risk, worth, and cost into a single, scalar-valued function for comparing alternative projects in a rational, objective manner.

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The Modeling Process

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Abstract—**Considerable interest currently exists in the application of the systems approach to the solution of societal, political, and environmental problems. The essence of this systems approach is modeling, the capability to describe large-scale complicated interactive systems by symbolic representations so that inferences regarding the effects of alternative system configurations can be easily and rapidly structured. The modeling process is itself becoming better understood as a direct extension of the scientific method. Furthermore, the applicability of statistical methodology to the design and analysis of experiments with computerized symbolic models is leading to wider acceptance of these representations as tools of considerably credible scientific stature. This paper presents a taxonomy of 24 model categories and, in a discussion of the scientific method and the modeling process, indicates the evaluations pertinent to the selection of a modeling medium appropriate to particular systems studies. The dynamic stochastic simulation model is shown to be the most general category of symbolic models which are amenable to facile organized experimentation. The application of such models to the understanding and solution of societal, political, psychological, medical, judicial, environmental, social, economic, and biological problems is indicated and is considered imminently practicable.**

INTRODUCTION

A SYSTEM may be defined as a collection of interdependent elements which act together in a collective effort to achieve some goal. The elements of systems are frequently referred to as entities, the fundamental components which interact with one another, positively or negatively, as the system seeks its goal.

For most systems the goal is self-evident or, at least, can be described precisely. The maximization of profit, the maximization of productivity, and the optimization of system performance are typical goals of managerial, productive, industrial, and military systems. However, for many systems goals are not so evident, such as is the case for biological evolution or ontological development.

Nonetheless, whether a system is organized and controlled or is self-adaptive and self-regulating, the *system scientist* (or operations research specialist, or system engineer, or management scientist, or operations analyst, as he is variously denominated) takes as axiomatic the assumptions that system goals can be defined and that systems are atomistic, capable of being dissociated into their component entities in such a way that their interactive behavior mechanisms can be described.

A TAXONOMY OF MODELS

In order to describe the phenomena internal to a system the systems scientist prepares a *model* of the system. The search for mathematical laws has been commonplace among scientists literally for centuries, and the use of replicas by architects and engineers has been equally well established. However, only relatively recently has an awareness of their common purpose been made manifest.

Rosenblueth and Wiener [27] were apparently among the first to note that both the scaled replica and the mathematical law are examples of models. Their initial categorization of models prescribed two types, each viewed as an aid to scientific enquiry: 1) *material models*—transformations of original physical objects, the representation of a complex system by another physical system assumed to be simpler than, yet similar to, the original system; and 2) *formal models*—symbolic assertions in logical terms of an idealized, relatively simple situation, the assertions representing the structural properties of the original factual system. Though no explicit statements of preferability regarding these two model categories were forthcoming, it seems apparent that, at the time, greater credibility and a more intricate representation would be associated with a material model if one were feasible in a particular instance.

Two other categorization schemes are also available in classifying models. First, a model is said to be *dynamic* or

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static depending upon whether its features or symbols do or do not, respectively, alter perceptibly with time. This classification is a cross-categorization scheme with respect to the Rosenblueth–Wiener taxonomy, so that one may have both dynamic and static material models as well as dynamic and static formal models. To wit, a statue of Benjamin Franklin is a quite static material model, as is a photograph and a weather map; whereas, a puppet show, a critical dosage test, a mobile orrery, and a planetarium show are all dynamic material models of physical phenomena. Exemplary static formal models include Newton's inverse square law and the expression for the equilibrium queue length in a nonpreemptive $M/M/1$ queue.[1] Typical dynamic formal models are Lanchester's laws[2] and the autoregressive time series.

One should note in passing that particular problem formulations, or classes of formulations, do not necessarily fall distinctly into the same cross-category; e.g., a linear programming model, though definitely of the formal model variety, may be either dynamic or static depending upon whether or not the optimization is conducted over time explicitly.

