assumption of equal stresses on each side slightly incorrect, and with this type of saddle care must be taken to provide against the wear produced by the motion of the chain. With the second type the use of rollers under the solid saddle leaves the motion of the saddle very free; the resultant pressure on the pier is always sensibly vertical, and the chains may leave the pier at any angle, equal or unequal. The chain must in no case be rigidly attached to the pier unless the support itself is free to rock on its base, as in fig. 29, where

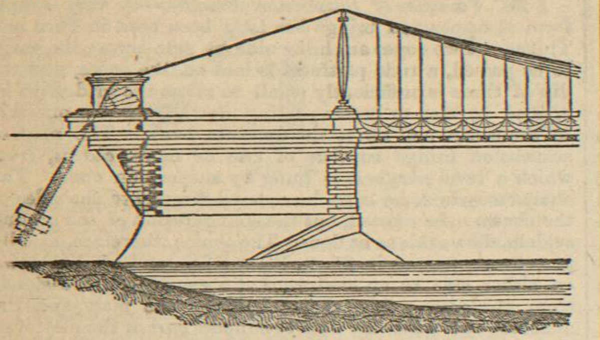

Fig. 29.

the place of the pier is taken by cast-iron struts, working on a horizontal axis.

Suspension bridges are chiefly used for very large spans, because, as we shall find, they can be constructed to carry the same load with a less weight of material than a beam or girder, subject, however, to the disadvantage of flexibility or deformation under a passing load,—a disadvantage which is very serious where, as in small bridges, the passing load is a large proportion of the whole load, but which is of less importance where the chief load to be carried is the weight of the structure itself. We will first consider the usual or simple suspension bridge, as shown in fig. 2, Plate XIX., and will then pass to the various modifications introduced to remedy its defects.

§ 29. *Form of Chain with given Load.*—Let the platform be hung from the chain by equidistant vertical rods; then the load may be treated as hanging from each joint where the rods are attached, and will consist at each joint of the weight of one subdivision of the chain and of one subdivision of the platform and its load. If the position of the vertical tie-rods be assumed as definite relatively to the points of suspension, so that the assumed loads on the joints act at known distances from the points of support, a form of chain, which will remain in equilibrium (or undisturbed) under these loads, can easily be found by the following graphic method:—

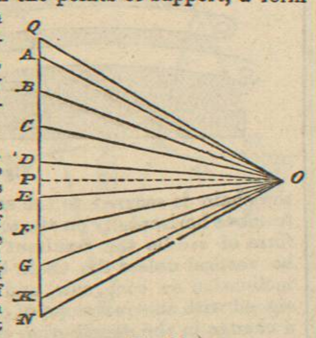

Fig. 30.

Let the vertical line QN (fig. 30) represent the whole load to be carried, and the subdivisions QA, AB, BC, CD, &c., the loads referred to each joint of the chain (QA and NK will be the portion of the load referred directly to the saddle or point of support, and will be simply half the weight of the piece of chain between the saddle and the joint pin). Let QP and NP be the weights carried by each pier,—equal if the distribution of load is symmetrical, otherwise to be determined as for a girder. Let a horizontal line, PO, represent the horizontal component of the tension to be allowed on the chain, or the whole tension on that part of the chain the tangent to which is horizontal; join O with Q, with A, B, &c.; then the lines OA, OB, &c., give the slopes of each successive link as shown in fig. 31, where the line parallel to OA in fig. 30 lies between the two spaces containing the letters O and A in fig. 31, similarly

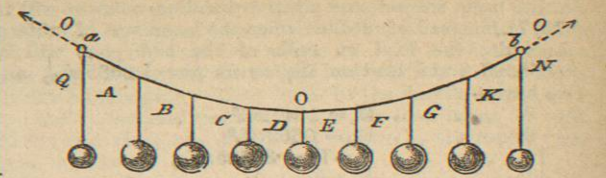

Fig. 31.

the line parallel to OB in fig. 30 is represented by the link between the two spaces lettered O and B in fig. 31, and so forth. The line in fig. 31 lying between O and Q parallel to OQ in fig. 30, represents the direction of the force on the point of support *a*, being equal and opposite to the resultant of the tension on the first link and the weight carried directly by the support. The triangle QOA (fig. 30) is the polygon of forces in equilibrium at the point of support *a* (fig. 31). The triangle OAB is the polygon of forces in equilibrium at the first joint, and similarly each component triangle of fig. 30 represents the equilibrated forces at one joint of the chain in fig. 31. This theorem is one example of the general theory of reciprocal figures, which will be treated hereafter under the general head of "Frames," § 53.

