impossible to name the inventor of the more perfect form of the instrument. But taking the precision balance in what is now considered its most perfected form, we may safely say that all which distinguishes it from the common balance proper is, in the main, the invention of the late Mr Robinson of London. In Robinson's, as in most modern precision balances, the beam consists of a perforated flat rhombus or isosceles triangle, made in one piece out of gun-metal or hard-hammered brass. The substitution for either of those materials of *hard steel* would greatly increase the relative inflexibility of the beam, but, unfortunately, steel is given to rusting, and, besides, is apt to become magnetic, and has therefore been almost entirely abandoned. The perforations in the beam are an important feature, as they considerably diminish its weight (as compared with what that would be if the perforations were filled up) without to any great extent reducing its relative solidity. In fact, the loss of carrying power which a solid rhombus suffers in consequence of the middle portions being cut out, is so slight that a very insignificant increase in the size of the minor diagonal is sufficient to compensate for it. *Why* a balance beam should be made as light as possible is easily seen; the object (and it is as well here to say at once, the *only* object) is to diminish the influence of the unavoidable imperfections of the central pivot. To reduce these imperfections to a minimum, the beam in all modern balances is supported on a polished horizontal plane of *agate* or *hard steel* fixed to the stand, by means of a perfectly straight "*knife-edge*," ground to a prism, of hard steel or agate, which is firmly connected with the beam, so that the edge coincides with the intended axis of rotation. In the best instruments the bearing plane is continuous, and the edge rests on it along its entire length; in less expensive instruments the bearing consists of two separate parts, of which the one supports the front end, the other the hind end of the edge. Every complete balance is provided with an "arrestment," one of the objects of which is, as the name indicates, to enable one to arrest the beam, and, if desired, to bring it back to its normal position; but the most important function of it is to secure to *every point of the central edge* a perfectly fixed position on its bearing. So far all modern precision balances agree; but the way in which the *point-pivots* A and B of our fictitious machine are sought to be realised varies very much in different instruments. In Robinson's, and in the best modern balances, the beam is provided at its two extremities with two knife-edges similar to the central one (except that they are turned upwards), which, in intention at least, are parallel to, and in the same plane as, the central edge; on each knife-edge rests a plane agate or steel bearing, with which is firmly connected a bent wire or stirrup, provided at its lower end with a circular hook, the plane of which stands perpendicular to the corresponding knife-edge; and from this hook the pan is suspended by means of a second hook crossing the first, matters being arranged so that, supposing both end-bearings to be in their proper places and to lie horizontally, the working points A′ and B′ of the two hook-and-eye arrangements are vertically below the intended point-pivots A and B on the edges. In this construction it is an important function of the arrestment to assign to each of the two terminal bearings a *perfectly constant* position on its knife-edge. How this is done a glance at figs. 3 and 4 (of which the former is taken from an excellent instrument constructed by L. Oertling of London, and the latter from an equally good balance, represented in fig. 5, made by Messrs Becker & Co., of New York) shows better than any verbal explanation. But what cannot be seen from these sketches is that the range of the arrestment is regulated, and its catching contrivances are placed, so that when the arrestment is at its highest place, the central edge is just barely lifted from its bearing, and the terminal bearings are similarly lifted from their respective knife-edges, so that the beam is now at rest in its normal position. In other balances, as, for instance, in the justly celebrated instruments of Mr Staudinger of Giessen, Robinson's plane terminal bearings are replaced by roof-shaped ones (fig. 6), so that their form alone suffices to secure to them a fixed position on their knife-edges. Another construction (which offers the great advantage of being easy of execution and facilitating the adjustment of the instrument) is to give to the terminal edges the form of circular rings, the planes of which stand parallel to the central edge, and from which the pans are suspended directly by sharp hooks, so that the points A′ and B′ coincide with A and B respectively. In either case the terminal bearings are independent of the arrestment, which must consequently be provided with some extra arrangement, by means of which the beam, when the central edge is lifted from its support, is steadied and held fast in its normal position. In second and third class instruments even the central edge is made independent of the arrestment, by letting it work in a semi-cylindrical or, what is better, a roof-shaped bearing, which, by its form, assigns to it (in intention at least) a definite position.

