

these lines can be observed with a good microscope. A cube, whose side is the 4000th of a millimetre, may be taken as the *minimum visibile* for observers of the present day. Such a cube would contain from 60 to 100 million molecules of oxygen or of nitrogen; but since the molecules of organised substances contain on an average about 50 of the more elementary atoms, we may assume that the smallest organised particle visible under the microscope contains about two million molecules of organic matter. At least half of every living organism consists of water, so that the smallest living being visible under the microscope does not contain more than about a million organic molecules. Some exceedingly simple organism may be supposed built up of not more than a million similar molecules. It is impossible, however, to conceive so small a number sufficient to form a being furnished with a whole system of specialised organs.

Thus molecular science sets us face to face with physiological theories. It forbids the physiologist from imagining that structural details of infinitely small dimensions can furnish an explanation of the infinite variety which exists in the properties and functions of the most minute organisms.

A microscopic germ is, we know, capable of development into a highly organised animal. Another germ, equally microscopic, becomes, when developed, an animal of a totally different kind. Do all the differences, infinite in number, which distinguish the one animal from the other, arise each from some difference in the structure of the respective germs? Even if we admit this as possible, we shall be called upon by the advocates of Pangenesis to admit still greater marvels. For the microscopic germ, according to this theory, is no mere individual, but a representative body, containing members collected from every rank of the long-drawn ramification of the ancestral tree, the number of these members being amply sufficient not only to furnish the hereditary characteristics of every organ of the body and every habit of the animal from birth to death, but also to afford a stock of latent gemmules to be passed on in an inactive state from germ to germ, till at last the ancestral peculiarity which it represents is revived in some remote descendant.

Some of the exponents of this theory of heredity have attempted to elude the difficulty of placing a whole world of wonders within a body so small and so devoid of visible structure as a germ, by using the phrase structureless germs.¹ Now, one material system can differ from another only in the configuration and motion which it has at a given instant. To explain differences of function and development of a germ without assuming differences of structure is, therefore, to admit that the properties of a germ are not those of a purely material system.

The evidence as to the nature and motion of molecules, with which we have hitherto been occupied, has been derived from experiments upon gaseous media, the smallest sensible portion of which contains millions of millions of molecules. The constancy and uniformity of the properties of the gaseous medium is the direct result of the inconceivable irregularity of the motion of agitation of its molecules. Any cause which could introduce regularity into the motion of agitation, and marshal the molecules into order and method in their evolutions, might check or even reverse that tendency to diffusion of matter, motion, and energy, which is one of the most invariable phenomena of nature, and to which Thomson has given the name of the dissipation of energy.

Thus, when a sound-wave is passing through a mass of

¹ See F. Galton, "On Blood Relationship," *Proc. Roy. Soc.*, June 13, 1872.

air, this motion is of a certain definite type, and if left to itself the whole motion is passed on to other masses of air, and the sound-wave passes on, leaving the air behind it at rest. Heat, on the other hand, never passes out of a hot body except to enter a colder body, so that the energy of sound-waves, or any other form of energy which is propagated so as to pass wholly out of one portion of the medium and into another, cannot be called heat.

We have now to turn our attention to a class of molecular motions, which are as remarkable for their regularity as the motion of agitation is for its irregularity.

It has been found, by means of the spectroscope, that the light emitted by incandescent substances is different according to their state of condensation. When they are in an extremely rarefied condition the spectrum of their light consists of a set of sharply-defined bright lines. As the substance approaches a denser condition the spectrum tends to become continuous, either by the lines becoming broader and less defined, or by new lines and bands appearing between them, till the spectrum at length loses all its characteristics and becomes identical with that of solid bodies when raised to the same temperature.

Hence the vibrating systems, which are the source of the emitted light, must be vibrating in a different manner in these two cases. When the spectrum consists of a number of bright lines, the motion of the system must be compounded of a corresponding number of types of harmonic vibration.

