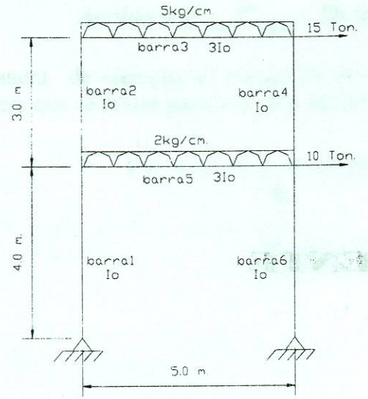
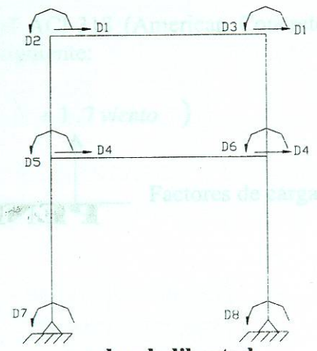
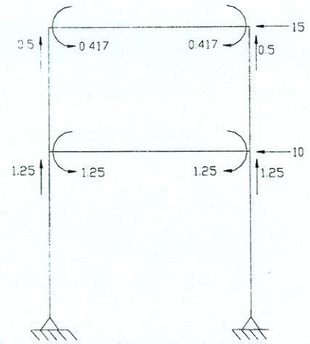
4.2.2.4. EJEMPLO ADICIONAL: Marco Con Un Claro, Dos Niveles

Analizar el marco:

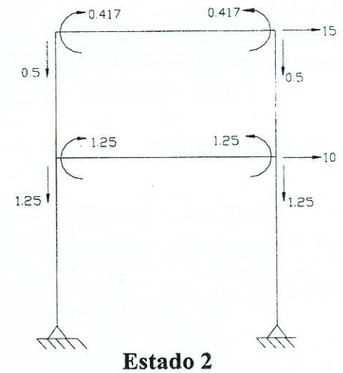




grados de libertad



Estado 1
"Estructura Restringida"



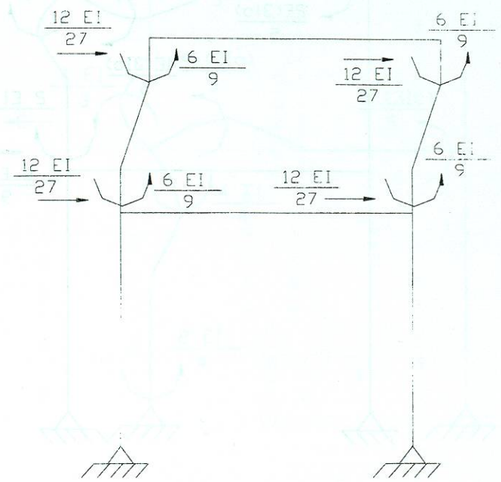
Estado 2
"Estructura liberada"

Formación del vector Fu:

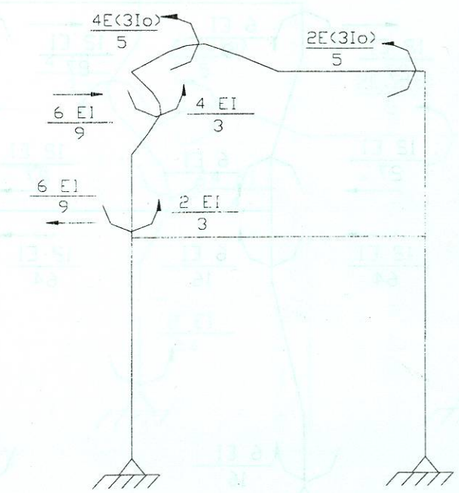
$$Fu = \begin{bmatrix} 15 \\ -0.417 \\ 0.417 \\ 10 \\ -1.25 \\ 1.25 \\ 0 \\ 0 \end{bmatrix}$$

Liberación de los grados de libertad:

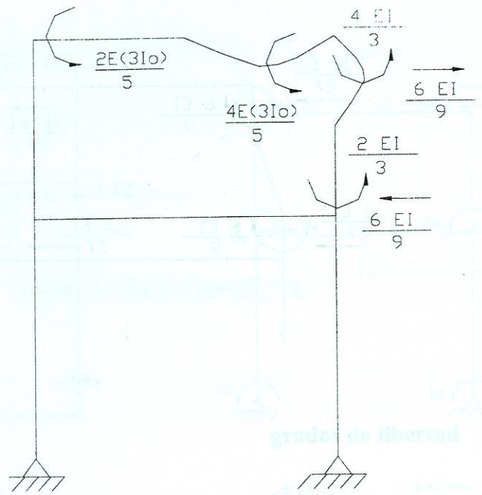
$D_1 = 1, D_n = 0$



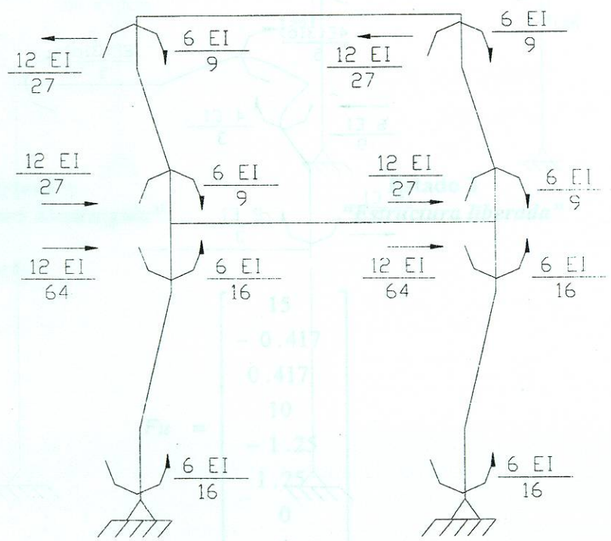
$D_2 = 1, D_n = 0.$



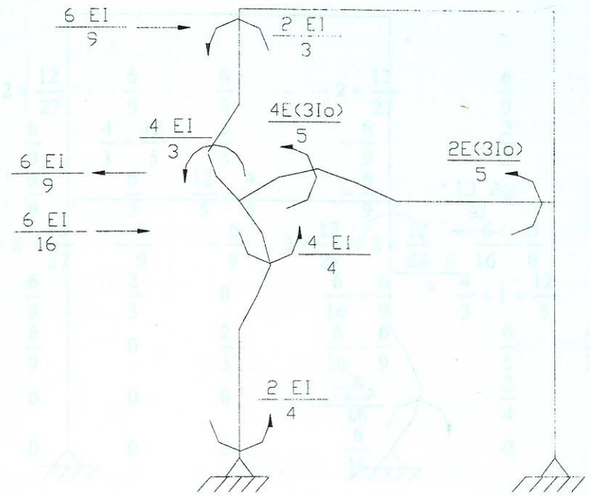
$D_3 = 1, D_n = 0$



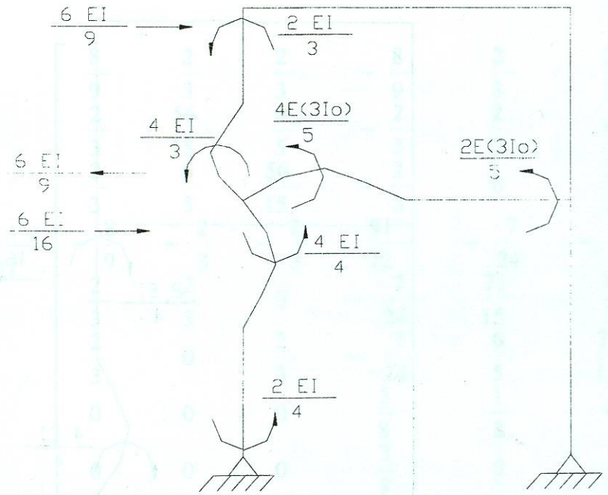
$D_4 = 1, D_n = 0$



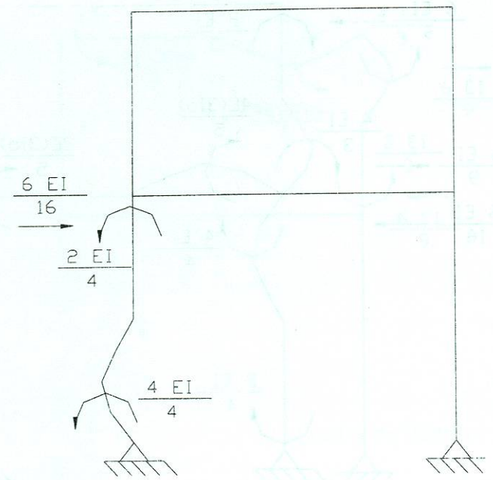
$D_5 = 1, D_n = 0$



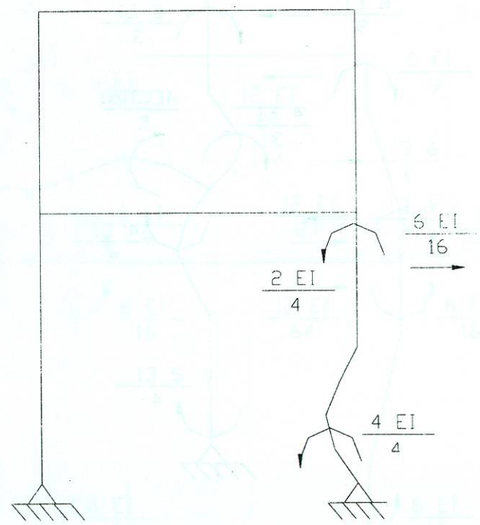
$D_6 = 1, D_n = 0$



$D_7 = 1, D_n = 0$



$D_8 = 1, D_n = 0$



Formación de la matriz K11u:

$$K11u = EI \begin{bmatrix} 2\times\frac{12}{27} & \frac{6}{9} & \frac{6}{9} & -2\times\frac{12}{27} & \frac{6}{9} & \frac{6}{9} & 0 & 0 \\ \frac{6}{9} & \frac{4}{3}+\frac{12}{5} & \frac{6}{5} & -\frac{6}{9} & \frac{2}{3} & 0 & 0 & 0 \\ \frac{6}{9} & \frac{6}{5} & \frac{12}{5}+\frac{4}{3} & -\frac{6}{9} & 0 & \frac{2}{3} & 0 & 0 \\ -2\times\frac{12}{27} & -\frac{6}{9} & -\frac{6}{9} & 2\times\frac{12}{27}+2\times\frac{12}{64} & \frac{6}{16}-\frac{6}{9} & \frac{6}{16}-\frac{6}{9} & \frac{6}{16} & \frac{6}{16} \\ \frac{6}{9} & \frac{2}{3} & 0 & \frac{6}{16}-\frac{6}{9} & \frac{4}{3}+1+\frac{12}{5} & \frac{6}{5} & \frac{2}{4} & 0 \\ \frac{6}{9} & 0 & \frac{2}{3} & \frac{6}{16}-\frac{6}{9} & \frac{6}{5} & \frac{4}{3}+\frac{12}{5}+1 & 0 & \frac{2}{4} \\ 0 & 0 & 0 & \frac{6}{16} & \frac{2}{4} & 0 & \frac{4}{4} & 0 \\ 0 & 0 & 0 & \frac{6}{16} & 0 & \frac{2}{4} & 0 & \frac{4}{4} \end{bmatrix}$$