A second cross-classification scheme for models is concerned with the predictability of the model's final state. A model is said to be *stochastic* if it contains intrinsic probabilistic or random elements which affect the outcome or response of the model; otherwise, the model is *deterministic*. The cross-classification of these model types with the four already mentioned implies a total of eight primary model categories. Examples of each include those which have been mentioned: the statue, road maps, or scale model (material–static–deterministic); the weather map or biological critical dosage test (material–static–stochastic); the model train set or planetarium shows (material–dynamic–deterministic); the puppet show or the genetic experiment with *Drosophila* (material–dynamic–stochastic); Ohm's law or Newton's inverse square law (formal–static–deterministic); the equilibrium queue length (formal–static–stochastic); Lanchester's laws (formal–dynamic–deterministic); and the autoregressive time series (formal–dynamic–stochastic).

EVOLUTION OF MODEL TYPES

At approximately the same time as the appearance of the initial categorization of models by Rosenblueth and Wiener, two important scientific fields were developing: computer science and a systems (or operational research) methodology. The computer was to have profound effect on models of both the material and formal varieties (as we shall see), and the development of the systems methodology has required an increasing need for greater detail and more realistic representations in models.

The systems methodology has relied more and more on the electronic computer. For example, the analog electronic computer, made practicable by the development of a sufficiently accurate electronic operational amplifier, permitted one to provide a physical representation for specific formalized expressions; thus a set of interrelated time-dependent differential equations (such as Lanchester's laws) could be "solved" by tracing appropriate electrical properties of an *ad hoc* electrical network as time evolved. (See Ragazzini *et al.* [26].)

That such analogue[3] models were possible was realized even as early as a century ago soon after pairs of formal models, each originally derived for a specific physical phenomenon, were noted to be of the same form; e.g., the applicability of an inverse square law in both gravitational and electrical field theories implied that one might well be capable of miming a gravitational phenomenon by an electrical device.

In their ground-breaking volume on operations research, Churchman *et al.* [6] defined models as "representations of the system under study, a representation which lends itself to use in predicting the effect on the system's effectiveness of possible changes in the system" and categorized models into three types: 1) *iconic*—those which pictorially or visually represent certain aspects of a system; 2) *analogue*—those which employ one set of properties to represent some other set of properties which the system being studied possesses; and 3) *symbolic*—those which require mathematical or logical operations and which can be used to formulate a solution to the problem at hand. From the discussion surrounding this classification, it seems apparent that the iconic models are material models in the Rosenblueth–Wiener sense and that the class of symbolic models was to be the equivalent of the formal class of models (*à la* Rosenblueth–Wiener). The analogue model was to have been a class "between" the other categories, which were themselves to be distinguished primarily from the view that iconic models were intended to be *descriptive*, yet symbolic models *explanatory*, of the phenomenon or system under study.

Subsequently, Sayre and Crosson [28], searching for a representational mode for an artificially intelligent device, categorized models as 1) *replications*—those which display a significant degree of physical similarity in all three dimensions between the model and the modeled; 2) *formalizations*—symbolic models in which none of the physical characteristics of the modeled are reproduced in the model itself *and* in which the symbols are manipulated by a well-formed discipline such as mathematical logic; and 3) *simulations*—the class of symbolic models, all of whose symbols are *not* manipulated by a well-formed discipline (such as mathematics or mathematical logic) in order to arrive at a particular numerical value or at an analytic solution or expression.

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[1] The shorthand refers to Kendall's classification of queues [14]. An $M/M/1$ queue is a single-server queue with Markovian arrival pattern and exponentially distributed service times.

[2] Lanchester's laws describe the dynamics of the battlefield, one of the earliest recognized contributions to the systems or operations research approach. (See Newman [23, pp. 2138–2159].)

[3] The French orthography is employed so as to distinguish between this type of physical model (*analogue*) and the *analog* type of computer, by which many analogue models are implemented.

There are several significant points regarding the Sayre–Crosson categorization scheme. First, no provision is made for the inclusion of the analogue model, though the fact that the many analog computer implementations were essentially "substitutions" for other physical phenomena (which happen to be described by the same formal model) could have led to the dismissal of this model type. Second, the class of replications specifically forbade dimensional reductions, thereby excluding photographs, maps, cinemas, and the planetarium show as replicas of modeled phenomena.