In order that a bright line may be sharply defined, the vibratory motion which produces it must be kept up in a perfectly regular manner for some hundreds or thousands of vibrations. If the motion of each of the vibrating bodies is kept up only during a small number of vibrations, then, however regular may be the vibrations of each body while it lasts, the resultant disturbance of the luminiferous medium, when analysed by the prism, will be found to contain, besides the part due to the regular vibrations, other motions, depending on the starting and stopping of each particular vibrating body, which will become manifest as a diffused luminosity scattered over the whole length of the spectrum. A spectrum of bright lines, therefore, indicates that the vibrating bodies when set in motion are allowed to vibrate in accordance with the conditions of their internal structure for some time before they are again interfered with by external forces.

It appears, therefore, from spectroscopic evidence that each molecule of a rarefied gas is, during the greater part of its existence, at such a distance from all other molecules that it executes its vibrations in an undisturbed and regular manner. This is the same conclusion to which we were led by considerations of another kind at p. 39.

We may therefore regard the bright lines in the spectrum of a gas as the result of the vibrations executed by the molecules while describing their free paths. When two molecules separate from one another after an encounter, each of them is in a state of vibration, arising from the unequal action on different parts of the same molecule during the encounter. Hence, though the centre of mass of the molecule describing its free path moves with uniform velocity, the parts of the molecule have a vibratory motion with respect to the centre of mass of the whole molecule, and it is the disturbance of the luminiferous medium communicated to it by the vibrating molecules which constitutes the emitted light.

We may compare the vibrating molecule to a bell. When struck, the bell is set in motion. This motion is compounded of harmonic vibrations of many different periods, each of which acts on the air, producing notes of as many different pitches. As the bell communicates its motion to the air, these vibrations necessarily decay, some

of them faster than others, so that the sound contains fewer and fewer notes, till at last it is reduced to the fundamental note of the bell.¹ If we suppose that there are a great many bells precisely similar to each other, and that they are struck, first one and then another, in a perfectly irregular manner, yet so that, on an average, as many bells are struck in one second of time as in another and also in such a way that, on an average, any one bell is not again struck till it has ceased to vibrate, then the audible result will appear a continuous sound, composed of the sound emitted by bells in all states of vibration, from the clang of the actual stroke to the final hum of the dying fundamental tone.

But now let the number of bells be reduced while the same number of strokes are given in a second. Each bell will now be struck before it has ceased to vibrate, so that in the resulting sound there will be less of the fundamental tone and more of the original clang, till at last, when the peal is reduced to one bell, on which innumerable hammers are continually plying their strokes all out of time, the sound will become a mere noise, in which no musical note can be distinguished.

In the case of a gas we have an immense number of molecules, each of which is set in vibration when it encounters another molecule, and continues to vibrate as it describes its free path. The molecule is a material system, the parts of which are connected in some definite way, and from the fact that the bright lines of the emitted light have always the same wave-lengths, we learn that the vibrations corresponding to these lines are always executed in the same periodic time, and therefore the force tending to restore any part of the molecule to its position of equilibrium in the molecule must be proportional to its displacement relative to that position.

From the mathematical theory of the motion of such a system, it appears that the whole motion may be analysed into the following parts, which may be considered each independently of the others:—In the first place, the centre of mass of the system moves with uniform velocity in a straight line. This velocity may have any value. In the second place, there may be a motion of rotation, the angular momentum of the system about its centre of mass remaining during the free path constant in magnitude and direction. This angular momentum may have any value whatever, and its axis may have any direction. In the third place, the remainder of the motion is made up of a number of component motions, each of which is a harmonic vibration of a given type. In each type of vibration the periodic time of vibration is determined by the nature of the system, and is invariable for the same system. The relative amount of motion in different parts of the system is also determinate for each type, but the absolute amount of motion and the phase of the vibration of each type are determined by the particular circumstances of the last encounter, and may vary in any manner from one encounter to another.