$$K11u = EI \cdot \begin{bmatrix} \frac{8}{9} & \frac{2}{3} & \frac{2}{3} & -\frac{8}{9} & \frac{2}{3} & \frac{2}{3} & 0 & 0 \\ \frac{2}{3} & \frac{56}{15} & \frac{6}{5} & -\frac{2}{3} & \frac{2}{3} & 0 & 0 & 0 \\ \frac{2}{3} & \frac{6}{5} & \frac{56}{15} & -\frac{2}{3} & 0 & \frac{2}{3} & 0 & 0 \\ -\frac{8}{9} & -\frac{2}{3} & -\frac{2}{3} & \frac{91}{72} & -\frac{7}{24} & -\frac{7}{24} & \frac{3}{8} & \frac{3}{8} \\ \frac{2}{3} & \frac{2}{3} & 0 & -\frac{7}{24} & \frac{71}{15} & \frac{6}{5} & \frac{1}{2} & 0 \\ \frac{2}{3} & 0 & \frac{2}{3} & -\frac{7}{24} & \frac{6}{5} & \frac{71}{15} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{3}{8} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{3}{8} & 0 & \frac{1}{2} & 0 & 1 \end{bmatrix}$$

Calculo de los desplazamientos:

$$Du = K\ 11\ u^{-1} \times Fu \qquad Du = \begin{bmatrix} 377.465 \\ -4.278 \\ -4.142 \\ 330.382 \\ -16.296 \\ -15.562 \\ -115.745 \\ -116.112 \end{bmatrix}$$

Calculo de los momentos:

Barra # 1

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = EI \begin{bmatrix} \frac{4}{4} & \frac{2}{4} \\ \frac{2}{4} & \frac{4}{4} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -115.745 + 0 - \frac{-330.382}{4} \\ -16.296 + 0 - \frac{-330.382}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \times \begin{bmatrix} -33.1495 \\ 66.2995 \end{bmatrix} = \begin{matrix} 0.00025 \\ 49.725 \end{matrix}$$

Barra # 2

$$\begin{bmatrix} M_2 \\ M_3 \end{bmatrix} = EI \begin{bmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -16.296 + \frac{-330.382}{3} - \frac{-377.465}{3} \\ -4.278 + \frac{-330.382}{3} - \frac{-377.465}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix} \times \begin{bmatrix} -0.6017 \\ 11.4163 \end{bmatrix} = \begin{matrix} 6.81 \\ 14.82 \end{matrix}$$

Barra # 3

$$\begin{bmatrix} M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} 0.417 \\ -0.417 \end{bmatrix} + EI \begin{bmatrix} \frac{12}{5} & \frac{6}{5} \\ \frac{6}{5} & \frac{12}{5} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -4.278 + 0 - 0 \\ -4.142 + 0 - 0 \end{bmatrix} = \begin{bmatrix} 0.417 \\ -0.417 \end{bmatrix} + \begin{bmatrix} 2.4 & 1.2 \\ 1.2 & 2.4 \end{bmatrix} \times \begin{bmatrix} -0.6017 \\ 11.4163 \end{bmatrix} =$$

Barra # 4

$$\begin{bmatrix} M_5 \\ M_4 \end{bmatrix} = EI \begin{bmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -15.562 + \frac{-330.382}{3} - \frac{-377.465}{3} \\ -4.142 + \frac{-330.382}{3} - \frac{-377.465}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} \end{bmatrix} \times \begin{bmatrix} 0.1323 \\ 11.5523 \end{bmatrix} = \begin{matrix} 7.88 \\ 15.49 \end{matrix}$$

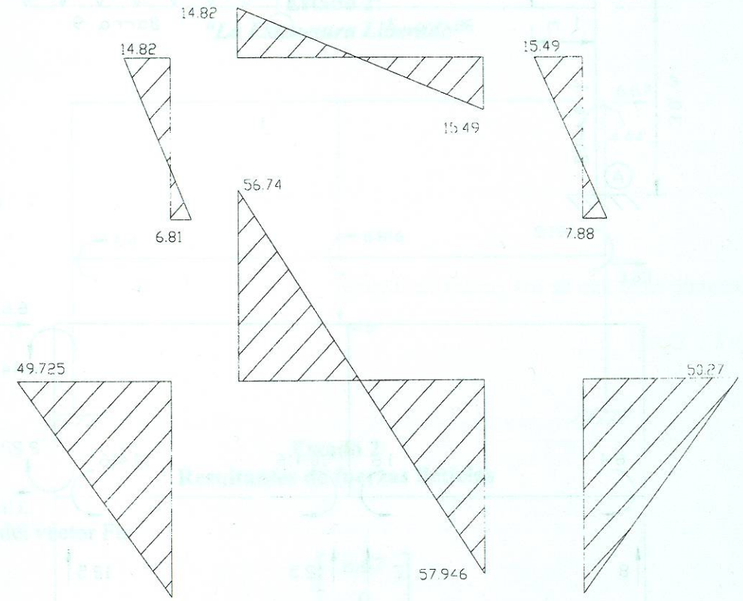
Barra # 5

$$\begin{bmatrix} M_2 \\ M_5 \end{bmatrix} = \begin{bmatrix} 1.042 \\ -1.042 \end{bmatrix} + EI \begin{bmatrix} \frac{12}{5} & \frac{6}{5} \\ \frac{6}{5} & \frac{12}{5} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -16.296 + 0 - 0 \\ -15.562 + 0 - 0 \end{bmatrix} = \begin{bmatrix} 1.042 \\ -1.042 \end{bmatrix} + \begin{bmatrix} 2.4 & 1.2 \\ 1.2 & 2.4 \end{bmatrix} \times \begin{bmatrix} -16.296 \\ -15.562 \end{bmatrix} = \begin{matrix} -56.74 \\ -57.946 \end{matrix}$$