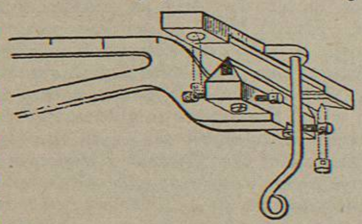

Fig. 3.—Oertling's Balance. End of Beam.

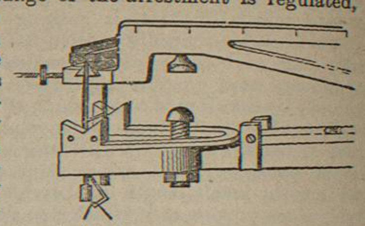

Fig. 4.—Becker's Balance. End of Beam.

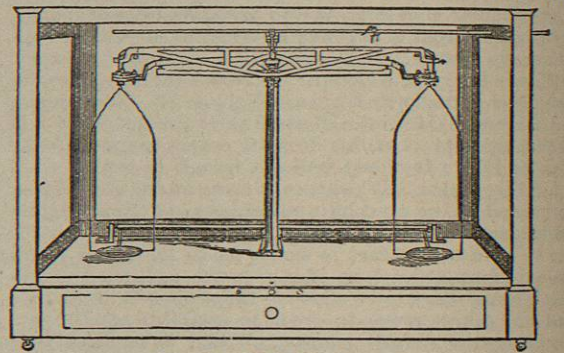

Fig. 5.—Becker's Balance.

Fig. 6.

In order now to develop a complete theory of the precision balance, let us first imagine an instrument, which, for distinctness, we will assume to be constructed on Robinson's model, the knife-edges and bearings, &c., being exactly and absolutely what they are *meant* to be, except that the terminal edges, while still parallel to the axis of rotation, are slightly shifted out of their proper places. Supposing such a balance were charged with $P' = p'_0 + p'$ from the left, and $P'' = p''_0 + p''$ from the right knife-edge,—and it is clear that in this case also the charges may be assumed to be concentrated,—P' in a certain fixed point A on the left, and P'' in a certain fixed point B on the right edge, and, consequently, the statical condition of the balance is the same as if the weights W, P', P'' were all concentrated in one fixed point C_0 (fig. 7), the position of which, in regard to the beam, is independent of the extent to which the latter may have turned, and independent of the direction of gravity. It is also easily seen that in a given beam the position of C_0 will depend only on P' and P'', and supposing P to remain constant it will change its position whenever P'' changes its value. The point C_0 will in general lie outside of the axis of rotation, and consequently there will in general be only two positions of the beam in which it can remain at rest, namely, first, that position in which C_0 lies vertically above, and, secondly, that position in which it lies vertically below the axis of rotation. Only one of these two positions can possibly lie within the angle of free play which the beam has at its disposal. The second of the two positions, if it is within this angle, can easily be found experimentally, because it is the position of stable equilibrium, which the beam, when left to itself in any but the first position, will always by itself tend to assume. The first position, viz., that of unstable equilibrium, is *practically* beyond the reach of experimental determination. Hence the points A, B, and S must be situated so that, at least whenever $P'l' = P''l''$ exactly or very nearly, the beam *has* a definite position of stable equilibrium, and that this position is within the angle of free play. To formulate these conditions mathematically, assume a system of rectangular co-ordinates, X, Y, Z, to be connected with the beam, so that the axis of the Z coincides with the central edge and the origin with the projection O of the centre of gravity on that edge, while the Y-axis passes through the centre of gravity. Let the values of the co-ordinates of the points A, B, S, C_0 (imagined to be situated as indicated by the figure) be as follows:—

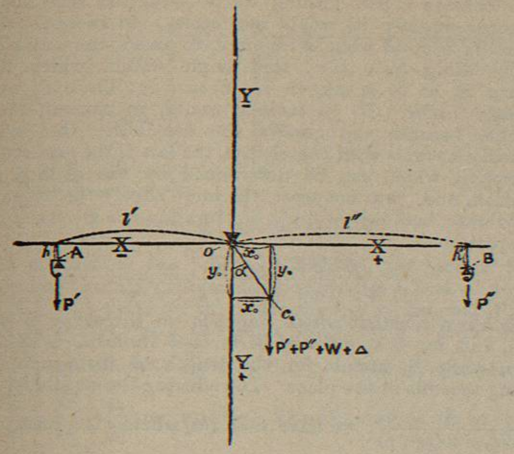

Fig. 7.—Diagram illustrating theory of Precision Balance.