The values of the periodic times of the different types of vibration are given by the roots of a certain equation, the form of which depends on the nature of the connections of the system. In certain exceptionally simple cases, as, for instance, in that of a uniform string stretched between two fixed points, the roots of the equation are connected by simple arithmetical relations, and if the internal structure of a molecule had an analogous kind of simplicity, we might expect to find in the spectrum of the molecule a

¹ Part of the energy of motion is, in the case of the bell, dissipated in the substance of the bell in virtue of the viscosity of the metal, and assumes the form of heat, but it is not necessary, for the purpose of illustration, to take this cause of the decay of vibrations into account.

series of bright lines, whose wave-lengths are in simple arithmetical ratios.

But if we suppose the molecule to be constituted according to some different type, as, for instance, if it is an elastic sphere, or if it consists of a finite number of atoms kept in their places by attractive and repulsive forces, the roots of the equation will not be connected with each other by any simple relations, but each may be made to vary independently of the others by a suitable change of the connections of the system. Hence, we have no right to expect any definite numerical relations among the wave-lengths of the bright lines of a gas.

The bright lines of the spectrum of an incandescent gas are therefore due to the harmonic vibrations of the molecules of the gas during their free paths. The only effect of the motion of the centre of mass of the molecule is to alter the time of vibration of the light as received by a stationary observer. When the molecule is coming towards the observer, each successive impulse will have a shorter distance to travel before it reaches his eye, and therefore the impulses will appear to succeed each other more rapidly than if the molecule were at rest, and the contrary will be the case if the molecule is receding from the observer. The bright line corresponding to the vibration will therefore be shifted in the spectrum towards the blue end when the molecule is approaching, and towards the red end when it is receding from the observer. By observations of the displacement of certain lines in the spectrum, Dr Huggins and others have measured the rate of approach or of recession of certain stars with respect to the earth, and Mr Lockyer has determined the rate of motion of tornadoes in the sun. But Lord Rayleigh has pointed out that according to the dynamical theory of gases the molecules are moving hither and thither with so great velocity that, however narrow and sharply-defined any bright line due to a single molecule may be, the displacement of the line towards the blue by the approaching molecules, and towards the red by the receding molecules, will produce a certain amount of widening and blurring of the line in the spectrum, so that there is a limit to the sharpness of definition of the lines of a gas. The widening of the lines due to this cause will be in proportion to the velocity of agitation of the molecules. It will be greatest for the molecules of smallest mass, as those of hydrogen, and it will increase with the temperature. Hence the measurement of the breadth of the hydrogen lines, such as C or F in the spectrum of the solar prominences, may furnish evidence that the temperature of the sun cannot exceed a certain value.

ON THE THEORY OF VORTEX ATOMS.

The equations which form the foundations of the mathematical theory of fluid motion were fully laid down by Lagrange and the great mathematicians of the end of last century, but the number of solutions of cases of fluid motion which had been actually worked out remained very small, and almost all of these belonged to a particular type of fluid motion, which has been since named the irrotational type. It had been shown, indeed, by Lagrange, that a perfect fluid, if its motion is at any time irrotational, will continue in all time coming to move in an irrotational manner, so that, by assuming that the fluid was at one time at rest, the calculation of its subsequent motion may be very much simplified.

It was reserved for Helmholtz to point out the very remarkable properties of rotational motion in a homogeneous incompressible fluid devoid of all viscosity. We must first define the physical properties of such a fluid. In the first place, it is a material substance. Its motion is

continuous in space and time, and if we follow any portion of it as it moves, the mass of that portion remains invariable. These properties it shares with all material substances. In the next place, it is incompressible. The form of a given portion of the fluid may change, but its volume remains invariable; in other words, the density of the fluid remains the same during its motion. Besides this, the fluid is homogeneous, or the density of all parts of the fluid is the same. It is also continuous, so that the mass of the fluid contained within any closed surface is always exactly proportional to the volume contained within that surface. This is equivalent to asserting that the fluid is not made up of molecules; for, if it were, the mass would vary in a discontinuous manner as the volume increases continuously, because first one and then another molecule would be included within the closed surface. Lastly, it is a perfect fluid, or, in other words, the stress between one portion and a contiguous portion is always normal to the surface which separates these portions, and this whether the fluid is at rest or in motion.

