190. **Diagrams and Table of Stresses for Equal Cross-sections.** — We may now draw a diagram for a single load W on any one joint, plotting the reactions, just obtained, and proceeding by the method of Part I., "Roofs," Fig. 21. Six diagrams, four of which are drawn, the scale being too small to make the other two clear, Fig. 64, will give all the stresses, as, by symmetry, loads on the right will cause stresses in pieces marked with unaccented letters equal to those now found in pieces marked with accents. The stresses are scaled in tons, tabulated, and marked with their proper signs, in the following table. They might be calculated by (1.), § 184, if preferred, and their sum might be checked by a diagram for complete load. The sums of the respective compressions and tensions are written below, and in the next line are found the differences of these quantities, or the stresses from steady load, marked S. L. Upon adding to these latter the tensions or compressions first referred to, we obtain the maximum stresses in the pieces for a moving load of the same intensity.

It will be seen that the horizontal member is always compressed; the curved rib may have at times a little tension in its middle portion, but the larger part of it is always compressed; the web members are struts and ties alternately, until we reach J K; the pieces from there to the middle may be exposed to a reversal of stress.

191. **Sections proportioned to Stresses.** — Guided by these stresses, we will now assume sections of the different pieces, which shall vary approximately as do the stresses just found. Of the web members, those under compression are intended to be proportionately heavier than those in tension, as they will not safely resist so large a unit stress. The assumed ratio of the sections is marked on the figure. The quantities $\frac{y^2}{p^2} \cdot \frac{l}{A}$ and $\frac{x\,y}{p^2} \cdot \frac{l}{A}$ are now found anew by simply dividing the previous similar quantities by the section ratios just referred to. The results follow on p. 184. $\Sigma \frac{y^2}{p^2} \cdot \frac{l}{A}$ is now 161.18.

STRESSES IN PIECES, ALL CROSS-SECTIONS EQUAL.

Load on	BO.	DO.	FO.	IO.	KO.	NO	AL.	CL.	EL.	GL.	JL.	ML.
1st.	+0.30	+0.30	+0.28	+0.24	+0.17	+0.09	−0.13	−0.13	−0.10	−0.07	−0.01	+0.07
2d.	+0.49	+0.89	+0.85	+0.74	+0.54	+0.29	+0.30	−0.39	−0.35	−0.25	−0.08	+0.14
3d.	+0.34	+0.96	+1.43	+1.26	+0.95	+0.53	+0.70	+0.17	−0.69	−0.57	−0.31	+0.06
4th.	+0.21	+0.62	+1.36	+1.85	+1.43	+0.86	+1.07	+0.72	+0.11	−0.89	−0.56	−0.04
5th.	+0.09	+0.33	+0.82	+1.68	+2.06	+1.34	+1.37	+1.15	+0.73	+0.02	−1.07	−0.44
6th.	0.00	+0.10	+0.38	+0.97	+1.91	+2.05	+1.53	+1.43	+1.20	+0.73	−0.07	−1.16
7th.	−0.06	−0.06	+0.07	+0.44	+1.13	+2.05	+1.54	+1.52	+1.40	+1.12	+0.56	−0.26
8th.	−0.09	−0.14	−0.10	+0.12	+0.61	+1.34	+1.45	+1.47	+1.45	+1.30	+0.93	+0.32
9th.	−0.09	−0.16	−0.16	−0.03	+0.31	+0.86	+1.22	+1.27	+1.29	+1.21	+0.98	+0.52
10th.	−0.07	−0.13	−0.16	−0.08	+0.14	+0.53	+0.90	+0.95	+0.97	+0.93	+0.78	+0.46
11th	−0.05	−0.09	−0.10	−0.07	+0.06	+0.29	+0.55	+0.60	+0.62	+0.62	+0.53	+0.37
12th.	−0.02	−0.03	−0.04	−0.03	0.00	+0.09	+0.19	+0.20	+0.22	+0.21	+0.19	+0.15
Σ +	1.43	3.20	5.19	7.30	9.31	10.32	10.82	9.48	7.99	6.14	3.97	2.09
Σ −	0.38	0.61	0.56	0.21	0.00	0.00	0.13	0.52	1.14	1.78	2.10	1.90
S. L.	+1.05	+2.59	+4.63	+7.09	+9.31	+10.32	+10.69	+8.96	+6.85	+4.36	+1.87	+0.19
Max.	+2.48	+5.79	+9.82	+14.39	+18.62	+20.64	+21.51	+18.44	+14.84	+10.50	+5.84	+2.28
											−0.23	−1.71