Point	A	B	S	C_0
$x =$	$-l'$	$+l''$	0	x_0
$y =$	h'	h''	s_0	y_0

(The z's are evidently of no practical consequence.) To find x_0 and y_0 we need only again apply the reasoning which helped us in the case of the similar problem regarding the ideal instrument. Assuming, then, first, gravity to act parallel to Y, we have $(P' + P'' + W)\,x_0 = P''l'' - P'l'$. Assuming, secondly, gravity to act parallel to X, we have $(P' + P'' + W)\,y_0 = P'h' + P''h'' + Ws$, $\therefore$ for the distance of the common centre of gravity C_0 of the system from the axis of rotation, $r = \sqrt{x_0^2 + y_0^2}$, and for the angle α through which the needle, supposing it to start from the zero-point, must turn to reach its position of stable equilibrium—

$$\tan\alpha = \frac{x_0}{y_0} = \frac{P''l'' - P'l'}{Ws_0 + P'h' + P''h''} \quad . \quad . \quad . \quad . \quad (3).$$

If, in particular cases, one or more of the points A, B, S should lie above the X-axis, we need only consider the respective ordinates as being in themselves negative, and the equations (as can easily be shown) remain in force. Taking equation 3, together with what was said before, we at once see that if a balance is to be at all available for what it has been made for, and supposing two of the co-ordinates h', h'' to have been chosen at random, the third must be chosen so that, at least whenever P' exactly or nearly counterpoises P'', $Ws_0 + P'h' + P''h'' > 0$. For if it were $= 0$, then, in case of $P'l' = P''l''$, the balance would have no definite position of equilibrium, and if it were negative, y_0 would be negative, and the position of stable equilibrium would lie outside the angle of free play. Obviously, the best thing the maker can do is so to adjust the balance that $h' = h'' = 0$ and $l' = l''$, because then the customary method of weighing (see above) assumes its greatest simplicity, and, especially, the factor with which the deviation of the needle has to be multiplied to convert it into the corresponding excess of weight present on the respective pan assumes its highest degree of relative constancy. We speak of a degree of constancy because this factor can never be absolutely constant, for the simple reason that no beam is absolutely inflexible, and consequently h' as well as h'' is a function of P', and P'' of the form $h = h_0 + \gamma P$, where γ has a very obvious meaning. What is actually done in the adjusting of the best instruments is so to place the terminal edges that, for a certain medium value of $P' + P''$, $h' + h'' = 0$, so that the sensibility of the balance is about the same when the pans are empty as when they are charged with the largest weights they are intended to carry. The condition $l' = l''$ also cannot be fulfilled absolutely in practice, but mechanicians nowadays have no difficulty in reducing the difference $\frac{l''}{l'} - 1$ to less than $\pm \frac{1}{50000}$, and even a greater value would create no serious inconvenience. We shall therefore now assume our balance to be exactly equal-armed; and, substituting for $h' + h''$ the symbol $2h$, and understanding it to be that (small) value which corresponds to the charge, substitute for equation 3 the simpler expression

$$\tan\alpha = \frac{\Delta . l}{Ws_0 + 2Ph} \quad . \quad . \quad . \quad . \quad (4),$$

which, on the understanding that $P'' = P' + \Delta$, and that Δ is a very small weight, gives the tangent-value corresponding to P and Δ. Sometimes it is convenient to look upon the pans (weighing p_0 each) as forming part and parcel of the beam; the equation then assumes the form—

$$\tan\alpha = \frac{\Delta l}{W's' + 2ph} \quad . \quad . \quad . \quad (5),$$

where $p = P - p_0$.

In a precision balance the sensibility, *i.e.*, the tangent-value of the deviation produced by $\Delta = 1$, which is

$$\frac{\tan\alpha}{\Delta} = \text{“}a\text{”} = \frac{l}{W's' + 2ph} \quad . \quad . \quad . \quad (6),$$