We have seen that in a molecular fluid the interdiffusion of the molecules causes an interdiffusion of motion of different parts of the fluid, so that the action between contiguous parts is no longer normal but in a direction tending to diminish their relative motion. Hence the perfect fluid cannot be molecular.

All that is necessary in order to form a correct mathematical theory of a material system is that its properties shall be clearly defined and shall be consistent with each other. This is essential; but whether a substance having such properties actually exists is a question which comes to be considered only when we propose to make some practical application of the results of the mathematical theory. The properties of our perfect liquid are clearly defined and consistent with each other, and from the mathematical theory we can deduce remarkable results, some of which may be illustrated in a rough way by means of fluids which are by no means perfect in the sense of not being viscous, such, for instance, as air and water.

The motion of a fluid is said to be irrotational when it is such that if a spherical portion of the fluid were suddenly solidified, the solid sphere so formed would not be rotating about any axis. When the motion of the fluid is rotational the axis and angular velocity of the rotation of any small part of the fluid are those of a small spherical portion suddenly solidified.

The mathematical expression of these definitions is as follows:—Let u, v, w be the components of the velocity of the fluid at the point (x, y, z) , and let

$$\alpha = \frac{dv}{dx} - \frac{dw}{dy}, \quad \beta = \frac{dw}{dx} - \frac{du}{dz}, \quad \gamma = \frac{du}{dy} - \frac{dv}{dx} \quad (1),$$

then α, β, γ are the components of the velocity of rotation of the fluid at the point (x, y, z) . The axis of rotation is in the direction of the resultant of $\alpha, \beta,$ and γ , and the velocity of rotation, ω , is measured by this resultant.

A line drawn in the fluid, so that at every point of the line

$$\frac{1}{\alpha} \frac{dx}{ds} = \frac{1}{\beta} \frac{dy}{ds} = \frac{1}{\gamma} \frac{dz}{ds} = \frac{1}{\omega} \quad (2),$$

where s is the length of the line up to the point x, y, z , is called a vortex line. Its direction coincides at every point with that of the axis of rotation of the fluid.

We may now prove the theorem of Helmholtz, that the points of the fluid which at any instant lie in the same vortex line continue to lie in the same vortex line during the whole motion of the fluid.

The equations of motion of a fluid are of the form

$$\rho \frac{\delta u}{\delta t} + \rho \frac{du}{dx} + \rho \frac{dV}{dx} = 0 \quad (3),$$

when ρ is the density, which in the case of our homogeneous incompressible fluid we may assume to be unity, the operator $\frac{\delta}{\delta t}$ represents the rate of variation of the symbol to which it is prefixed at a point which is carried forward with the fluid, so that

$$\frac{\delta u}{\delta t} = \frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz} \quad (4)$$

p is the pressure, and V is the potential of external forces. There are two other equations of similar form in y and z . Differentiating the equation in y with respect to z , and that in z with respect to y , and subtracting the second from the first, we find

$$\frac{d}{dz} \frac{\delta v}{\delta t} - \frac{d}{dy} \frac{\delta w}{\delta t} = 0 \quad (5).$$

Performing the differentiations and remembering equations (1) and also the condition of incompressibility,

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad (6),$$

we find

$$\frac{\delta \alpha}{\delta t} = \alpha \frac{du}{dx} + \beta \frac{dv}{dy} + \gamma \frac{dw}{dz} \quad (7).$$