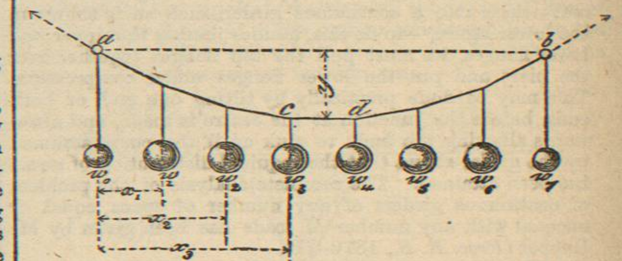

Fig. 32.

When the maximum dip is given instead of the horizontal component of the stress, it is easy to find the latter from the former by the method of moments when the point is known where the chain will be horizontal; for then, let the link *cd*, fig. 32, be horizontal; let the dip be called y, and let the distances of weights w_1, w_2, w_3, &c., from the point of support *a* be called x_1, x_2, x_3, &c., and let the horizontal tension represented by PO in fig. 30 be called H. Then taking moments round the point *a*, we have—

1. . . . . $Hy = \Sigma wx$,

from which H can always be found.

When the length is given of each link (or portion of the chain between the joints where the platform is suspended), and consequently the length of the whole chain, the problem of determining the form assumed under any distribution of load is difficult, for the proportion of the load carried by each pier and the position of each load relatively to the piers vary when the form of the chain varies. The problem may be solved tentatively, but it is seldom attempted.

The converse problem of finding the load which will keep a chain in equilibrium when the dimensions and curve are given is perfectly easy.

From a point O, fig. 30, draw a series of lines parallel to the given links. At any convenient distance, OP, draw a vertical line cutting the lines diverging from O at the points Q, A, B, C, D, &c. The vertical loads required to keep the chain in equilibrium are proportional to the lengths QA, AB, BC, &c.

§ 30. *Relation between the Curve of Bending Moments and the Curve assumed by a Loaded Chain.*—The vertical ordinates at the joints of an equilibrated chain, measured from a horizontal axis passing through the two points of support (these being at the same level), are proportional to the bending moments for similarly chosen sections of a girder similarly loaded.

Let us consider any joint, say that at which w_3 is hanging in fig. 33. Let V be the vertical component of the resultant pull on the left

hand pier, and H the horizontal component. We know that the vertical component is equal to the whole reaction at the pier

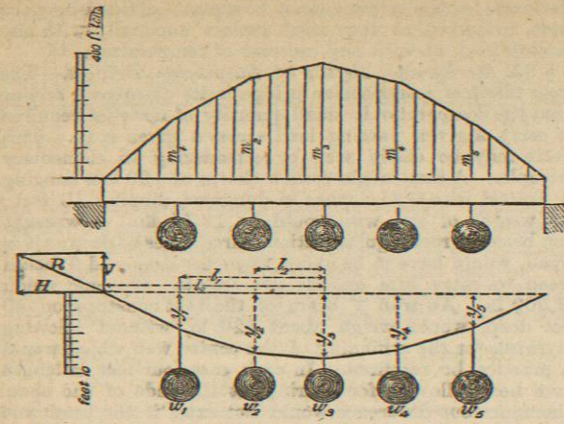

Fig. 33.

when the same load is carried by a girder; then, taking moments about the joint in question, we have—

1. . . . . $Hy_3 = Vl_0 - w_1 l_1 - w_2 l_2$,

but the second member of the equation is the bending moment m_3 for the section at a distance l_0 from the pier in a girder of similar span and similarly loaded, therefore, whatever may be the value of H, the values of y_1, y_2, y_3, &c., are proportional to the bending moments. If, then, a curve of bending moments with the ordinates m_1, m_2, m_3, &c., be drawn for a given distribution of load, we can, with a pair of proportional compasses, construct any number of equilibrated curves, by making the values of y in these curves simply proportional to the values of m in the curve of bending moments, and by selection among these a curve of any required length may be found. If H be unity, the ordinates y_1, y_2, y_3, &c., are equal to the bending moments.