must have a pretty considerable value, and at the same time ought to be as nearly as possible independent of the charge. Hence what the equation (4) indicates with reference to a balance to be constructed is, that, so far as these two qualities are concerned, we may choose the weight of the beam as we like; and in regard to the sensibility which the instrument is meant to have when charged to a certain extent, we have even the free choice of the arm-length, because, whatever l or W be, if only the centre of gravity of the empty beam is brought to the proper distance from the central edge, we can give to the sensibility any value we please. What is actually done is so to construct the beam that its centre of gravity lies decidedly lower than one would ever care to have it, and then to connect with the beam a small movable weight (called the "bob") in such a manner that it can be shifted up and down along a wire, the axis of which coincides with the Y-axis, and thus the value s_0 of the distance of the centre of gravity of the beam from the central edge be caused to assume any value, from a certain maximum down to nothing, and even a little beyond nothing. As to the relative independence of the sensibility of the charge, equation 5 shows that a given balance will possess this quality in the higher a degree the less the distance h of the central edge is from the plane of the two terminal ones, and, supposing h to be constant (*i.e.*, the adjustment to be finished), the less the initial sensibility a_0 exhibited by the empty instrument. Passing from one balance to the other, but supposing h and a_0 to remain constant, we readily see that the sensibility is the more nearly independent of the charge p in the pans, the greater the *arm-length* l is. From what has been said above, it would appear that by means of a balance provided with a gravity-bob, we could attain any degree of precision we liked, but evidently this is not possible practically, because in the actual instrument neither the knife-edges and their bearings nor the arrestment are what we have hitherto supposed them to be; and, consequently, both l' and l'' as well as h, instead of being constants, are *variable quantities.* Obviously, the non-constancy of the ratio $l' : l''$ is the most important point, and to this point we shall therefore confine our attention. Let us imagine that the imaginary balance hitherto considered has been charged equally on both sides (with $P = p_0 + p$), so that its normal position is its position of rest, and then assume, first, that the *middle edge* (which hitherto has been an absolutely rigid line) is now a narrow and slightly, but irregularly, curved rough surface. The effect will be, that, supposing the balance to be repeatedly arrested and made to vibrate, the axis of rotation, instead of being constant, will shift irregularly between $x = +\lambda$ and $x = -\lambda$ where λ means a small length. But this comes to the same as if the central pivot *were* absolutely perfect, but had the common centre of gravity C_0, instead of being fixed at $x = 0$, oscillating between $x = \pm \lambda_0$. In other words, the balance may possibly come to rest at any position within a certain angle $\pm \beta$, which, as an angle of deviation, corresponds to the overweight

$$\epsilon_0 = \{2(p_0 + p) + W\}\frac{\lambda_0}{l}.$$

Assume now, secondly, that, say, the right terminal edge was slightly turned so as no longer to be parallel to the middle edge. This *in itself* would not matter much; because although it might produce a change in the length of the right arm, this change would be permanent, and the arm-length again be constant, provided the hook-and-eye arrangement for the suspension of the pan, and the arrestment, were ideally perfect. But, practically, they are *not*, and, moreover, the knife-edge and its bearing are not what theory supposes them to be; and the effect is the same as if the virtual point of application A of the charge $p_0 + p$, instead of being at the constant distance l from the centre, oscillated irregularly between $l + \lambda'$ and $l - \lambda'$, where λ' has a similar meaning to that of λ_0. The joint effect of the imperfections of the three pivots is that the indications of the balance, instead of being constant, are variable within $\pm \epsilon$, where ϵ means a small weight determined approximately by the equation—

$$\epsilon = \frac{1}{l}\{[2(p_0 + p) + W]\lambda_0 + 2(p_0 + p)\lambda\} \quad . \quad (7) -$$

Hence, in a balance to be constructed for a given purpose, l must be made long enough to make sure of its compensating the effects of the λ's, which, for a given set of knife-edges, and a given degree of absolute exactitude in their adjustment, may be assumed to have constant values. Evidently in a given balance ϵ has nothing to do with the sensibility, and consequently it would be useless to increase the sensibility beyond what is required to make the angle β, corresponding to ϵ (*i.e.*, that angle within which the balance is, so to speak, in indifferent equilibrium), *conveniently visible.* To go further would, in general, be a mistake, because the greater the sensibility the more markedly it varies with the charge, the less is the maximum overweight which can be determined by the method of vibration, and, last not least, the more slowly the balance will vibrate, because the time of vibration t is governed by the equation—

$$t = \sqrt{\frac{kW + 2(p_0 + p)}{Ws_0 + 2h(p_0 + p)}} \cdot \frac{l}{\sqrt{R_0}};$$

where k is a constant which depends on the shape of the beam, and for the ordinary perforated rhombus is about $= \frac{1}{3}$, while R_0 stands for the length of the pendulum beating seconds at the place. Introducing the sensibility—

$a = \dfrac{l}{Ws_0 + 2h(p_0 + p)}$, we have $t = c\sqrt{a}$, where c is a constant.