Finally, and most importantly, Sayre and Crosson distinguished between two types of symbolic models. Whereas Churchman *et al.* had essentially equated their symbolic category of models with the formal models of Rosenblueth and Wiener, Sayre and Crosson recognized that the programmed electronic digital computer was making possible the construction and implementation of *algorithmic* or *operational* models more general than the formal symbolic models which required the use of well-formed mathematical disciplines for the manipulation of the model's symbols. These more general models, typified by a logically connected sequence of machine-comprehensible statements (or *algorithmic programs*), no longer required the use of mathematical (or mathematical logical) operations for their manipulation, but merely a coherent consistent outline of procedures to be followed, either in manipulating or in assigning successively values to the symbols constituting the model. These algorithmic models were termed *simulations* by Sayre and Crosson.

Unfortunately, the term "simulation" had been previously adopted by users of analog electronic computers to describe their use of such devices in miming formalized equivalent expressions for other time-dependent physical phenomena. (See, e.g., McLeod [18].) Moreover, many authors tend to identify the term simulation with any computerized dynamic model (cf: Kiviat [15] and Gordon [13]).

The semantic difficulties become further confounded whenever one attempts to substitute for "simulations" (*à la* Sayre–Crosson) either the term "algorithmic models" or, as suggested by Sisson [30], the denominator "procedural models." Each of these terms carries a connotation, at least so among computer scientists, which would permit one to include certain algorithmic procedures, such as numerical integration techniques, as simulations. Some might argue, in those cases where the variable of integration is time, that such numerically analytic integration techniques indeed constitute a dynamic mimicry of the phenomenon described by the formalized differential equation. Again, the semantic difficulties of the term "simulation" become apparent. "Simulation" simply has come to mean different things to different people!

However, the essential distinction made by Sayre and Crosson needs to be emphasized. There does exist a class of symbolic models whose component symbols are not entirely manipulated by the operations of well-formed disciplines such as mathematics, mathematical logic, or numerical analysis. A few examples should suffice to illustrate the distinction which the author prefers to make by referring to the Sayre–Crosson "simulation" category as the class of *simular models*.

One of the oldest simular models is the static deterministic model for a cake (or other culinary delicacy): the housewife's recipe, an algorithmic model for the finished product, a model which certainly requires no mathematical operations. Another such simular model, though of the stochastic static variety, is a nonadaptive memoryless chess-playing program, which decides randomly among alternative, equally highly considered, moves. More interesting among simular models are the dynamic varieties such as the algorithm for determining the critical path in an acyclic connected graph (deterministic simulation) or the digital computer program describing the passenger-by-passenger activities of a bank of elevators subject to random demands (stochastic simulation).

A PROPOSED UNIFIED CATEGORIZATION OF MODELS

In summary, one is able to unify the previous efforts to classify models as follows. Basic cross-categorization schemes are the dynamic–static and the deterministic–stochastic, as defined in the preceding section. In addition, the third categorization criterion (that of Rosenblueth and Wiener) will also be retained, although their formal models will be henceforth referred to as symbolic models.

By reference to Table I, one sees that the following refinement of the material class of models is proposed: 1) *replicas*—spatial transformations of original physical objects in which the dimensionality of the modeled is retained in the replica; 2) *quasi-replicas*—physical models in which one or more of the dimensions of the modeled object is missing; and 3) *analogues*—physical models which bear no direct resemblance to the modeled phenomena, yet whose essential properties may be placed in a one-to-one correspondence with pertinent properties of the modeled.

At the extreme position of precision, then, among the material models are the replicas, for they incorporate the same materials (though perhaps dimensionally scaled or reduced) as does the original physical object. For example, a scale model of a riverine–estuarine system may not be scaled in depth commensurately with the scale reduction in the system's width and length, yet the presence of all three dimensions ensures that such a model be a replica. The most precise model, then, is the *duplicate*, such as presumably arises in manufacturing processes and in biological (identical) twinning.

Less precise among the material models are the quasi-replicas in which one or more dimensions of the modeled have been eliminated. Characteristic quasi-replicas are photographs, road maps, planetarium shows, cinemas, CRT displays of the performance of an endurance or flight test, and weather maps.