§ 31. *Chain Loaded uniformly along a Horizontal Line.*—If the lengths of the links be assumed indefinitely short, the chain under given simple distributions of load will take the form of comparatively simple mathematical curves known as catenaries. The true catenary is that assumed by a chain of uniform weight per unit of length, but the form generally adopted for suspension bridges is that assumed by a chain under a weight uniformly distributed relatively to a horizontal line. This curve is a parabola.

From equation 1, § 30, remembering that Σwl in this case will be equal to $\frac{wL^2}{8}$, we see that the horizontal tension H at the vertex for a span L (the points of support being at equal heights) is given by the expression—

1. . . . . $H = \frac{wL^2}{8y}$,

or, calling x the distance from the vertex to the point of support,

$$H = \frac{wx^2}{2y}.$$

The value of H is equal to the maximum tension on the bottom flange, or compression on the top flange, of a girder of equal span, equally and similarly loaded, and having a depth equal to the dip of the suspension bridge.

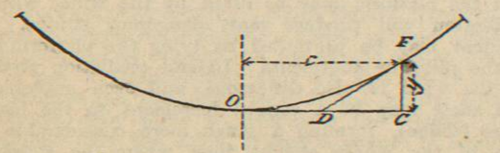

Fig. 34.

Consider any other point F of the curve, fig. 34, at a distance x from the vertex, the horizontal component of the resultant (tangent to the curve) will be unaltered; the vertical component V will be simply the sum of the loads between O and F, or wx. In the triangle FDC, let FD be tangent to the curve, FC vertical, and DC horizontal; these three sides will necessarily be proportional respectively to the resultant tension along the chain at F, the vertical force V passing through the point D, and the horizontal tension at O; hence

$$H : V = DC : FC = \frac{wx^2}{2y} : wx = \frac{x}{2} : y,$$

hence DC is the half of OC, proving the curve to be a parabola.

The value of R, the tension at any point at a distance x from the vertex, is obtained from the equation—

$$R^2 = H^2 + V^2 = \frac{w^2x^4}{4y^2} + w^2x^2,$$

or,

2. . . . . . . $R = wx\sqrt{1 + \frac{x^2}{4y^2}}$.

Let i be the angle between the tangent at any point having the co-ordinates x and y measured from the vertex, then

3. . . . . $\tan i = \frac{2y}{x}$.

Let the length of half the parabolic chain be called s, then

4. . . . . $s = x + \frac{2y^2}{3x}$.

The following is the approximate expression for the relation between a change Δs in the length of the half chain and the corresponding change Δy in the dip:—

$$s + \Delta s = x + \frac{2}{3x}\left\{y^2 + 2y\Delta y + (\Delta y)^2\right\} = x + \frac{2y^2}{3x} + \frac{4y\Delta y}{3x} + \frac{2\Delta y^2}{3x},$$

or, neglecting the last term,

5. . . . . $\Delta s = \frac{4}{3}\frac{y\Delta y}{x}$

and

6. . . . . $\Delta y = \frac{3}{4}\frac{x}{y}\Delta s$.

From these equations the deflection produced by any given stress on the chains or by a change of temperature can be calculated.

If the points of support are not at equal height (fig. 35) call the heights above the vertex y and y_1, and the horizontal distances of

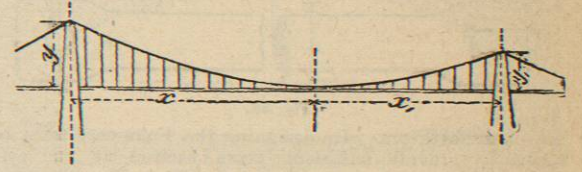

Fig. 35.

the vertex from the points of support x and x_1; let y and y_1 be given, and x, x_1 unknown.

