening.** — That the curve of equilibrium for the cable of a suspension-bridge, when the load is supposed uniform per horizontal foot, and covers the entire span, is a parabola, was proved in § 28, Fig. 6. The steady load will always be carried by the cable. When, however, a moving load is upon the structure, the cable will tend to become flatter in curvature over the lightly-loaded portion, and more curved over the heavily-loaded portion, thus throwing the roadway from its proper line. Some means of stiffening the roadway or chain against distortion is therefore needed. Bridges subjected to travelling loads of but moderate amount may be stiffened by the longitudinal beams of the roadway; but heavy loads necessitate the employment of trusses or girders in some form.

161. **Inverted Arch.** — If the cable is divided into two parallel members, braced together as shown in Fig. 46, it becomes an inverted arch, and follows the treatment already given in either Chap. II., III., or IV., depending upon whether hinges are or are not introduced at the piers and the middle. From the fact that the cables are carried over the towers to anchorages, and that movement over the top of the tower will take place both from change of load and change of temperature, the span cannot be assumed invariable: hence there is greater liability to alteration of stress in the several members from unavoidable causes; and a larger factor of safety than is commonly employed





in structures will be appropriate. The introduction of three hinges will do away with these sources of error. This type of stiffening truss will be discussed further in connection with the one which follows.

162. **Horizontal Girder.** — It is much more common to employ a horizontal truss or girder, as shown in Fig. 47, to stiffen the suspension-bridge. If we note that the office of the arch or inverted arch is twofold, — first to resist the direct stress, and, second, to resist the bending moments at successive sections, — we see that the horizontal girder of this figure will be subject to the same bending moments at similar sections as the inverted arch or braced rib of Fig. 46, while the chain will here carry the direct stress, which in the former case was also resisted by the rib.

If the truss is hinged at the middle as well as at the abutments, it comes under the class of Chap. II.; and the effect of one or more loads is easily determined. We may draw Fig. 48, if desired, and find by inspection the extent of rolling load required to produce the maximum bending moment of either kind at any point. See § 32. Thus, at one-fourth the span from one abutment, the maximum bending moment of one kind occurs when the rolling load covers four-tenths of the span on the same side; and the maximum bending moment of the opposite kind, when the rolling load covers the other six-tenths of the span. The maximum moment at a point near the abutment is found when the head of the load is at one-third the span from that abutment. These values are easily deduced by finding the horizontal distance of the point of intersection D, in Fig. 48, on A F, of that line, which, starting from B, passes through E, the extremity of a certain ordinate. Those authors who make maximum bending moments at all points occur, for a stiffening girder hinged at ends and middle, when the half-span is covered, are in error. The shear diagrams are constructed as explained in the earlier chapters. The construction for normal shear will be applicable to Fig. 46, and the vertical shear diagram to the stiffening truss of Fig. 47.

163. **Distribution of Rolling Load between Cable and Truss.** — It may be well to call more particular attention to the distribution of the rolling load between the truss and cable of Fig. 47, and the way in which bending moments are caused in the unloaded portion of the horizontal girder. If the bridge is unequally loaded, and no stiffening appliances are used, a distortion is produced, as explained in the first section of this chapter. When a weight W is applied on a suspension-bridge of half-span $c$, at any point distant $b$ from the middle hinge, we know, in the first place, that the *total* reaction at A, Fig. 47, the end farthest from the weight, is $W\frac{c-b}{2c}$, and at B is $W\frac{c+b}{2c}$; and, in the second place, as there can be no shear in the cable, we see, from the equilibrium polygon of Fig. 48, and the lines 0–4 and 0–3, drawn in the stress diagram parallel to the tangents to the cable at the tops of the towers, that $5\text{–}4 : H = 2k : c$, or $5\text{–}4 = \frac{2k}{c}H$. By § 23, $H = \frac{c-b}{2k}W$; therefore the amount of vertical force combined with H of the cable is $W\frac{c-b}{c}$. Hence at A and at B the cable itself produces a reaction of $W\frac{c-b}{c}$, the balance of the reaction comes from the truss; the reaction of the truss at A will therefore be $-W\frac{c-b}{2c}$, and at B will be $W\left(\frac{c+b}{2c}-\frac{c-b}{c}\right) = W\frac{3b-c}{2c}$. This reaction also will be negative when $b$ is less than $\frac{1}{3}c$. Such is the case in Fig. 48, for the polygon A D B; and we have a corroboration in the negative bending moments near each end.