Now, let us suppose a vortex line drawn in the fluid so as always to begin at the same particle of the fluid. The components of the velocity of this point are u, v, w . Let us find those of a point on the moving vortex line at a distance ds from this point where

$$ds = \omega d\sigma \quad (8).$$

The co-ordinates of this point are

$$x + \alpha d\sigma, \quad y + \beta d\sigma, \quad z + \gamma d\sigma \quad (9),$$

and the components of its velocity are

$$u + \frac{\delta \alpha}{\delta t} d\sigma, \quad v + \frac{\delta \beta}{\delta t} d\sigma, \quad w + \frac{\delta \gamma}{\delta t} d\sigma \quad (10).$$

Consider the first of these components. In virtue of equation (7) we may write it

$$u + \frac{du}{dx} \alpha d\sigma + \frac{dv}{dy} \beta d\sigma + \frac{dw}{dz} \gamma d\sigma \quad (11),$$

or

$$u + \frac{u\alpha}{dx} d\sigma + \frac{u\beta}{dy} d\sigma + \frac{u\gamma}{dz} d\sigma \quad (12),$$

or

$$u + \frac{d\sigma}{d\sigma} \quad (13).$$

But this represents the value of the component u of the velocity of the fluid itself at the same point, and the same thing may be proved of the other components.

Hence the velocity of the second point on the vortex line is identical with that of the fluid at that point. In other words, the vortex line swims along with the fluid, and is always formed of the same row of fluid particles. The vortex line is therefore no mere mathematical symbol, but has a physical existence continuous in time and space.

By differentiating equations (1) with respect to $x, y,$ and z respectively, and adding the results, we obtain the equation—

$$\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} = 0 \quad (14).$$

This is an equation of the same form with (6), which expresses the condition of flow of a fluid of invariable density. Hence, if we imagine a fluid, quite independent of the original fluid, whose components of velocity are α, β, γ , this imaginary fluid will flow without altering its density.

Now, consider a closed curve in space, and let vortex

lines be drawn in both directions from every point of this curve. These vortex lines will form a tubular surface, which is called a vortex tube or a vortex filament. Since the imaginary fluid flows along the vortex lines without change of density, the quantity which in unit of time flows through any section of the same vortex tube must be the same. Hence, at any section of a vortex tube the product of the area of the section into the mean velocity of rotation is the same. This quantity is called the *strength* of the vortex tube.

A vortex tube cannot begin or end within the fluid; for, if it did, the imaginary fluid, whose velocity components are α, β, γ , would be generated from nothing at the beginning of the tube, and reduced to nothing at the end of it. Hence, if the tube has a beginning and an end, they must lie on the surface of the fluid mass. If the fluid is infinite the vortex tube must be infinite, or else it must return into itself.

We have thus arrived at the following remarkable theorems relating to a finite vortex tube in an infinite fluid:—(1.) It returns into itself, forming a closed ring. We may therefore describe it as a vortex ring. (2.) It always consists of the same portion of the fluid. Hence its volume is invariable. (3.) Its strength remains always the same. Hence the velocity of rotation at any section varies inversely as the area of that section, and that of any segment varies directly as the length of that segment. (4.) No part of the fluid which is not originally in a state of rotational motion can ever enter into that state, and no part of the fluid whose motion is rotational can ever cease to move rotationally. (5.) No vortex tube can ever pass through any other vortex tube, or through any of its own convolutions. Hence, if two vortex tubes are linked together, they can never be separated, and if a single vortex tube is knotted on itself, it can never become untied. (6.) The motion at any instant of every part of the fluid, including the vortex rings themselves, may be accurately represented by conceiving an electric current to occupy the place of each vortex ring, the strength of the current being proportional to that of the ring. The magnetic force at any point of space will then represent in direction and magnitude the velocity of the fluid at the corresponding point of the fluid.

These properties of vortex rings suggested to Sir William Thomson¹ the possibility of founding on them a new form of the atomic theory. The conditions which must be satisfied by an atom are—permanence in magnitude, capability of internal motion or vibration, and a sufficient amount of possible characteristics to account for the difference between atoms of different kinds.

