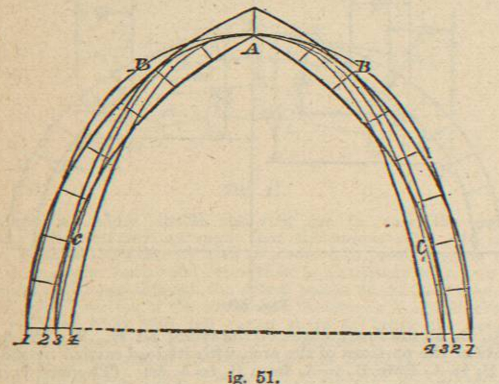
the ring itself, keeping the masonry joints radial. The portion of the arch below the joint of rupture B is often considered as part of the abutment.

If the load at the crown of an arch were very light, and the load at the haunches comparatively very heavy, the series of tentative curves drawn with various values of h would assume the character shown in fig. 51. The longest curve which can now be fitted into the ring (drawn with a thick black line in the figure) will probably approach the soffit at the crown and be tangent to the upper surface of the ring at the haunch. In fig. 51 the longest linear arch is shown as tangent to the soffit at C. This condition could seldom be secured; with most loads the linear arch tangent to the back of the ring at B will *cut* the soffit at C. Nevertheless, the value of h to be provided for will be that given by the linear arch tangent to the soffit. If this arch leaves the middle third at B, the ring must be thickened or efficient backing provided at this point. If the abutment yield an arch thus loaded would fail, as in fig. 51a, but the case very seldom arises in practice. If the arch were not pointed at A, but curved so as to contain the linear arch near the crown, the piece BAB would be lifted up as a whole without breaking at A.



ig. 51.

The joints of rupture can be found for unsymmetrical loads as well as for symmetrical loads, but these joints will then not be at equal distances from the crown.

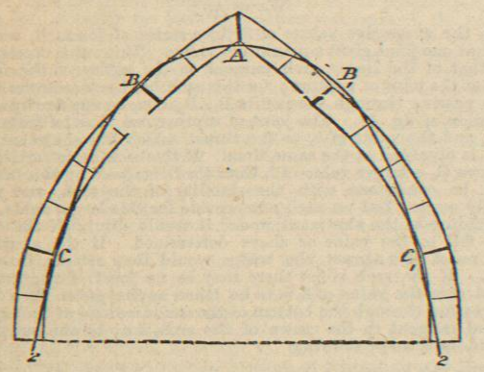


Fig. 51a.

If the middle third of the ring be alone treated as effective, the designer, after finding the joint of rupture for a bridge of the usual form and with usual loads, need make no further calculation as to the arch *above* that joint. A linear arch which is tangent to the soffit at the joint of rupture, and to the upper surface of the ring at the crown, will probably lie within the ring at intermediate joints, and will cut them at an angle not differing much from a right angle; but the linear arch must be carried on below the joint of rupture, through the backing and the abutments, to see that it is nowhere too much inclined to the bedding joints, and never comes too near the edge of the effective masonry. The horizontal thrust determined by finding the joint of rupture on the hypothesis that the middle third of the ring is the only effective part will be a safe value; but the actual value may be considerably less, since the actual linear arch called into play may lie outside the middle third. Since we do not know the actual position of the resultant pressures on each voussoir, any refinement in calculating the maximum intensity of stress due to these resultants would be useless. If the actual horizontal thrust were known, it would be easy to determine the couple acting on each joint and due to the distance between the resultant pressure and the centre of resistance of the joints; then knowing this couple and the total thrust it would be equally easy by the principles in § 8 to determine the maximum intensity of stress. Practically the thickness of the arch ring is determined by rules derived from experience, and the chief use of the above theory is to determine the dimensions of the abutments; if, however, with a given load the joint of rupture were found much nearer the crown than the positions indicated above, it would be well to rearrange the permanent loads or to alter the form of the ring.