As the vertical force at A or B from the cable is the load on the half-span of the cable, and this load must be uniformly distributed horizontally to keep the cable in its curve, the intensity of vertical pull exerted between the cable and the rods per horizontal foot is found by dividing the above force by the half-span: hence it is $W\frac{c-b}{c^2}$. This will be the *upward* pull on



the girder per horizontal foot at all points and the cause of the bending moments. Of course at the point of application of W the resultant force acts downward. The action of a continuous load over a greater or less portion of the girder will follow the same law; and we shall have downward forces on the loaded portion of the girder equal to the difference between the imposed load and the pull of the vertical rods, and upward forces on the unloaded portion.

It is convenient to notice that the amount of W carried by either half of the cable is that portion which would be carried by the middle hinge if the half-girder alone supported W. As the girder reaction at the farther abutment is one-half of this amount, and the half-girder on the unloaded side is subjected to a uniform upward force, the shear on the middle hinge will also be one-half of this amount, or $W\frac{c-b}{2c}$. The shear diagram is given in Fig. 48. For any extent of load it will now be easy to find the amount carried by the cable; for we have only to calculate the portion which would come upon the middle hinge, were that a point of support of a simple truss of span $c$, and this portion will be the load on the half-cable.

164. **Comparison of Inverted Arch and Horizontal Girder.** — All statements in regard to the horizontal stiffening girder are equally true of the two parallel chains with bracing. While, in the bridge formed of cable and horizontal girder, the girder resists bending moments, and the chain takes up the direct stress, in the latter case the cables have to resist both moment and direct stress. But the maximum direct stress at any section, half of which is borne by each cable, occurs when the bridge is fully loaded: the maximum bending moment is found with a partial load, at which time the direct stress is less. Hence less material is theoretically required for the cables and truss of the type of Fig. 46 than for one like Fig. 47, — perhaps three-fourths as much. The introduction of the middle hinge in the axis of the rib of Fig. 46, with connections of sufficient strength to transmit the cable stresses, is attended with a little difficulty, which does not exist in the other case.

The three-hinged girder or rib may have the third hinge removed from the middle towards one end, as shown in Fig. 50, where one portion of the girder takes the form of a short link, extending to the first suspending rod. The same device may be introduced in an arch. The effect on the equilibrium polygon and the derived quantities may at once be seen.

165. **Horizontal Stiffening Girder hinged at Ends only.** — In case the middle hinge is omitted, the girder will be exposed to bending moments, as explained in Chap. III. Here, again, an inspection of Fig. 8 will show the extent of load required to produce maximum M of either kind; and an examination of the table of bending moments in the chapter referred to will show that an absolute maximum M occurs at one-fourth of the span from either abutment for a continuous load extending from one end to a point distant 0.43 of the span from the end nearer to the point of maximum M. Its amount is about $\frac{1}{7.5}$, or .133 of the maximum moment at the middle of an unassisted girder of the entire span. The stretching of the cables on both sides of the towers impairs the accuracy of these deductions. With a truss hinged at the middle, the sagging of the main cable, as well as the change of temperature, is of little consequence. From the value of $Y_1$, § 50, it is evident that $\frac{5}{32}(1 - n^2)(5 - n^2)$ W is carried by either half-chain, and this quantity divided by $c$ will give the intensity of upward pull on the truss from a load W at one point. The above amount is again that which would be carried to the point of contraflexure of the truss, if that were the point of support of the unassisted truss, and the truss were discontinuous over the support. (Compare Rankine's "Applied Mechanics," p. 375, *note.*)

If the ends of the girder are fixed in direction, we have the case of Chap. IV. Enough has been said to plainly indicate the treatment.