The horizontal stress at the vertex will be the same as if the bridge were composed of two symmetrical halves, each having a span $2x$ and a dip y, or of two symmetrical halves, with a span $2x_1$ and a dip y_1; in other words—

$$H = \frac{wx^2}{2y} = \frac{wx_1^2}{2y_1},$$

hence

7. . . . . $\frac{x^2}{y} = \frac{x_1^2}{y_1}$,

or,

$$x : x_1 = \sqrt{y} : \sqrt{y_1};$$

thus, to find the horizontal position of the vertex, we have only to subdivide the span in the ratio $\sqrt{y} : \sqrt{y_1}$; we may then calculate the strains on one side of the vertex as for half a bridge with the span $2x$ and the dip y, and on the other side of the vertex as for half a bridge with the span $2x_1$ and the dip y_1. The device of piers of different heights may be used with advantage when it is desired to throw a larger portion of the weight of the bridge on one pier than the other, because of a difference in the soundness of the foundations, or for other reasons. The stresses on the loaded and unloaded portion of the chains between the piers and the anchorage are easily determined by methods similar to those which have been given for the stresses on each part of the main span. The same methods also give the direction of each successive link, and of the final links leading to the anchorage.

§ 32. *Practical Details.*—The chains of suspension bridges are either long wire ropes or true chains made of links pinned together. Wire ropes allow the strongest known material to be adopted, namely steel wire, which can be bought in large quantities of a quality which does not break with less than a stress of from 55 to 60 tons per square inch of section; charcoal iron wire of the sizes used will bear 40 tons per square inch; common sizes of wire for the purpose are from 0·16 to 0·14 inches, or say, No. 9 or 10 Birmingham wire gauge. Three or four thousand wires are not unfrequently used in one cable, and it is very essential that each wire shall take an equal part of the whole stress. It used to be thought necessary to ensure this by straining each wire separately either over the actual piers, or piers similarly placed, and binding them together when hanging, strained by their own weight with the dip proposed for the bridge. It was also thought essential that each rope should be an aggregate of parallel wires, not spun as in a hempen rope. Experiment has shown, however, that wire ropes spun with machines which do not put a twist into each wire, but lay it helically and untwisted, and with no straight central wire, are as strong as wire ropes of equal weight made with straight wires. They are, however, much more easily made. A number of ropes of this kind may, therefore, with more convenience and economy be bound together into one cable in the manner previously practised for single wires. Care should be taken to fill every interstice of the ropes with a bituminous compound.

When the chains are made of links of iron their ultimate strength cannot be taken as more than 30 tons per square inch, even if the very best material is secured. It is doubtful if this ultimate strength can at present be surpassed by steel links, for although many steel links of greater strength could certainly be obtained, occasionally a comparatively weak link will be produced even by the

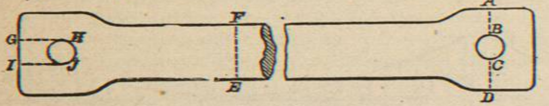

Fig. 36.

best manufacturers. In designing the links care must be taken to provide sufficient cross section at the eye, AB+CD, fig. 36, as well as at EF. The diameter of the pin BC must be such as will allow it to resist the shearing stress on it, and the surface of the pin and eye from B to C must be sufficient to bear the crushing stress. Otherwise, although the pin may not be shorn it may be squeezed flat, and the head of the link may bulge out and be much distorted under the stress. To obtain the necessary surface, without unduly increasing the diameter of the pin. the link may be rolled with a head

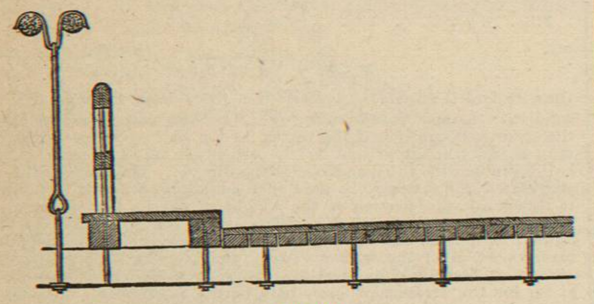

Fig. 37.

broader than the body of the link. The section at GH and IJ must also be sufficient to resist shearing. When two or more parallel chains are used, care must be taken that the rods suspending the platform bear equally on the several chains. Fig. 37 shows a plan of securing this. Chains of unequal dip should not be used to support one platform, for the strain cannot be equally divided between them, inasmuch as they must deflect unequally with any passing load, or with any increase of temperature.