§ 42. Professor George Fuller of Belfast has communicated the following novel and very neat method of finding the linear arch of maximum rise (and therefore of minimum thrust) which can be drawn within the middle third of a given ring.

In fig. 52 let the dotted curves GI and HK bound the middle third of the ring. Let the span be divided into any convenient

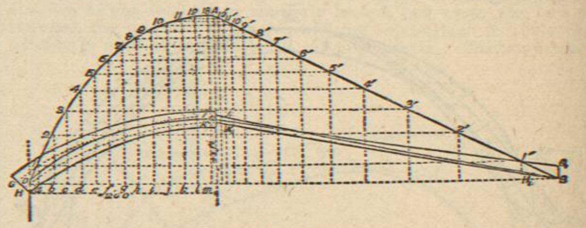


Fig. 52.

number of parts at a, b, c . . . &c. Let the load on the half arch be subdivided into a corresponding number of parts, and each partial load referred to the vertical line passing through a, b, c . . . &c. Let the curve D 1 2 3 . . . A be a curve of bending moments for these loads, drawn to any convenient scale. This curve will also (§ 30) be a linear arch for the given loads. Draw the straight line AB at any convenient inclination, cutting the horizontal line DB at B. Raise the verticals $a1$, $b2$, $c3$, . . . &c., from the points 1, 2, 3, &c. Where these cut the curve DA draw horizontal lines, cutting AB at 1′, 2′, 3′, . . . &c. Since the ordinates of all possible linear arches are merely multiples or submultiples of the curve of bending moments, it follows that any other straight line from B to the vertical through A will have ordinates, which, if measured from DB along the verticals passing through 1′, 2′, 3′, &c., will be the ordinates of a linear arch, set off on the corresponding verticals passing through 1 and a, 2 and b, 3 and c, &c. AB might be called the development of the linear arch DA. Now let the curves CI and HK be developed in a similar way, so that, for instance, the ordinates measured from a to these curves are equal to the ordinates measured on the vertical passing through 1′ from DB to the developments I_1 G_1 and H_1 K_1; then it is clear that for the given loads any linear arch which lies within the middle third of the ring must, when developed, be represented by a straight line lying within the area I_1 G_1 H_1 K_1, and consequently that the straight line BC, which starts from the lowest point B in this area, and is tangent to the curve G_1 I_1, will be the development of the curve of maximum rise and minimum thrust which can be drawn within the given middle third with the given loads. The line BC determines the point C, and the ordinates of BC give the ordinates of the curve DC, the ordinates being measured on corresponding verticals.

The example shown is the diagram for an arch of 52 feet span and 10 feet 4 inches rise, the depth of the ring at the crown being 2 feet 6 inches, and at the springing 3 feet 8 inches. The loads per foot of breadth, beginning at a, are—23·04, 19·45, 16·35, 13·94, 12·01, 10·35, 8·98, 7·93, 6·32, 5·79, 5·52, 5·39 cwts The rise of the linear arch found is 10·2 feet, and h per foot of breadth of the arch 129 cwt. $= \frac{1315 \text{ ft. cwt.}}{10·2 \text{ ft.}}$

§ 43. *Empirical Expression for the Thickness of the Ring.*—The ring when not of equal thickness is always made of least depth at the crown. The depth of the key stone is therefore the thickness of the ring at its smallest part.

Let D be this depth in feet, and r the radius of the arch in feet at the crown. Then we may take (Trautwine)—

1. . . . . $D = C\sqrt{r}$.

According to Rankine, C may be taken as ·346 for a single arch, and ·413 for one of a series of arches. The reason for making one of a series thicker than a single arch is, that the former has, when not loaded itself, to bear part of the thrust from its neighbours when these are loaded; this thrust tends to throw the linear arch in the unloaded span low down in the keystone. The following is another series of values of C in practical use:—

For first class stonework	C = ·36
,, second class stonework	C = ·4
,, brick and rubble	C = ·45

Perronet gives the following rule:—Let L be the span in feet—

2. . . . . $D = 1 + \frac{L}{30}$.