166. **Stiffening Girder of Varying Depth.** — Returning anew to the case of the stiffening girder with three hinges, it is evident, that if the girder has a variable depth, greatest at the points of maximum bending moment, the stresses in the flanges or chords will be diminished proportionally, with an economy of material. If, at the same time, the girder is itself the suspension cable, we can so adjust the depth, that the flange stresses for a partial load shall never exceed those arising from an entire load. Modifications having this end more or less in view have been suggested and carried out. Let us first draw, in Fig. 49, the equilibrium curve for a rolling load alone over half the span. While this curve will not give maximum bending moments, it will not differ greatly from the curves of maximum M, and it offers a very convenient and sufficiently accurate basis of comparison. Its form will be a straight line over the unloaded half of the span, and a parabola tangent to that line for the remaining portion. As the tangent at the abutment end of this parabola meets the tangent from the other end in the vertical through the centre of gravity of the load, the tangent A D is at once drawn. Draw the chord A C. The parabola cuts the middle vertical ordinate E D from the chord A C at its middle point F. If the height of the original parabola of the cable is $k$, the ordinate at one-fourth the span is $\frac{3}{4}k$. G D $= \frac{3}{2}k$; G E $= \frac{1}{2}k$; therefore E D $= k$; E F $= \frac{1}{2}k$; and F G $= k$. Hence the remaining ordinate for bending moment at one-fourth the span is $\frac{1}{4}k$ on either side, and of opposite signs.



167. **Ead's Arch, or Lenticular Stiffening Girder.** — If the two half-ribs of the arch of Fig. 51, or of the stiffened suspension-bridge, are each made of two equal parabolas, the outer ones being the continuous equilibrium curve for a complete load, the vertical depth of the semi-girders at their middle sections E and F will be one-half the rise or height, $k$. Let us denote the horizontal thrust or tension from steady load $w$ by H; that from a full rolling load $w'$, by H′. The horizontal stress due to a rolling load extending from one abutment over half the span will be $\frac{1}{2}$ H′; for a similar load over the other half-span must give an equal stress, and both combined must equal H′. When the above bridge is fully covered with mov-

ing load. the equilibrium curve will coincide with the continuous curve, and the stress at each section of the main cable will be that due to $H + H'$. The auxiliary ribs and bracing will experience no stress. When the bridge is half loaded, say from C to B, the equilibrium polygon for rolling load will be the one sketched in our figure; it passes at I, $\frac{1}{4} k$ below the main cable at D, and through the middle or axis of the truss A C. The horizontal component of the stress at D, due to $\frac{1}{2} H'$ at I, is, from the equation of moments about E, $\frac{3}{4} H'$; that is, $\frac{1}{2} H' . \frac{3}{4} k =$ hor. comp. at D $\times \frac{1}{2} k$. Taking moments about D, $\frac{1}{2} H' . \frac{1}{4} k = -$ hor. comp. at E $\times \frac{1}{2} k$; or horizontal component at E is $- \frac{1}{4} H'$. At F and G the horizontal component is, in each member, $\frac{1}{4} H'$. The minus-sign denotes opposite stress, here compression; in the arch, tension. We may therefore write the following table of cases:

| Horizontal component of stress at . | E | D | F | G. |
|---|---|---|---|---|
| With steady load only . . . . . . | 0 | H | 0 | H, |
| " " and one-half rolling load | $-\frac{1}{4}H'$ | $H+\frac{3}{4}H'$ | $+\frac{1}{4}H'$ | $H+\frac{1}{4}H'$, |
| " " " complete " " | 0 | $H+H'$ | 0 | $H+H'$. |

Since F and G change places with E and D for a load on the other half-span, we see that the lower member, or main cable, experiences a horizontal component which fluctuates from H to $H + H'$, always tension; while the auxiliary rib has a stress whose horizontal component ranges between $\frac{1}{4} H'$, tension, and $\frac{1}{4} H'$, compression. The bracing will undergo no stress from a full load. The stress in the bracing for partial loads may be worked out by the method of the previous chapters for finding the amount of shear remaining after subtracting the vertical components for the two cables at a section, by the method of Part II., "Bridges," Chap. V., or by drawing stress diagrams as given in Part I., "Roofs."

As the parabola through I is a *projection* of that through D, the above deductions for the points D and E are true for the other points of the girder. Although, as pointed out in § 162, the bending moments are a little greater for loads which cover not quite half the span, it is evident that the horizontal component of the stress in the main cable can never exceed $H + H'$, and in the counter-rib will but slightly exceed $\pm \frac{1}{4} H'$. This form of arch was designed and patented by James B. Eads: a paper upon it by him may be found in the "Transactions of the American Society of Civil Engineers," vol. iii., No. 6, October, 1874.