§ 33. *Merits and Defects of Suspension Bridges.*—The great merit of a suspension bridge is its cheapness, arising from the comparatively small quantity of material required to carry a given passing load across a given span. This merit may be easily seen by considering an elementary example. A man might cross a chasm of 100 feet hanging to a steel wire 0·21 inches in diameter, dipping 10 feet; the weight of the wire would be 12·75 ℔. A wrought iron beam of rectangular section, three times as deep as it is broad, would have to be about 27 inches deep and 9 inches broad to carry him *and its own weight.* It would weigh 87,500 ℔. An iron I beam of the best construction, 10 feet deep, would weigh about 120 ℔, without allowing anything for the stiffening of the centre web which would in practice be required. In each case four feet in length have been allowed for bearings at the ends of the span. The enormous difference would not exist if the beam and wire had only to carry the man, although even then there would be a great difference in favour of the wire; the main difference arises from the fact that the bridge has to carry *its own weight.* The chief merit of the suspension bridge does not, therefore, come into play until the weight of the rope or beam is considerable when compared with the platform and rolling load; for although the chain will for any given load be lighter than a beam, the saving in this respect will for small spans be more than compensated by the expense of the anchorages. In large spans the advantage of the suspension bridge is so great that we find bridges on this principle of 800 or 900 feet span constructed at much less cost per foot run than girder bridges of half the span. The disadvantages of the suspension bridge are, however, very great. A change in the distribution of the load causes a very sensible deformation of the structure; for the chain of the suspension bridge must adapt its form to the new position of the load, whereas in the beam the deformation is hardly sensible, equilibrium being attained by a new distribution of the stresses through the material. This flexibility of the suspension bridge renders it unsuitable for the passage of a railway train at any considerable speed. The platform rises up as a wave in front of any rapidly advancing load, and the masses in motion produce stresses much greater than those which could result from the same weights when at rest; moreover, the kinetic effect of the oscillations produced by bodies of men marching, or even by impulses due to wind, may give rise to strains which cannot be foreseen, and which have actually caused the failure of some suspension bridges. On the 16th of April 1850 a suspension bridge at Angers gave way when 487 soldiers were passing, and of these 226 were killed by the accident. Another danger peculiar to suspension bridges is that the platform may be lifted by the wind, when its oscillation will produce most dangerous strains. This accident may be prevented by tying the platform down to the piers or abutments. Lateral oscillation produced by the wind is also dangerous, and even gathered ice and snow may be a serious increment to the load on these bridges, forming a much more considerable fraction of the whole weight than where the supporting structure is itself massive. Suspension bridges must be well cross-braced to resist the action of the wind. They can be much stiffened laterally by placing the chains in inclined planes, converging downwards to the platform.

§ 34. *Modifications of the Simple Suspension Bridge.*—Many efforts have been made to design a bridge which

shall combine the lightness of the true suspension bridge with the stiffness of the girder. Mr Dredge's design with sloping rods (fig. 38) gives a somewhat stiffer structure

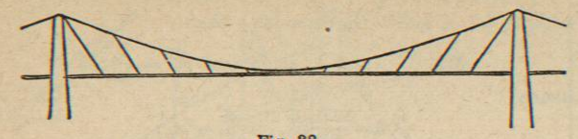

Fig. 38.

than the bridge with vertical suspension rods; the inclined rods throw a strain on the platform, which must be resisted either by ties along the central portion, or by struts abutting against the piers. The stresses on each part will be shown under the heading "Compound Structures" (§ 62).

The design fig. 39 has been proposed by many, but is

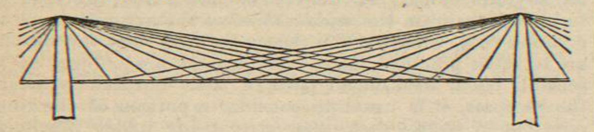

Fig. 39.

worthless. The object of the proposers is to support each part of the platform by rods which are quite independent of other parts of the structure, and which, being originally straight, do not alter their form under stress. The unequal stretching of the long and short rods under a stress, or with a rise of temperature, is a radical defect. Mr Ordish has proposed a plan in which the road is supported by sloping tie rods, arranged like the struts in fig. 87, inverted. Flexible chains, like that of an ordinary suspension bridge, carry the weight of these tie rods by vertical rods, which keep the sloping rods straight. The chain in this form is not subjected to unequal loading. Various forms of bridge have been proposed, in which, as in figs. 76 and 78, two chains are braced together. These may be made thoroughly stiff bridges, with a moderate increase in the amount of metal required for the flexible bridge. They will be described under the head of "Frames." Stiffness has also been obtained in some structures by using an auxiliary girder to stiffen the platform. This is best effected by the use for each chain of two girders, each half the length of the platform. These girders are placed as in fig. 40, being hinged together by a strong pin at B, and