Considerably less realistic (though still quite useful) among the material models are those which replace the material representation or behavior of one physical object or system with that of another: the *analogue* model. In this

TABLE I
Exemplary Models

		Material			Symbolic		
		Replication	Quasi-replica	Analogue	Descriptive	Simular	Formalization
STATIC	Deterministic	Earthen Relief Map	Road Map	Statue of B. Franklin	Ten Commandments	Decision Logic Tables (See [33])	Ohm's Law
	Stochastic	Critical Dosage Test	Weather Map	Die Toss for Russian Roulette	Weather Report	Non-Adaptive, Random, Chess Playing Program	Equilibrium Queue Length
DYNAMIC	Deterministic	Model Train Set	Planetarium Show	Analog Computer Circuitry for $\dot{y} = -y$	Constitution of U.S.A.	Critical Path Algorithm	Lanchester's Laws
	Stochastic	Drosophila Genetic Experiment	CRT Display of Endurance Test	White Noise Generator	Text on Darwinian Evolution	Vehicle-By-Vehicle Transportation Model	Stochastic Differential Equation

Increasing Generality

Increasing Abstraction
Increasing Inferential Facility
Decreasing Reality

regard, the aforementioned bronze statue of Benjamin Franklin becomes an *analogue* model (of the *deterministic static* variety), since Dr. Franklin was of flesh and bone, rather than copper and tin. Other analogue models include: the analog electronic computer "solution" for a time-dependent differential equation (*deterministic dynamic analogue* type); the use of a hybrid computer with shift-register random numbers so as to mime a stochastic time-dependent differential equation (see Korn [17] for details regarding this type of *stochastic dynamic analogue* model); and the throw of a single die with a black ace as a substitute for the revolver in a game of Russian roulette (a *stochastic static analogue* model).

Further removed from the physical objects, phenomena, and systems which one seeks to represent are the *symbolic (or literal) models*. Taking the cue of Black [1], the most general among symbolic models is the metaphorical or *descriptive* model. Such models, being expressed in terms of one of man's natural languages, are the most "natural" among symbolic models, but are subjected to manipulation and transformation only by means of the accepted rules of grammar. Exemplary descriptive models include: a twentieth-century text on Darwinian evolution (a *dynamic stochastic descriptive* model of life on the planet Earth); the *Constitution of the United States of America* (a *deterministic descriptive* model for social and political organization, the symbolic model's being of the *dynamic* variety due to its inclusion of an amending process); the Ten Commandments of Moses (a *static deterministic descriptive* model); and a weather report (a *static stochastic descriptive* model).

The use of natural language as a means of modeling is probably man's most elaborate (and perhaps most realistic) method of conveying a meaningful symbolic (nonmaterial) representation of a system or phenomenon. The descriptive model is a verbal (written) explanation of a process and is subject only to the rules of grammar applicable to the natural language in which it is expressed.

Somewhat more precise, though perhaps providing less reality in its representation, is the class of symbolic models defined, in the Sayre–Crosson sense, as simulations. Exemplary simular models have been discussed previously and are also provided for facile reference in Table I. These simular models may include grammatical structures (indeed, consider the typical simulation program written in an *ad hoc* simulation programming language), but may also incorporate mathematical expressions and formulas. In fact, one might speculate that the constitution for a republic such as that intended for the United States is quite nearly a simular type of model for social structure since most of its statements are clearly delineated procedures or constraints.

Finally, there remains the highly considered class of formalized models or *formalizations*: symbolic models that, in the Sayre–Crosson sense, consist of symbols which are manipulated entirely by the operations of a well-formed mathematical discipline such as the integral calculus, algebra, numerical analysis, or mathematical logic. Exemplary formalizations are also found cross-categorized in Table I.

CHOICES AND IMPLEMENTATIONS OF MODELS

A perusal of Table I reveals that, as one moves from left to right in the table, an increase in abstraction and a concomitant deviation from reality is encountered. The quasi-replica is a dimensionally reduced material model and therefore less representative of the original phenomenon than is a proper replication or duplicate. The analogue model provides a greater departure from reality in that it represents a *change of medium* from the modeled to the model. All the symbolic models require a similar change of medium from the physical phenomenon of interest to the written expression or model; many persons, especially those who have had significant training and experience in the use of analog computational devices, would feel that, of the two types of changes in modeling media, the analogue representation constitutes the less drastic alteration. (See,