Barra # 6

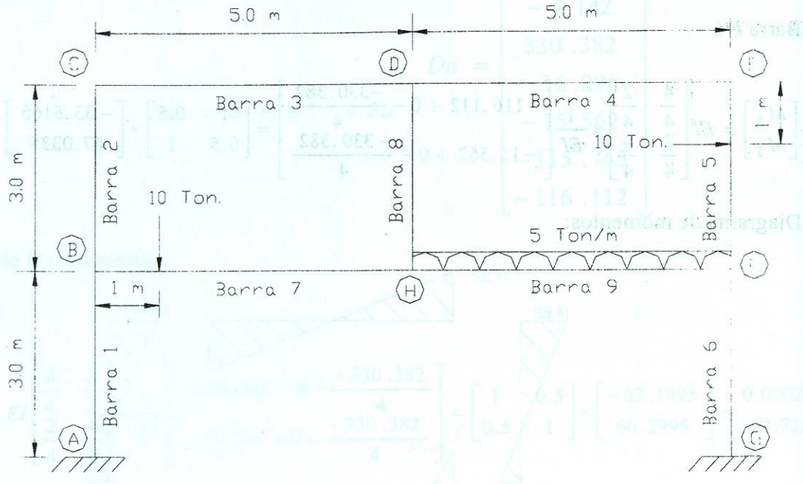
$$\begin{bmatrix} M_6 \\ M_5 \end{bmatrix} = EI \begin{bmatrix} \frac{4}{4} & \frac{2}{4} \\ \frac{2}{4} & \frac{4}{4} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -116.112 + 0 - \frac{-330.382}{4} \\ -15.562 + 0 - \frac{-330.382}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \times \begin{bmatrix} -33.5165 \\ 67.0335 \end{bmatrix} = \begin{matrix} -0.00025 \\ 50.27 \end{matrix}$$

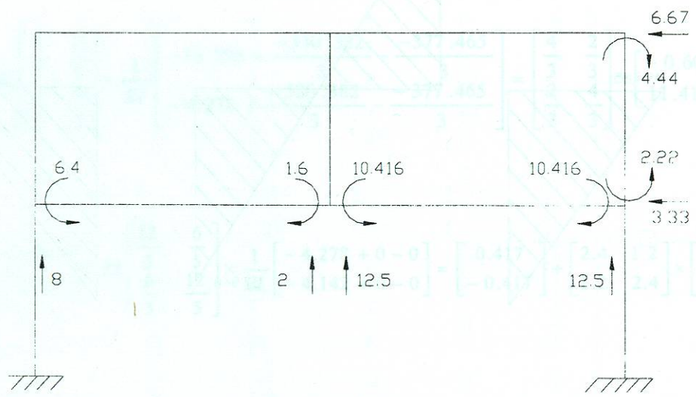
Diagrama de momentos:



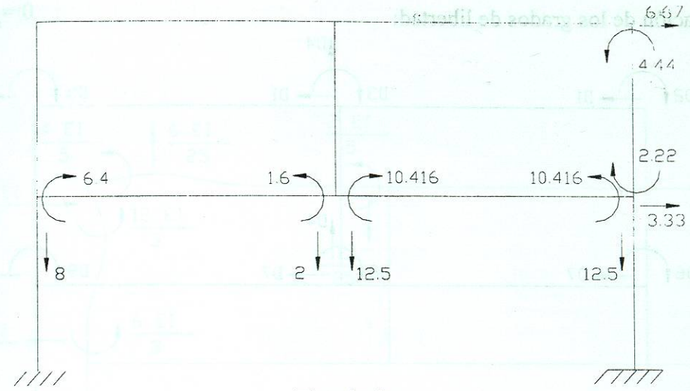
4.2.2.5. EJEMPLO ADICIONAL: Marco Con Grado De Libertad Vertical

Analizar y dibujar el diagrama de momentos para el marco:

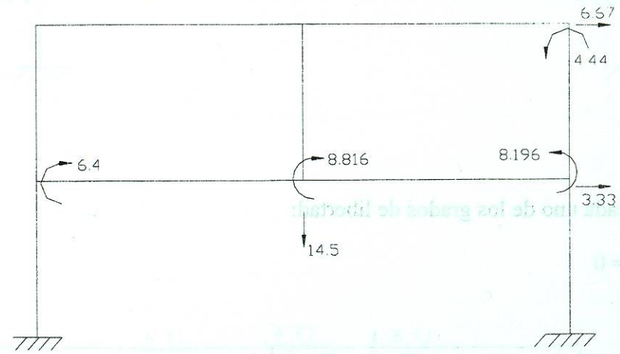




Estado 1
"La Estructura Restringida"



Estado 2:
"La Estructura Liberada"

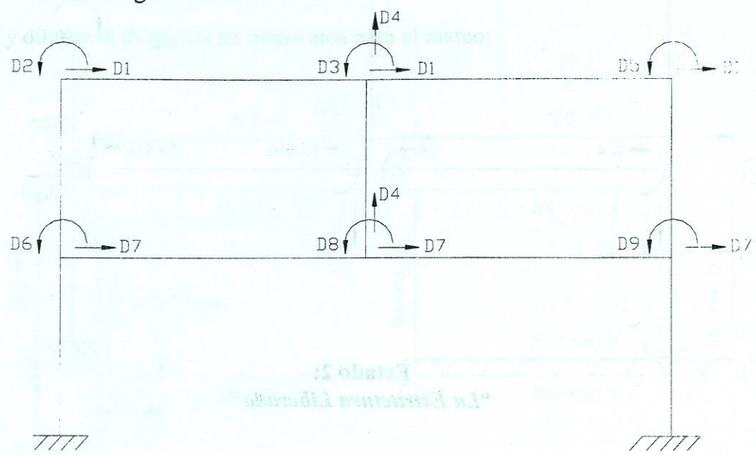


Estado 2
Resultantes de fuerzas ficticias

Formación del vector Fu:

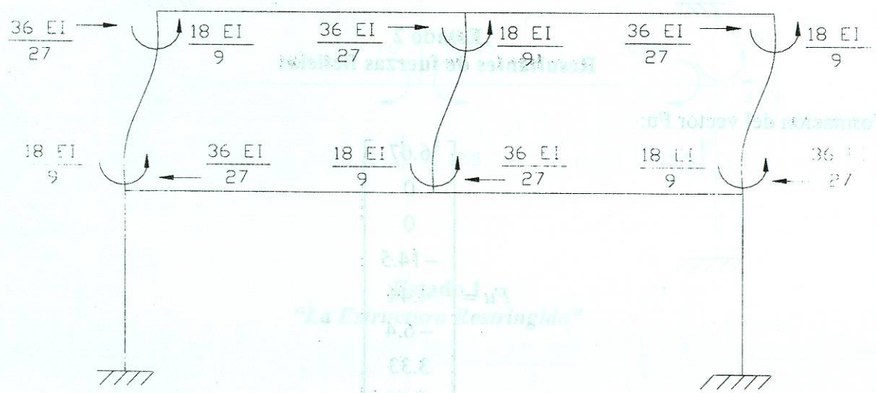
$$Fu = \begin{bmatrix} 6.67 \\ 0 \\ 0 \\ -14.5 \\ 4.44 \\ -6.4 \\ 3.33 \\ -8.816 \\ 8.196 \end{bmatrix}$$

Identificación de los grados de libertad:

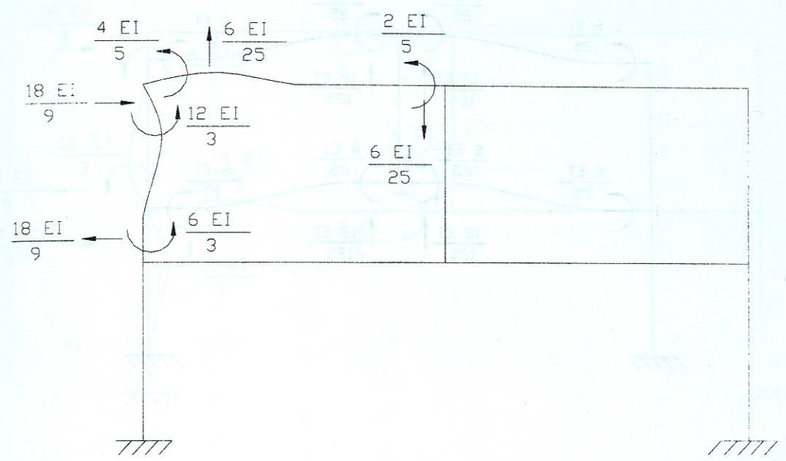


Liberando cada uno de los grados de libertad:

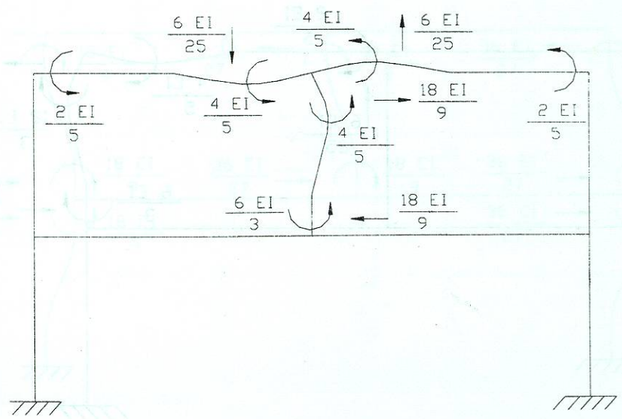
$D_1 = 1$; $D_n = 0$



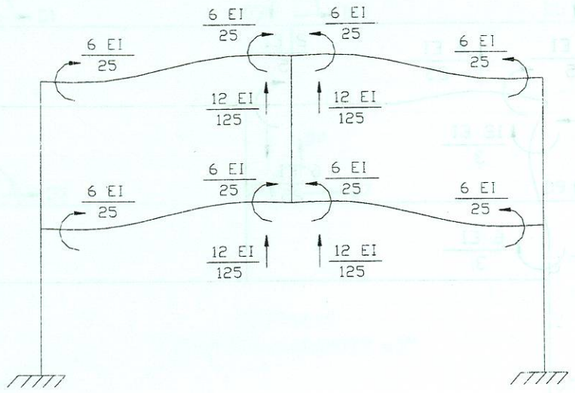
$D_2 = 1$; $D_n = 0$



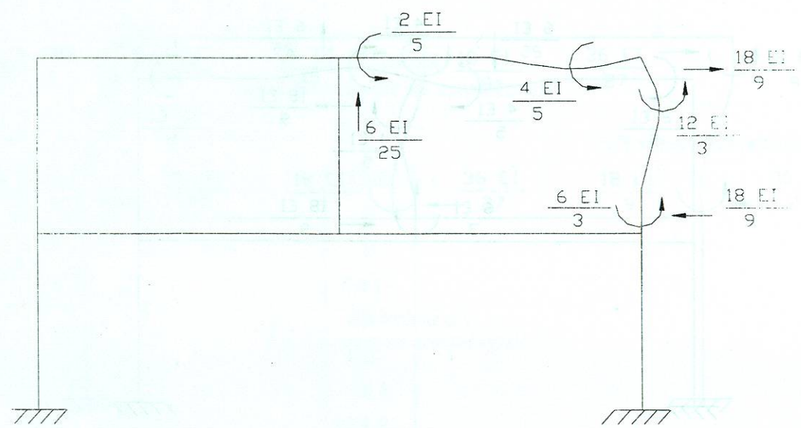
$D_3 = 1$, $D_n = 0$



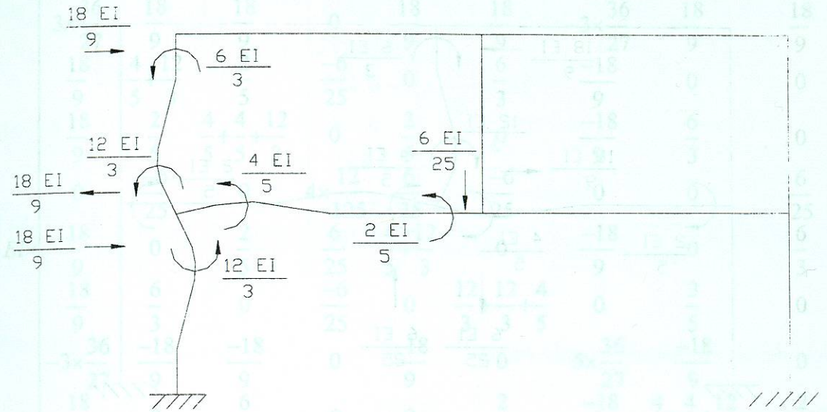
$D_4 = 1, D_n = 0$



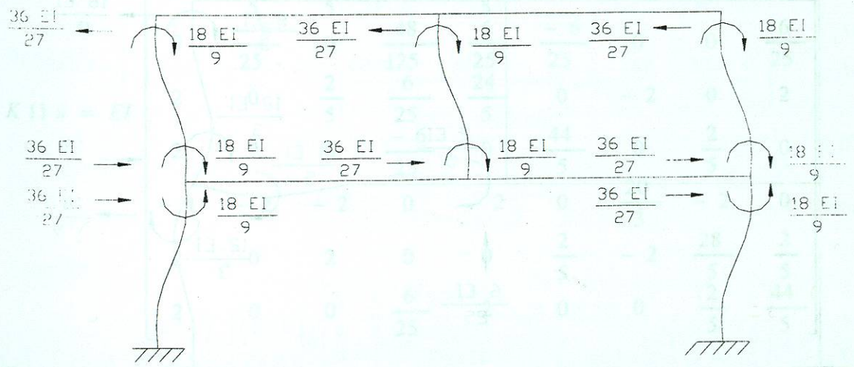
$D_5 = 1, D_n = 0$



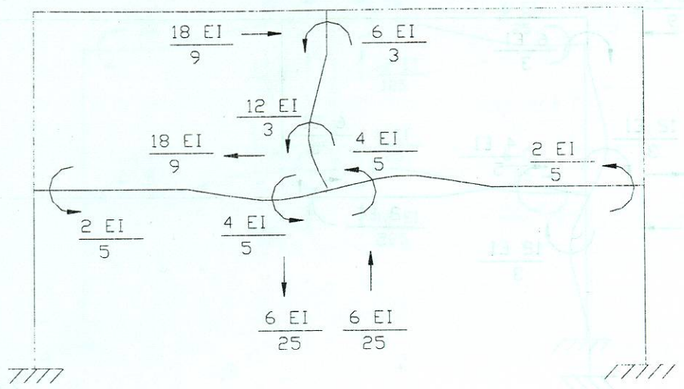
$D_6 = 1, D_n = 0$



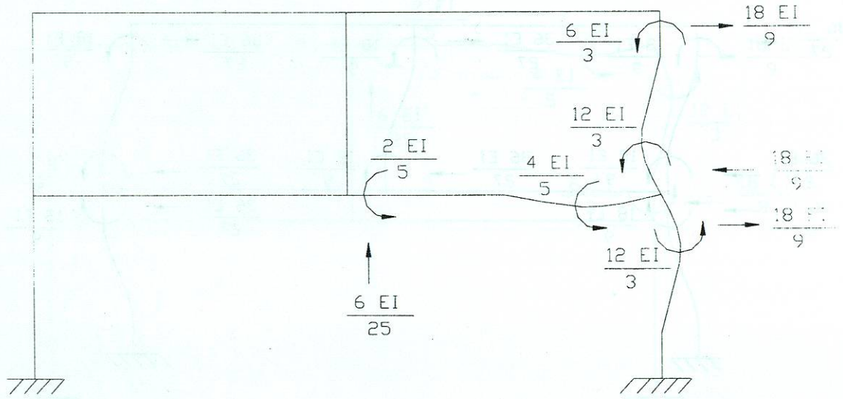
$D_7 = 1, D_n = 0$



$D_8 = 1, D_n = 0$



$D_9 = 1, D_n = 0$



Formación de la matriz K11u:

$$K11u = EI\times\begin{bmatrix}
3\times\frac{36}{27} & \frac{18}{9} & \frac{18}{9} & 0 & \frac{18}{9} & \frac{18}{9} & -3\times\frac{36}{27} & \frac{18}{9} & \frac{18}{9}\\
\frac{18}{9} & \frac{4}{5}+\frac{12}{3} & \frac{2}{5} & \frac{-6}{25} & 0 & \frac{6}{3} & \frac{-18}{9} & 0 & 0\\
\frac{18}{9} & \frac{2}{5} & \frac{4}{5}+\frac{4}{5}+\frac{12}{3} & 0 & \frac{2}{5} & 0 & \frac{-18}{9} & \frac{6}{3} & 0\\
0 & \frac{-6}{25} & 0 & 4\times\frac{12}{125} & \frac{6}{25} & \frac{-6}{25} & 0 & 0 & \frac{6}{25}\\
\frac{18}{9} & 0 & \frac{2}{5} & \frac{6}{25} & \frac{4}{5}+\frac{12}{3} & 0 & \frac{-18}{9} & 0 & \frac{6}{3}\\
\frac{18}{9} & \frac{6}{3} & 0 & \frac{-6}{25} & 0 & \frac{12}{3}+\frac{12}{3}+\frac{4}{5} & 0 & \frac{2}{5} & 0\\
-3\times\frac{36}{27} & \frac{-18}{9} & \frac{-18}{9} & 0 & \frac{-18}{9} & 0 & 5\times\frac{36}{27} & \frac{-18}{9} & 0\\
\frac{18}{9} & 0 & \frac{6}{3} & 0 & 0 & \frac{2}{5} & \frac{-18}{9} & \frac{4}{5}+\frac{4}{5}+\frac{12}{3} & \frac{2}{5}\\
\frac{18}{9} & 0 & 0 & \frac{6}{25} & \frac{6}{3} & 0 & 0 & \frac{2}{5} & \frac{12}{3}+\frac{12}{3}+\frac{4}{5}
\end{bmatrix}$$