Load on	OA.	AB.	BC.	CD.	DE.	EF.	FG.	GI.	IJ.	JK.	KM.	MN.
1st.	+1.05	+0.01	−0.01	+0.01	−0.03	+0.03	−0.04	+0.04	−0.06	+0.06	−0.06	+0.06
2d.	+0.80	−0.82	+1.07	+0.02	−0.07	+0.07	−0.09	+0.09	−0.16	+0.16	−0.18	+0.19
3d.	+0.58	−0.58	+0.81	−0.82	+1.07	+0.07	−0.14	+0.15	−0.25	+0.25	−0.29	+0.30
4th.	+0.33	−0.34	+0.53	−0.54	+0.77	−0.79	+1.04	+0.17	−0.32	+0.33	−0.42	+0.42
5th.	+0.16	−0.16	+0.32	−0.33	+0.53	−0.54	+0.76	−0.78	+0.95	+0.39	−0.52	+0.52
6th.	0.00	0.00	+0.14	−0.15	+0.31	−0.32	+0.51	−0.52	+0.72	−0.74	+0.84	+0.60
7th.	−0.06	+0.06	+0.03	−0.03	+0.15	−0.16	+0.33	−0.34	+0.52	−0.55	+0.65	−0.66
8th.	−0.13	+0.13	−0.04	+0.04	+0.04	−0.04	+0.18	−0.19	+0.37	−0.38	+0.51	−0.53
9th.	−0.13	+0.14	−0.07	+0.07	−0.02	+0.02	+0.11	−0.12	+0.25	−0.26	+0.38	−0.39
10th.	−0.10	+0.10	−0.07	+0.07	−0.02	+0.02	+0.07	−0.07	+0.17	−0.18	+0.27	−0.28
11th.	−0.07	+0.08	−0.05	+0.05	−0.03	+0.03	+0.02	−0.02	+0.09	−0.09	+0.15	−0.15
12th.	−0.02	+0.02	−0.01	+0.01	0.00	0.00	+0.01	−0.01	+0.02	−0.02	+0.05	−0.05
Σ +	2.92	0.54	2.90	0.27	2.87	0.24	3.03	0.45	3.09	1.19	2.85	2.09
Σ −	0.51	1.90	0.25	1.87	0.17	1.85	0.27	2.05	0.79	2.22	1.47	2.06
S. L.	+2.41	−1.36	+2.65	−1.60	+2.70	−1.61	+2.76	−1.60	+2.30	−1.03	+1.38	0.00
Max.	+5.33		+5.55		+5.57		+5.79		+5.39	+0.16	+4.23	+2.09
		−3.26		−3.47		−3.46		−3.65		−3.25	−0.09	−2.06

VALUES OF $\frac{y^2}{p^2} \cdot \frac{l}{A}$.

B O	0.296	A L	0.069	O A	0.582	A B	0.948	
D O	0.680	C L	0.128	B C	0.544	C D	0.642	15.971
F O	1.248	E L	0.248	D E	0.483	E F	0.577	134.527
I O	1.877	G L	0.535	F G	0.377	G I	0.461	4.544
K O	2.513	J L	1.236	I J	0.243	J K	0.356	6.138
N O	2.743	M L	4.338	K M	0.043	M N	0.085	161.180
	9.357		6.554		2.272		3.069	24
	6.614		2		2		2	3868.320
	15.971		13.108 × 10.263		4.544		6.138	

VALUES OF $\frac{xy}{p^2} \cdot \frac{l}{A}$.