168. **Bowstring Stiffening Girder.** — If the auxiliary members connecting the hinges A, C, and B, Fig. 52, are straight, we have a variation in the method of stiffening and a change in the stresses. The equilibrium curve A F C I B, for a rolling load over one-half the span, is also drawn here, coinciding with A C, and passing through I, $\frac{1}{4} k$ below D. The steady load will be entirely carried by the main cable as before, as will also a complete rolling load. The half rolling load, being entirely supported on the left by A F C, will cause in that member a tension whose horizontal component is $\frac{1}{2} H'$; a horizontal tension in D, of H', and a horizontal compression in E, of $\frac{1}{2} H'$, as is found by similar equations of moments to those in the last section. There results, then, for this type the following cases: —

| Horizontal component of stress at . . | E | D | F | G, |
|---|---|---|---|---|
| With steady load only . . . . . . . | 0 | H | 0 | H, |
| " " " and one-half rolling load | $-\frac{1}{2}H'$ | $H+H'$ | $+\frac{1}{2}H'$ | H, |
| " " " " complete " " | 0 | $H+H'$ | 0 | $H+H'$. |

The stress on the main cables will be very slightly increased for some partial loads, as shown before. The increase will, however, be small, for the direct stress is decreased at the time the bending moment is increased; so that the absolute maximum may be called $H + H'$ without any error of importance. The stress in the straight stiffening rib ranges from a tension of $\frac{1}{2} H'$ to a compression of $\frac{1}{2} H'$. While the member A C or C B has to resist double the force of the preceding case, and that force also completely reversed for a moving load over one-half of the bridge, the unbraced lengths are shorter than in Fig. 51, the construction of a straight member is simpler, and the web members are only one-half as long: the cost may therefore be sufficiently influenced to cause this design to commend itself more to the practical builder than does the former. A notable example of this type is the Point Bridge at Pittsburgh, Penn., eight hundred feet span, built by the American Bridge Company of Chicago, in 1876.

169. **Fidler's Stiffened Suspension-Bridge.** — Again, let us conceive of two cables, A F C D B and B E C G A, Fig. 53, each separately subject to, and in equilibrium under, a rolling load over one-half the span, and then let their places be taken by the two girders shown. A C and C B will be straight, as in the last figure; A G C and C D B will be parabolas, each tangent at C to the chord of the other; and the equilibrium curve for a complete load will pass through the middle of each truss, as shown by the dotted line. These trusses are, therefore, of the form of Fig. 52; but they have a depth equal to that of the trusses of Fig. 51. The horizontal component H, of steady load, and H′, of complete rolling load, will be carried equally by both members of each truss, $\frac{1}{2}$ H and $\frac{1}{2}$ H′ on each. A rolling load on the right half of the span will cause a horizontal tension of $\frac{1}{2}$ H′ at D and at F. We may, then, write, for this type,

| Horizontal component of stress at | E | D | F | G. |
|---|---|---|---|---|
| With steady load only . . . . | $\frac{1}{2}$H | $\frac{1}{2}$H | $\frac{1}{2}$H | $\frac{1}{2}$H, |
| " " " and one-half rolling load . . . . . . . . | $\frac{1}{2}$H | $\frac{1}{2}$H+$\frac{1}{2}$H′ | $\frac{1}{2}$H+$\frac{1}{2}$H′ | $\frac{1}{2}$H, |
| with steady load and complete rolling load . . . . . . . . | $\frac{1}{2}$H+$\frac{1}{2}$H′ | " | " | $\frac{1}{2}$H+$\frac{1}{2}$H′. |

The stresses will, therefore, always be tension, and the horizontal component will vary in each member from $\frac{1}{2}$ H to $\frac{1}{2}$ (H + H′), a most favorable showing for the structure. The



remark of § 162 in regard to maximum bending moments applies here also. The maximum stresses in the bracing can be worked up in the way thought most convenient. This type may also be analyzed as two inverted bowstring girders, a weight on one causing simply a tension in the tie of the other and an inclined reaction in its line at the middle hinge. Hence the investigation of the bowstring girder in Part II. may be applied here. A very interesting analytical discussion of the types of bridges and arches of this chapter may be found in "Engineering," vol. xx. for 1875, from the pen of Mr. T. Claxton Fidler, the inventor and patentee of the type discussed in this section.