$$K11u = EI\begin{bmatrix}
4 & 2 & 2 & 0 & 2 & 2 & -4 & 2 & 2\\
2 & \frac{24}{5} & \frac{2}{5} & \frac{-6}{25} & 0 & 2 & -2 & 0 & 0\\
2 & \frac{2}{5} & \frac{28}{5} & 0 & \frac{2}{5} & 0 & -2 & 2 & 0\\
0 & \frac{-6}{25} & 0 & \frac{48}{125} & \frac{6}{25} & \frac{-6}{25} & 0 & 0 & \frac{6}{25}\\
2 & 0 & \frac{2}{5} & \frac{6}{25} & \frac{24}{5} & 0 & -2 & 0 & 2\\
2 & 2 & 0 & \frac{-6}{25} & 0 & \frac{44}{5} & 0 & \frac{2}{5} & 0\\
-4 & -2 & -2 & 0 & -2 & 0 & \frac{20}{3} & -2 & 0\\
2 & 0 & 2 & 0 & 0 & \frac{2}{5} & -2 & \frac{28}{5} & \frac{2}{5}\\
2 & 0 & 0 & \frac{6}{25} & 2 & 0 & 0 & \frac{2}{5} & \frac{44}{5}
\end{bmatrix}$$

Calculo de los desplazamientos:

$$[Du] = [K11u]^{-1}\times[Fu]$$

$$Du = \frac{1}{EI} \times \begin{bmatrix} 20.485 \\ -4.274 \\ -2.088 \\ -42.092 \\ -0.423 \\ -5.368 \\ 9.493 \\ -4.207 \\ -2.289 \end{bmatrix}$$

Calculo de los momentos en cada barra:

Barra # 1:

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = EI \begin{bmatrix} \frac{4\times3}{3} & \frac{2\times3}{3} \\ \frac{2\times3}{3} & \frac{4\times3}{3} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 0+0-\frac{-9.493}{3} \\ -5.368+0-\frac{-9.493}{3} \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 3.16 \\ -2.203 \end{bmatrix} = \begin{bmatrix} 8.234 \\ -2.492 \end{bmatrix}$$

Barra # 2:

$$\begin{bmatrix} M_B \\ M_C \end{bmatrix} = EI \begin{bmatrix} \frac{12}{3} & \frac{6}{3} \\ \frac{6}{3} & \frac{12}{3} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -5.368+\frac{-9.493}{3}-\frac{-20.485}{3} \\ -4.274+\frac{-9.493}{3}-\frac{-20.485}{3} \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} -1.704 \\ -0.61 \end{bmatrix} = \begin{bmatrix} -8.036 \\ -5.848 \end{bmatrix}$$

Barra # 3:

$$\begin{bmatrix} M_C \\ M_D \end{bmatrix} = EI \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -4.274+0-\frac{-42.092}{5} \\ -2.088+0-\frac{-42.092}{5} \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \begin{bmatrix} 4.144 \\ 6.33 \end{bmatrix} = \begin{bmatrix} 5.847 \\ 6.722 \end{bmatrix}$$

Barra # 4:

$$\begin{bmatrix} M_D \\ M_E \end{bmatrix} = EI \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -2.088+\frac{-42.092}{5}-0 \\ -0.423+\frac{-42.092}{5}-0 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \begin{bmatrix} -10.506 \\ -8.841 \end{bmatrix} = \begin{bmatrix} -11.941 \\ -11.275 \end{bmatrix}$$

Barra # 5:

$$\begin{bmatrix} M_F \\ M_E \end{bmatrix} = \begin{bmatrix} 2.22 \\ -4.44 \end{bmatrix} + EI \begin{bmatrix} \frac{12}{3} & \frac{6}{3} \\ \frac{6}{3} & \frac{12}{3} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -2.289+\frac{-9.493}{3}-\frac{-20.485}{3} \\ -0.423+\frac{-9.493}{3}-\frac{-20.485}{3} \end{bmatrix} = \begin{bmatrix} 2.22 \\ -4.44 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} -1.704 \\ -0.61 \end{bmatrix} = \begin{bmatrix} -8.036 \\ -5.848 \end{bmatrix}$$

Barra # 6:

$$\begin{bmatrix} M_G \\ M_F \end{bmatrix} = EI \begin{bmatrix} \frac{12}{3} & \frac{6}{3} \\ \frac{6}{3} & \frac{12}{3} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 0+0-\frac{-9.493}{3} \\ -2.289+0-\frac{-9.493}{3} \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 3.164 \\ 0.875 \end{bmatrix} = \begin{bmatrix} 14.406 \\ 9.828 \end{bmatrix}$$

Barra # 7:

$$\begin{bmatrix} M_B \\ M_H \end{bmatrix} = \begin{bmatrix} 6.4 \\ -1.6 \end{bmatrix} + EI \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -5.368+0-\frac{-42.092}{5} \\ -4.207+0-\frac{-42.092}{5} \end{bmatrix} = \begin{bmatrix} 6.4 \\ -1.6 \end{bmatrix} + \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \begin{bmatrix} 3.05 \\ 4.211 \end{bmatrix} = \begin{bmatrix} 10.52 \\ 2.98 \end{bmatrix}$$