Rankine, *Civil Engineering*, p. 427, shows that Trautwine's rule is rational. Perronet's can only be so when the usual proportion of rise to span is adopted.

Brickwork arches of 24 feet span and less are made 1 foot 6 inches deep at the crown; 30 feet span, 1 foot 10½ inches; 40 feet span, 2 feet 3 inches. The usual flat arch of these dimensions has its ring increased by two rings of bricks towards the haunches. These do not show on the face being concealed by the spandrils. Rubble arches are made a little thicker.

§ 44. *Practical Details.*—The strongest and simplest form of arch is a flat circular arc, having a rise of about one quarter of the span. In these arches the springing is above the place where the joint of rupture would occur if the ring were prolonged. Those parts of an elliptical or semicircular arch which lie below the joint C, fig. 49, are of use chiefly to improve the appearance of the arch. They are virtually part of the abutment, which is sometimes even considered as extending to the joint B. In a very flat arch the linear arch may be brought to coincide more truly with the axis of the ring by lightening the haunch, with which object the roadway is sometimes carried on small flat arches turned at right angles to the main arch, and having the spandrils of the main arch as abutments.

The joints between the voussoirs should be very evenly worked, so that the pressure may be evenly distributed. In brick joints the layers of mortar should be thin. Great care should be taken to provide for the drainage of the roadway above the arch. With this object the masonry should be covered with a sheet of asphalt sloping down to the piers or abutments, and suitable drains must be provided to collect the water and discharge it through the pier or abutment.

SKEW ARCHES have already been treated of under the general head ARCH (vol. ii. p. 330).

Considerable attention must be given to the construction of the *centres* or wooden frames on which the voussoirs rest while the ring is in process of being built. Extreme rigidity is necessary, and this rigidity is best attained by adopting one of the three following plans (Rankine):—1. Direct supports as in fig. 53, illustrating Hartley's centre for the bridge over the Dee at Chester (total span 200 feet); 2. Inclined struts in pairs as shown in fig. 54, being a diagram of the centre used in the erection of Waterloo Bridge; 3. Trussed wooden girders, of which an example is afforded by the truss used in the erection of London Bridge, fig. 55.

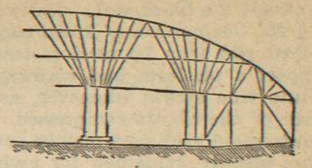


Fig. 53.

Figure 55 shows the *striking plates* and *wedges* by which the centre is lowered after the completion of the arch. The upper and lower plates A and B are strong

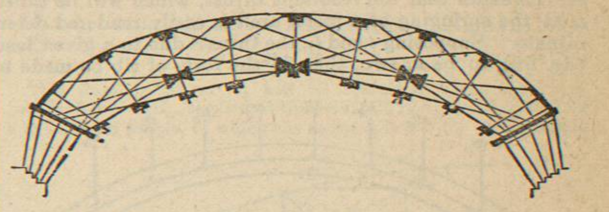


Fig. 54.

beams suitably notched, and are separated by the compound wedge C; this wedge is kept in its place by cross wedges shown in section in the figure. When the centre

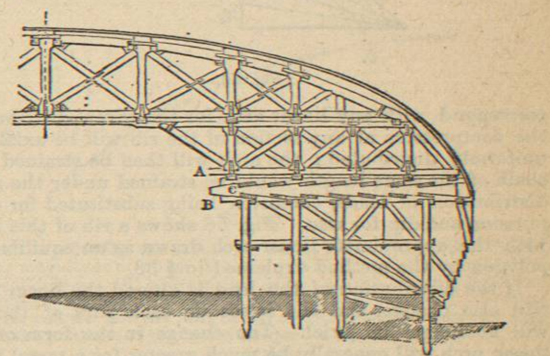


Fig. 55.