		Σ			Σ			Σ			Σ
B O	− 0.78	0.78	A L	− 0.21	0.21	O A	−1.54	1.54	A B	−2.51	2.51
D O	− 1.99	2.77	C L	− 1.16	1.37	B C	−1.72	3.26	C D	−2.38	4.89
F O	− 4.09	6.86	E L	− 3.74	5.11	D E	−1.81	5.07	E F	−2.16	7.05
I O	− 6.96	13.82	G L	− 11.31	16.42	F G	−1.73	6.80	G I	−2.11	9.16
K O	−10.72	24.54	J L	− 33.59	50.01	I J	−1.48	8.28	J K	−2.17	11.33
N O	−13.71	38.25	M L	−144.04	194.05	K M	−0.55	8.83	M N	−1.05	12.38
K′O	−15.14	53.39	M′L	−170.22	364.27	N M′	+0.48	8.35	M′K′	+0.02	12.36
I′ O	−14.18	67.57	J′ L	− 55.98	420.25	K′J′	−0.39	8.74	J′ I′	−0.17	12.53
F′O	−12.60	80.17	G′ L	− 27.48	447.73	I′ G′	−1.42	10.16	G′F′	−0.76	13.29
D′O	−10.33	90.50	E′ L	− 14.20	461.93	F′E′	−2.40	12.56	E′D′	−1.27	14.56
B′O	− 8.99	99.49	C′ L	− 8.13	470.06	D′C′	−3.21	15.77	C′B′	−1.68	16.24
			A′ L	− 4.78	474.84	B′A′	−3.58	19.35	A′O′ fixed.		

The above *summations* are negative.

Next follow, as before, the computations of H (p. 185).

It will be seen that the change in the sections of the pieces has made but little change in the values of H; the thrust now being 10.820 W for a steady load of W on each joint. We may therefore proceed to draw anew the diagrams for a single load W on any one joint, or we may, by the use of lines of another color, alter the figures already drawn. As H has been changed so little, the new stresses will determine the final

VALUES OF H.

W on 1st Joint.		W on 2d Joint.		W on 3d Joint.	
0.00	99.49	0.78	90.50	2.77	80.17
0.00	474.84	0.21	470.06	1.37	461.93
1.54	19.35	3.26	15.77	5.07	12.56
1.54	16.24	2.51	14.56	4.89	13.29
23	609.92	6.76	590.89	14.10	567.95
35.42		21	3	19	5
609.92		141.96	1772.67	267.90	2839.75
645.34÷3868=.167 W.		1772.67		2839.75	
		1914.63÷3868=.495 W.		3107.65÷3868=.803 W.	

W on 4th Joint,		W on 5th Joint.		W on 6th Joint.	
6.86	67.57	13.82	53.39	24.54	38.25
5.11	447.73	16.42	420.25	50.01	364.27
6.80	10.16	8.28	8.74	8.83	8.35
7.05	12.53	9.16	12.36	11.33	12.38
25.82	537.99	47.68	494.74	94.71	423.25
17	7	15	9	13	11
438.94	3765.93	715.20	4452.66	1231.23	4655.75
3765.93		4452.66		4655.75	
4204.87÷3868=1.087W.		5167.86÷3868=1.336 W.		5886.98÷3868=1.522 W.	

dimensions of the pieces. A sample of the stresses obtained in the upper chord is given below for comparison.

	B O.	D O.	F O.	I O.	K O.	N O.
Σ +	1.45	3.18	5.10	7.08	9.23	10.20
Σ −	0.42	0.63	0.51	0.07	0.00	0.00
S. L.	1.03	2.55	4.59	7.01	9.23	10.20
Max.	+2.48	+5.73	+9.69	+14.09	+18.46	+20.40

A certain allowance in section may be made for the stresses from change of temperature, or the effect of the change of length in each piece may be worked out separately.

192. **Bracing with Vertical Struts.** — The bracing of the arch just described is of the Warren or triangular type. The design of Fig. 65 has been used with success, is probably more economical of material, and is, in our judgment, more pleasing to the eye. The inclined braces are ties, and the introduction of the counters at the crown obviates the reversal of stress in the braces. When the upper member approaches the curved member closely at the crown, the web may be made of a plate for a distance of two panels: sometimes the two members are brought into contact at the crown.