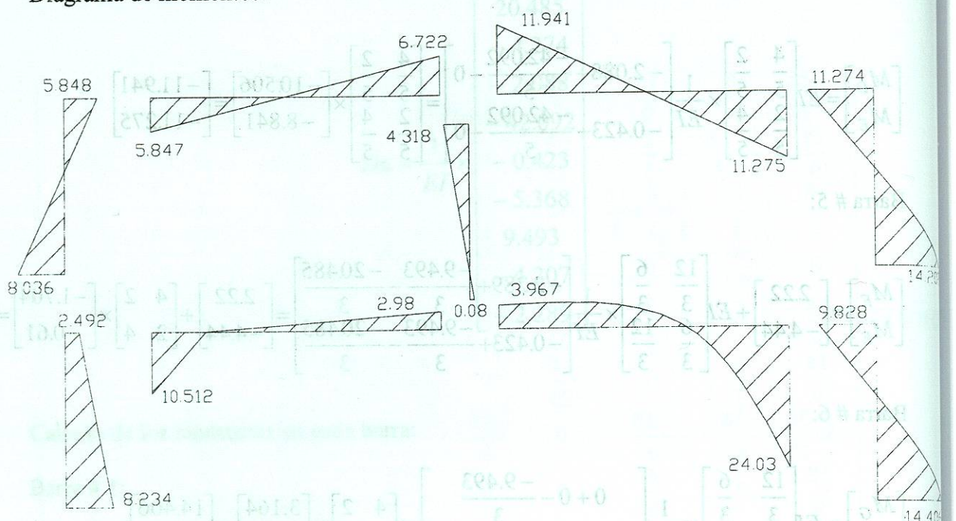
Barra # 8:

$$\begin{bmatrix} M_H \\ M_D \end{bmatrix} = EI \begin{bmatrix} \frac{12}{3} & \frac{6}{3} \\ \frac{6}{3} & \frac{12}{3} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -4.207+\frac{-9.493}{3}-\frac{-20.485}{3} \\ -2.088+\frac{-9.493}{3}-\frac{-20.485}{3} \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} -0.693 \\ 1.426 \end{bmatrix} = \begin{bmatrix} 0.08 \\ 4.318 \end{bmatrix}$$

Barra # 9:

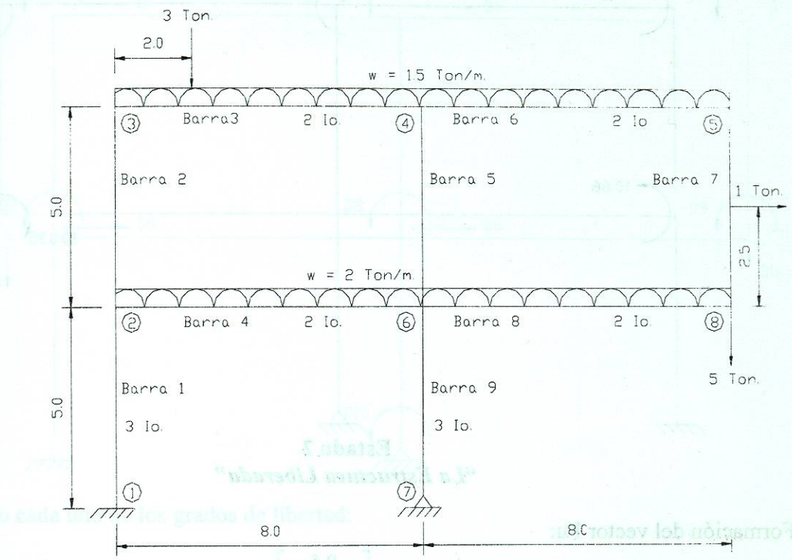
$$\begin{bmatrix} M_H \\ M_F \end{bmatrix} = \begin{bmatrix} 10.416 \\ -10.416 \end{bmatrix} + EI \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -4.207+\frac{-42.092}{5}-0 \\ -2.289+\frac{-42.092}{5}-0 \end{bmatrix} = \begin{bmatrix} 10.416 \\ -10.416 \end{bmatrix} + \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \times \begin{bmatrix} -12.625 \\ -10.707 \end{bmatrix} = \begin{bmatrix} -3.967 \\ -24.03 \end{bmatrix}$$

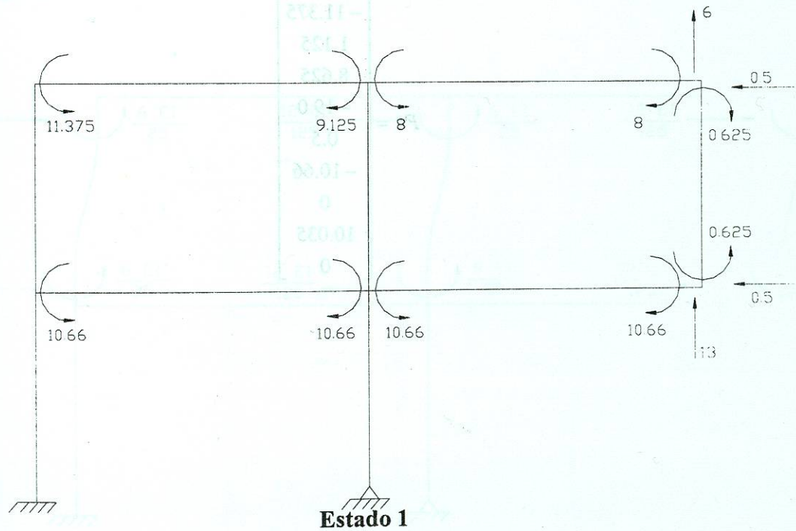
Diagrama de momentos:



4.2.2.6. EJEMPLO ADICIONAL: Marco Con "Volado"

Analizar y dibujar el diagrama de momentos para el marco:





Estado 1

"La Estructura Restringida"