4. *Compound Lever Balances.*—Of these numerous inventions—in all of which a high degree of practical convenience is obtained at the expense of precision—we must content ourselves with noticing two which, on account of their extensive use, cannot be passed over. We here allude, in the first place, to that particular kind of equal-armed lever balances, in which the pans are situated above the beam, and which are known as "*Roberval's balances;*" and secondly, to those peculiar complex *steel-yards* which are used for the weighing of heavy loads by means of comparatively small weights.

In *Roberval's* balance (fig. 8), the beam consists of a parallelogram, in which each of the four corners A, B, A', B' is a joint, and which by means of two joints situated in the centres of the two longer sides AB and A'B' is suspended from a vertical rod so that the two shorter sides AA and BB' under all circumstances stand vertical. With these two sides the pans are rigidly connected; and the main feature in the machine is, that wherever the charge in the pan may lie, *i.e.*, whatever may be the virtual point of application of the whole charge P in regard to the vertical side of the beam, its statical effect is the same as if P was concentrated in a point D in the axis of the rod AA' or BB'. That this really is so is easily proved. Imagine the particle weighing P units to be rigidly connected with, say, AA,' but situated to the left of that line, and, whatever may be its distance from AA', when the beam descends through a certain angle, the vertical projection of the path described by the point D, *i.e.*, its fall h, has the same value whatever its distance from AA'. Hence the work done, say, against an elastic string tending to hold the beam in its place, invariably is $= Ph$ as it would be if D was situated in AA'.

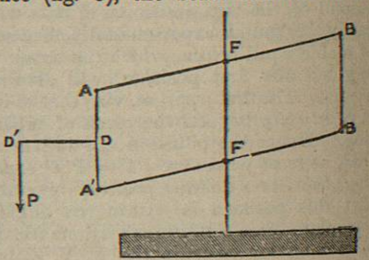

Fig. 8.—Roberval's Balance.

The ordinary *Decimal Balance* is a combination of levers illustrated by fig. 9. a, c, b, d, e, g, h, f, are all joints or pivots; a and h rest on the fixed framework of the machine, and consequently indirectly on the ground; c rests on the lever ab. In the actual machine cd supports the "*bridge,*" which accommodates the load, while at f is suspended a pan for the weight. The pan is so adjusted

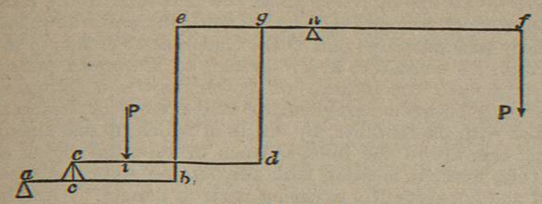

Fig. 9.—Decimal Balance.

that it balances the bridge. Suppose the load P to be placed so that its centre of gravity is at i, and a portion P_c of P will press on the knife-edge at c, the rest P_d will pull at d and, with the same force, at g. Now, $P_c = P.\dfrac{\overline{id}}{\overline{cd}}$, equivalent to $\dfrac{\overline{ac}}{\overline{ab}}\,P_c$ pulling at b or e, equivalent to $P.\dfrac{\overline{ac}}{\overline{ab}}.\dfrac{\overline{id}}{\overline{cd}}.\dfrac{\overline{eh}}{\overline{gh}}$ pulling at g. The dimensions are so chosen that $\dfrac{\overline{ac}}{\overline{ab}} = \dfrac{\overline{gh}}{\overline{eh}}$, hence the effect of P_c at g is equivalent to a weight $P.\dfrac{\overline{id}}{\overline{cd}}$. The other portion of P, viz., P_d, pulls at d, and consequently also at g, with a force $P.\dfrac{\overline{ic}}{\overline{cd}}$. Hence the effect of the total load is equivalent to $P.\left(\dfrac{\overline{ic} + \overline{id}}{\overline{cd}}\right) = P$ units suspended at g, and if, for instance, $gh = \frac{1}{10}hf$, one pound in the pan will counterpoise ten pounds at *any* point of the bridge.