193. **Cast-Iron Arch as a Breast-Summer.** — Builders sometimes employ a cast-iron member, shaped like Fig. 66, for spanning openings of considerable size, and carrying the weight of a brick wall. Aside from the fact that cast-iron in large masses is of very uncertain strength, by reason of internal stresses produced by contraction in cooling, an additional element of uncertainty is introduced by the method of constructing these ribs. The thrust of the arch is resisted by a wrought-iron rod, represented by a straight line in the figure, which, in place of being fastened by bolts or nuts, is fitted into recesses in the casting at its ends. In order to have the rod tight, it is made shorter than the distance between bearings, then heated, and shrunk into place. The rod is therefore under an initial tension, and the rib under initial compression, both of which are likely to be of uncertain amount, and detrimental; for, when the arch is loaded, its horizontal thrust will be added to the tension in the bar, and the compression of the rib will be increased. As, however, the bar elongates under the pull, it would be well, were it possible, to have the bar so much shorter than the normal span of the arch, that the value of H proper to the arch under the proposed load should elongate the rod to that normal span; then the initial bending moments produced in the rib by shrinking on the rod will be removed. It would seem possible, by a careful measurement of the extension of the rod between two marks some ten or twenty feet apart, especially if the stretch has been previously tested, to determine the initial tension.

If the arch is well built into the masonry at the ends, and if the bearings are long, the rib may be considered as fixed at the ends. If not so built, and in preliminary testing on two supports under an applied weight, the rib must be considered as pivoted at the ends. From the small rise, such arches may be assumed, in either case, to be parabolic. In testing, therefore, under a single weight W applied at the middle, by § 40 $H = \frac{25}{64} \frac{c}{k} W$. At that time temporary bearings ought to be placed at A to prevent the arch from bearing at C when loaded. Under the load of the wall, unless the latter is cut by large openings, so that a pier concentrates the weight on a small portion of the rib, there will be no bending moments, as the load is uniformly distributed.

194. **Gothic Rib for Roofs.** — The rib which supports the roof of the Grand Central Depot in New-York City is probably circular, and will be analyzed readily by the principles already laid down; but the Gothic rib requires some special treatment. Fig. 67 is a sketch of the rib which sustains the roof over the train-house of the Boston and Providence Railroad Depot in Boston, Mass. The span is 125 feet between walls, and the height is 55 feet to the axis of the rib. As height impresses one more than horizontal distance, it is evident that this roof appears lofty when viewed from the inside. In order to give height quickly near the walls, the half-rib is struck with two radii, as indicated in the figure. The lower portion is built with a solid web; while most of the upper portion has a uniform depth of three feet. If the junction at the crown or apex of the roof allows any movement, if the ribs can rock or turn on castings at their bases, and if they are independent of the side walls, they may be treated as hinged at three points, and discussed like any three-hinged arch. If there is no opportunity for movement at the bases, and especially if the ribs abut closely against the side walls and buttresses, while still a joint is provided at the crown, the condition of invariability of span must be applied, and also the condition that the deflection of

the crown when measured by area moments from the tangent at one abutment shall equal the deflection of the crown from the tangent at the other abutment. The integration will then be between limits which will appear from the discussion of the third supposition.

The rib may be fixed at the ends and crown, and will then offer a troublesome case for treatment by reason of the great depth at the haunches, unless we assume that it is well buttressed by the wall. In this case, the portion below the top of the wall and the wall itself will act as an abutment; and, as it will only require a moderate tension in the inside flange at the springing to resist the overturning moment, such an assumption seems entirely warrantable. Above the wall, then, some 25 feet high, where the horizontal mark is made on the left-hand side, we assume the springing line of the arch, and consider the remainder as a rib fixed at the ends, and continuous at the crown. In applying the conditions for a rib with fixed ends to this case, we must change the derived equations, as the curve is not continuous at the crown. A parabola drawn through the middle of the depth of the rib at crown, springing, and a third point near the upper end of the straight portion of the rafter, will agree very closely with the axis of the rib throughout. We must first determine k and c for this parabola. In Fig. 68 let h be the height or rise of the arch at the apex, a the horizontal distance from h to the point where the parabola would become horizontal; then

$$h = \frac{k}{c^2}(c^2 - a^2); \quad \text{or} \quad k = h\frac{c^2}{c^2 - a^2}.$$