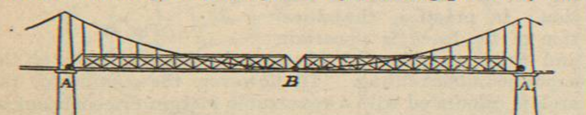

Fig. 40.

held down by pins at A and A_1, which should, however, be left free to move horizontally. These girders are not sensibly strained by the rise and fall of the chains due to a fall or rise of temperature; they can also deflect freely as a whole when the chain is deflected under strain; nevertheless, they serve to distribute the weight of a passing load over the chain, so that it cannot be sensibly distorted. Rankine has given the following rule for designing these stiffening girders. Let w_1 be the greatest rolling load per foot run; let x be the half span of the chain; let M be the greatest bending moment which the auxiliary girders will have to resist (*i.e.*, at the centre of each); let F be the greatest shearing force (at the end and central pins), then

1. . . . . . $M=\frac{1}{16}w_1x^2$,

and

2. . . . . . $F=\frac{1}{4}w_1x$

Each auxiliary half girder is in fact to be designed as a beam of half the span of the bridge, and capable of carrying half the passing load per foot run (but not its own weight). This plan of stiffening is quite effective, but adds considerably to the weight and cost of the whole structure; for not only have we to provide these extra girders, but extra material in the chains to carry this extra dead load.

§ 35. *Maximum Span.*—If we assume that wire can be obtained which will safely bear 15 tons per square inch, a rope (or single wire) with a dip of $\frac{1}{14}$th of the span would safely bear its own weight over a span of about one mile, and would not break till the span exceeded 4 miles. With a dip of $\frac{1}{7}$th of the span a steel wire rope of the best quality would not break until the span exceeded 7 miles. These lengths are not given as indicating practical spans for bridges, but to show the limits which with our present materials cannot be exceeded, however light the passing load may be.

IV. The Arch.

§ 36. *General Description.*—An arch may be of stone, brick, wood, or metal. The oldest arches are of stone or brick. They differ from metal or wooden arches, inasmuch as the compressed arc of materials called the *ring* (fig. 41, London Bridge), is built of a number of separate pieces

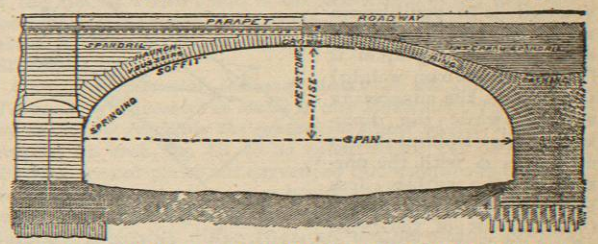

Fig. 41.—Half Elevation and Half Section of Arch of London Bridge.

having little or no cohesion. Each separate stone used in building the *ring* has received the name of *voussoir*, or archstone. The lower surface of the ring is called the *soffit* of the arch. The *joints*, or bed-joints, are the surfaces separating the voussoirs, and are normal to the soffit. A brick arch is usually built in numerous rings, so that it cannot be conceived as built of voussoirs with plane joints passing straight through the ring. The bed-joints of a brick arch may be considered as stepped and interlocked. This interlocking will affect the stability of the arch only in those cases where one voussoir tends to slip along its neighbour. The ring springs from a course of stones in the abutments, called *quoins*. The plane of demarcation between the ring and the abutment is called the *springing* of the arch. The *crown* of the arch is the summit of the ring. The voussoirs at the crown are called *keystones*. The *haunches* of the arch are the parts midway between the springing and the crown. The upper surface of the ring is sometimes improperly called the extrados, and the lower surface is more properly called the intrados. These terms, when properly employed, have reference to a mathematical theory of the arch little used by engineers. The walls which rest upon the ring along the arch, and rise either to the parapet or roadway, are called *spandrils*. There are necessarily two outer spandrils forming the faces of the bridge; there may be one or more inner spandrils. The *backing* of an arch is the