The small hard body imagined by Lucretius, and adopted by Newton, was invented for the express purpose of accounting for the permanence of the properties of bodies. But it fails to account for the vibrations of a molecule as revealed by the spectroscope. We may indeed suppose the atom elastic, but this is to endow it with the very property for the explanation of which, as exhibited in aggregate bodies, the atomic constitution was originally assumed. The massive centres of force imagined by Boscovich may have more to recommend them to the mathematician, who has no scruple in supposing them to be invested with the power of attracting and repelling according to any law of the distance which it may please him to assign. Such centres of force are no doubt in their own nature indivisible, but then they are also, singly, incapable of vibration. To obtain vibrations we must imagine molecules consisting of many such centres, but, in so doing, the possibility of these centres being separated altogether is again introduced.

¹ "On Vortex Atoms," *Proc. Roy. Soc. Edin.*, 18th February 1867.

Besides, it is in questionable scientific taste, after using atoms so freely to get rid of forces acting at sensible distances, to make the whole function of the atoms an action at insensible distances.

On the other hand, the vortex ring of Helmholtz, imagined as the true form of the atom by Thomson, satisfies more of the conditions than any atom hitherto imagined. In the first place, it is quantitatively permanent, as regards its volume and its strength,—two independent quantities. It is also qualitatively permanent as regards its degree of implication, whether "knottedness" on itself or "linkedness" with other vortex rings. At the same time, it is capable of infinite changes of form, and may execute vibrations of different periods, as we know that molecules do. And the number of essentially different implications of vortex rings may be very great without supposing the degree of implication of any of them very high.

But the greatest recommendation of this theory, from a philosophical point of view, is that its success in explaining phenomena does not depend on the ingenuity with which its contrivers "save appearances," by introducing first one hypothetical force and then another. When the vortex atom is once set in motion, all its properties are absolutely fixed and determined by the laws of motion of the primitive fluid, which are fully expressed in the fundamental equations. The disciple of Lucretius may cut and carve his solid atoms in the hope of getting them to combine into worlds; the follower of Boscovich may imagine new laws of force to meet the requirements of each new phenomenon; but he who dares to plant his feet in the path opened up by Helmholtz and Thomson has no such resources. His primitive fluid has no other properties than inertia, invariable density, and perfect mobility, and the method by which the motion of this fluid is to be traced is pure mathematical analysis. The difficulties of this method are enormous, but the glory of surmounting them would be unique.

There seems to be little doubt that an encounter between two vortex atoms would be in its general character similar to those which we have already described. Indeed, the encounter between two smoke rings in air gives a very lively illustration of the elasticity of vortex rings.

But one of the first, if not the very first desideratum in a complete theory of matter is to explain—first, mass, and second, gravitation. To explain mass may seem an absurd achievement. We generally suppose that it is of the essence of matter to be the receptacle of momentum and energy, and even Thomson, in his definition of his primitive fluid, attributes to it the possession of mass. But according to Thomson, though the primitive fluid is the only true matter, yet that which we call matter is not the primitive fluid itself, but a mode of motion of that primitive fluid. It is the mode of motion which constitutes the vortex rings, and which furnishes us with examples of that permanence and continuity of existence which we are accustomed to attribute to matter itself. The primitive fluid, the only true matter, entirely eludes our perceptions when it is not endued with the mode of motion which converts certain portions of it into vortex rings, and thus renders it molecular.

In Thomson's theory, therefore, the mass of bodies requires explanation. We have to explain the inertia of what is only a mode of motion, and inertia is a property of matter, not of modes of motion. It is true that a vortex ring at any given instant has a definite momentum and a definite energy, but to show that bodies built up of vortex rings would have such momentum and energy as we know them to have is, in the present state of the theory, a very difficult task.

It may seem hard to say of an infant theory that it is bound to explain gravitation. Since the time of Newton,