For another ordinate h', distant $c - a'$ from the springing, we write

$$k = h'\frac{c^2}{c^2 - a'^2}.$$

In this case $c - a = 55.75$ feet, $h = 30.3$ feet, $c - a' = 22.5$ feet, and $h' = 17$ feet: hence we find that $k = 31.68$ feet, $c = 70.48$ feet, and $a = 14.73$ feet.

In place of performing the integrations of §§ 58–59 between the limits there given, we must omit or subtract from the equations the integrals between the limits $+a$ and $-a$, as this portion is cut out of the parabola. Thus the equation (1.) of § 58 will be written

$$\int_0^{2c} \mathrm{D\,E}^2 - \int_{c-a}^{c+a} \mathrm{D\,E}^2 = \int_0^{c+b} \mathrm{D\,F\,.\,D\,E} - \int_{c-a}^{c+a} \mathrm{D\,F\,.\,D\,E} + \int_0^{c-b} \mathrm{D\,F\,.\,D\,E}.$$

As limits $c + a$ and $c - a$ will yield terms similar to limits $c + b$ and $c - b$, the subtractive quantities above can be written from inspection of (2.), § 58, and (2.), § 39. A similar treatment of the other equations of condition will be required. The solution will then proceed as usual.

If the weight at the apex of the roof, arising from the ventilator, &c., is sufficiently great, it will take the place of the omitted portion of breadth $2a$, so that the rib will be very nearly in equilibrium under steady load.

195. **Remarks on Designing.** — The examples which have been given in the preceding pages will indicate the steps to be pursued in working out a specific design. The type of structure having been determined upon, the moving load must be taken of an intensity in harmony with the position of the bridge, or we must decide upon the weight of snow and pressure of wind to which the roof will be liable. The dead weight of the structure must then be assumed, of such an amount as our judgment and experience dictate, to be afterwards verified and corrected from the actual sections. The abutment reactions and bending moments from the applied forces will then be found, after which, stress diagrams may be constructed, or equilibrium polygons drawn; from the first we obtain stresses directly, as in Part I.; from the second, bending moments, with shears and direct thrusts, from which the stresses in the several pieces will be found, as in Part II. The first method is probably the shorter for roofs, unless the rib is solid, or has a plate web, as all of the load of one kind may be included at one operation: the second method will be preferred where a moving load has to be

considered. The stresses will then be tabulated, and the maximum compression and tension on each piece found.

A point which may call for a little explanation is illustrated by Fig. 69. We desire to draw a stress diagram for an arched rib, which is fixed at the end A B, the equilibrium curve beginning with the line G D, and the bending moment at A B being T . *p*, or its equivalent. The flanges at A and B will transmit direct force only: therefore decompose T into C, the compression parallel to the flanges, at the springing, and F, the shear at right angles. Then, by moments about A, Thrust at B . A B = C . A G, or Thrust at $B = \frac{C \cdot AG}{AB}$; by moments about B, Tension at $A = \frac{C \cdot BG}{AB}$. The shear F will be resisted either at A or B, depending upon which of the braces is designed to carry it: if the braces are ties, it must pass through the one at A. Thus we obtain the forces with which to begin the stress diagram. In case of a hinge at the abutment, the point G is found midway between A and B, and there will be ½ C, compression, at each flange. F will be found in the proper brace as above.

The arched rib must be thoroughly stayed laterally; for so much of either flange as is compressed is in unstable equilibrium; between lateral stays, the breadth of a compressed flange must be determined from the formulæ for columns. For a few formulæ and directions for detailing, see the closing chapter of Part I.