5. *Torsion Balances.*—Of the several instruments bearing this name, the majority are no balances at all, but machines for measuring horizontal forces (electric, magnetic, &c.), by the extent to which they are able to distort an elastic wire vertically suspended and fixed at its upper end. In the torsion *balances* proper the wire is stretched out horizontally, and supports a beam so fixed to it that the wire passes through its centre of gravity. Hence the elasticity of the wire here plays the same part as the weight of the beam does in the common balance. An instrument of this sort was invented by Ritchie for the measurement of very small weights, and for this purpose it may offer certain advantages; but, clearly, if it were ever to be used for measuring larger weights, the beam would have to be supported by knife-edges and bearings, and in regard to such application therefore (*i.e.*, as a means for serious gravimetric work), it has no *raison d'être.* See ELECTRICITY and MAGNETISM.

6. For *Hydrostatic weighing machines* see the article HYDROMETER. (W. D.)

BALANCE OF POWER. The theory of the Balance of Power may be said to have exercised a preponderating influence over the policy of European statesmen for more than two hundred years, that is, from the Treaty of Westphalia until the middle of the present century; and to have been the principal element in the political combinations, negotiations, and wars which marked that long and eventful period of modern history. It deserves, therefore, the attentive consideration of the historical student, and, indeed, the motive cause of many of the greatest occurrences would be unintelligible without a due estimate of its effects. Even down to our own times it has not been without an important influence; for the Crimean War of 1854 was undertaken by England and France for no other object than to maintain the balance of power in Eastern Europe, and to prevent the aggrandisement of Russia by the dismemberment of the Ottoman empire and the conquest of Constantinople. Nevertheless there is, perhaps, no principle of political science, long and universally accepted by the wisest statesmen, on which modern opinion has, within the last twenty years, undergone a greater change; and this change of opinion is not merely speculative, it has regulated and controlled the policy of the most powerful states, and of none more than of Great Britain, in her dealings with the continent of Europe. At the date of the publication of the last edition of this work, the theory of the balance of power was believed to be so firmly established, both by reason and experience, that it was laid down, in the forcible words of Earl Grey, that "the poorest peasant in England is interested in the balance of power, and that this country ought to interfere whenever that balance appeared to be really in danger." At the present time no English statesman would lay down that proposition categorically; and probably no European statesman would be prepared to act upon it. In proportion as the theory of the balance of power has lost much of its former authority, the doctrine of non-intervention has gained strength and influence, and this has been accepted at the present day both by Whig and Tory ministers, so that no strong difference of opinion can at the present time be said to exist in the British nation on the subject. Within the last fifteen years political changes of extraordinary magnitude have been brought about in Europe by force of arms and by revolutions. In former times such changes would certainly have led to a general war, on the principle that it was essential to maintain the relative strength and independence of states, and to support the fabric of European policy. But, under the policy of non-intervention, the effects of these contests have been confined to the states which were directly engaged in them; and the other powers of Europe have maintained a cautious neutrality, which has probably not lessened their own strength, and which has saved the world from a general conflagration.

The theory of the balance of power rested on several assumptions. It was held, more especially from the time of Grotius, in the early part of the 17th century, that the states of Europe formed one grand community or federal league, of which the fundamental principle and condition was the preservation of the balance of power; that by this balance (in the words of Vattel) was to be understood such a disposition of things, as that no one potentate or state shall be able absolutely to predominate and prescribe laws to the others; that all were equally interested in maintaining this common settlement, and that it was the interest, the right, and the duty of every power to interfere, even by force of arms, when any of the conditions of this settlement were infringed or assailed by any other member of the community. The principle can hardly be more tersely expressed than in the words of Polybius (lib. i. cap. 83): "Neque enim ejusmodi principia contemnere oportet, neque tanta cuiquam astruenda est potentia, ut cum eo postea de tuo quamvis manifesto jure disceptare ex æquo non queas." Or, to borrow the language of Fénelon in his *Instructions*, drawn up by him for the guidance of the Duc de Bourgogne, "This attention to maintain a sort of equality and equipoise between neighbouring nations is the security of the general tranquillity. In this respect all neighbouring nations, trading with each other, form one great body and a sort of community. Thus, Christendom is a kind of universal republic, which has its interests, its fears, and its precautions to be taken. All the members