is to be lowered these cross wedges are knocked out, and the main wedge C driven back. Owing to defective centering some large French arches sank much during construction, and owing partly to this cause, and partly, as it would appear, to defective mason-work, the total deformation after the centres had been struck was most extraordinary. In Perronet's bridge at Neuilly (*vide* Table XVII., § 84) the sinking, while the centre was in its place, amounted to 13 inches, and after the centre was struck a further sinking took place of 9½ inches. The crown of the centering had a radius of 150 feet, but the sinking of the arch was such that for 60 feet it assumed the form of an arc of a circle with a radius of 244 feet. It is remarkable that the bridge, built in 1774, of very bold design and so imperfectly executed, still stands. When the centres of Waterloo Bridge were removed no arch sank more than 1½ inches. Centres have occasionally been supported on strong sacks full of sand. To lower the centre the sand was allowed to escape through apertures in the sack. It is believed that this method was first employed by a French engineer, M. Beaudemoulin. The canvas sack has been advantageously replaced by wrought iron boxes or troughs; the block supporting the centre acts as a lid resting on the sand inside; when the sand is allowed to escape the block sinks slowly down inside the box.

§ 45. *Comparison of Metal with Masonry Arches.*—Metal arched ribs may be used instead of rings of masonry to support a platform and roadway. These arched ribs constitute true arches whenever, as is generally the case, all parts of the rib are compressed. The principles by which the stress on each part may be computed do not differ from those already explained for arches of masonry, but it is possible to calculate the stresses with much greater exactitude for continuous metal ribs than for voussoirs. With voussoirs we have seen that the resultant thrust at the springing is indeterminate both in magnitude and position, but we shall see hereafter that the resultant thrust, which will be called t, at the springing of a metal arch is easily rendered determinate. Supposing t and t_1, the thrusts due to a given load (fig. 56), to be known, then if the form of rib be made to

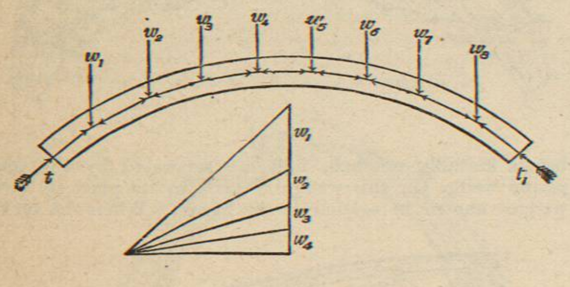


Fig. 56.

correspond with any linear arch for the given distribution the compression at any section of the rib will be axial and uniformly distributed; the arch will then be strained as a chain of the same length would be strained under the same distribution of loads, extension being substituted for compression and dip for rise. Fig. 56 shows a rib of this kind with the approximate linear arch drawn as an equilibrated polygon by the method explained in § 38.

If the distribution of the load is altered the linear arch will also change, and the stress on each part of the rib will no longer be axial. The change in the form of the linear arch will generally be much greater for a metal than for a masonry arch, because most metal arches have light open spandrils and a light roadway, so that the passing load is considerable in comparison with the permanent load. Not improbably the linear arch, when only one haunch of a metal rib is loaded, may pass quite outside the rib for a portion of its length if this rib is made, as is usually the case, of a form containing the linear arch for a symmetrically distributed load. On the other hand, it does not follow, as with masonry, that because the linear arch passes outside the rib the bridge will fail. The bending couple then produced can be resisted by the moment of the elastic forces of the cross section of the rib if the rib is made strong enough. In masonry the joints open so soon as the resultant pressure passes outside the middle third of the ring; the couple required to produce equilibrium would then require a negative force or tension at the opposite edge, and masonry cannot supply this tension, but in a metal rib the couple of bending moment produced by the excentricity of the stress may be resisted by the stiffness of the rib acting as a beam subject to a bending moment. Thus the strength of an arch to resist flexure is a more important element in the metal rib than in the masonry structure. It would be false to say that the ring of voussoirs had *no strength* to resist flexure, for we have on the contrary seen that the moment of the elastic forces at any section of a stone ring does resist any distorting action produced by the load; but in masonry this moment should never exceed the comparatively small value consistent with the absence of tension on any part of any joint. The metal rib may with safety be subjected to considerable tension in parts, and its strength to resist flexure can be easily increased and can be calculated with certainty. Moreover, by hinging the rib at one or both springings, as can be done with metal, the problem of determining the horizontal thrust (or total thrust) is simplified, the position of the thrust being thereby rendered certainly axial at this point, and then by taking into account the actual deformation of each part of the rib a complete solution of the problem of its strength can be obtained.