170. **Ordish's Suspension-Bridge.**—Another stiffened suspension-bridge, in which the problem of resisting distortion from a partial load is solved in quite a different way, is what is known as Ordish's, shown in Fig. 55. The Albert Bridge over the Thames, at Chelsea, Eng., is of this type; and one of moderate span has been erected over the Pennsylvania Railroad, at 40th Street, Philadelphia. Here a certain initial stiffness is given to the platform itself, and it is then directly supported at several points from the tops of the towers. It is intended that the weight shall be entirely carried by the inclined ties. As these ties, from their length, would sag considerably under their own weight, a passing load would cause the roadway to move vertically; for an increased pull on a tie would tend to straighten it. They are, therefore, suspended, at the joints in the several bars which make up the ties, from a light cable, which is designed simply to carry the weight of the ties; and the suspending rods are so adjusted, that the ties shall be straight. No movement of the roadway of any importance can then take place. The analysis is very simple.

171. **Erect and Inverted Arch combined.**—The bridge over the Elbe, at Hamburg, one span of which is shown in Fig. 54, is a combination of the erect and inverted arch. This construction dispenses with abutments to withstand a thrust, as the thrust of the upper rib will at all times be balanced by the

tension of the lower rib. If the ribs are of equal stiffness, any load may be considered as divided equally between the two systems: if the ribs, while having the same curvature, are not alike in cross-section, the load will probably be distributed in the ratio of their moments of inertia. As the erect arch always tends to move away from its equilibrium curve, and the inverted arch to approach the equilibrium curve, the tangents at the abutment ends will move in the same direction, and therefore the structure should be treated as hinged at the ends, unless each flange is firmly bolted to the skew-back. If the structure is carried on columns or a pier, it appears to us that the ends cannot be rigid, and we judge that the two ribs will begin to turn about the middle of the depth without the introduction of a pivot or hinge.

The effect of temperature is annulled. Also the shortening of the erect arch under the direct compression being opposite to the extension of the inverted arch under the direct tension, the span will tend to remain unaltered; but the ribs themselves will be changed in form, one rib flattening as the other becomes more convex. If, in making such a design, the section of the arch is found to differ much from the section of the inverted rib, it will be well to calculate the relative deflections of the two ribs at the middle. The amount of load each will carry varies inversely as the deflection under equal loads, since they must deflect equally; and hence, if the arch is first designed of such shape, for the purpose of resisting compression, that it is stiffer or has less deflection than the chain, when each has one-half the load, the cross-section of the arch must be increased, and that of the chain may be diminished. This type of structure must not be confounded with a lenticular girder: the absence of bracing between the ribs makes them independent.

# CHAPTER XI.

## BENDING MOMENTS FROM CHANGE OF FORM.[1]

172. **Displacement from Bending Moments.**—It follows, from the fact that the arched rib moves away from the equilibrium polygon or curve, that the bending moments and chord stresses will have a slight tendency to increase. When the rib changes in shape, however, the equilibrium polygon must also move enough to still satisfy for the new form the equations of condition by which it was first established, and this movement will in some measure counteract the former. Besides, the equilibrium curve for steady load generally runs so close to the axis of the rib, that the change of shape from bending moments is very slight; and, even when the influence of rolling load is added, the increments of the bending moment ordinates are too small to be of material consequence.

The vertical displacement at any point E, Fig. 56, produced by any load, will be found, for the parabolic rib, by taking *area moments*, as explained in Part II., "Bridges," Chap. VI., or for the circular rib by summing the ordinates as usual along the rib. As was done in the treatment of beams, it will here be necessary to find the point D where the tangent to the rib in its new form is horizontal, which point will not be at the crown,

[1] Many of the deductions in this chapter are only intended as guides in practical construction, to indicate where, and to show approximately how much, additional stress may be anticipated from change of form. Exact results are not attempted.