§ 46. *Horizontal Thrust of a Metal Arch or Rib hinged at the Abutments.*—By supporting a rib on pins or in cylindrical bearings (*vide* fig. 62) at the abutments we determine two points traversed by the thrust. The effect of allowing free rotation is necessarily to render the bending moment *nil* round the centre of rotation. Hence the resultant thrust must traverse the centre of the pin, or the centre of curvature of the bearing. Knowing the point of application of the thrust we have now to determine its magnitude. The vertical component v is the same as the load on the pier of a girder of the same span equally and similarly loaded, so that the problem reduces itself to the determination of h the horizontal component.

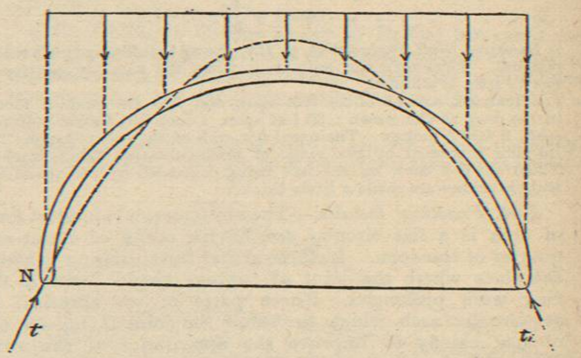


Fig. 57.

Let us first consider a semicircular rib (fig. 57), bearing a load uniformly distributed along the horizontal platform of the bridge (neglecting the weight of the rib). The linear arch will pass through the centre of the bearings N and Q, and will be a parabola. Moreover, it will be that parabola which requires the rib to exert no internal forces due to its own elasticity, and tending either to push out or draw in the springings; in other words, the rib, being supposed in equilibrium before the application of the weights, will not tend to act as a spring to increase or diminish the opening between N and Q. Nevertheless, as the semicircle cannot coincide with the parabola, most parts of the rib must be subject to bending moments, against which it will react as a bent spring. When the linear arch, as shown in fig. 57, passes above the axis of the rib at the crown, and below it at the haunches, the upper portion of the bent rib will act as a spring, tending by its reaction to diminish the distance between the ends N and Q, while the portions near the springing will be so bent as by their reaction to tend to increase that distance; now, if, as is necessarily the case, the whole rib is not to act as a spring, tending either to close or open the ends N and Q, then the effect of the bending near the haunches must exactly neutralize the effect of the bending near the crown. We have now to find what direction of thrust at the springing will give a linear arch such that the above condition may be fulfilled.

Let M be the bending moment acting at any given section, the centre or neutral axis of which is at a height y (fig. 58) above the horizontal line joining the springings; let this moment be considered constant for a short length ΔL of the rib measured axially along the rib; let Δs be the short distance measured horizontally by which the moment M acting throughout the length ΔL would

increase or diminish the span of the rib at the springings, the rest of the rib being assumed free from strain. Then calling I the moment of inertia of the cross section of the rib, and E the modulus of elasticity, we shall have—

$$1. \quad . \; . \; . \; . \; . \; . \quad \Delta s = \frac{My.\Delta L}{IE},$$

as appears from the following considerations:—

In fig. 58, let $Oo = Aa = Bb = \Delta L$; conceive the surface at AB as fixed, and let the action of the couple M be such as to extend the

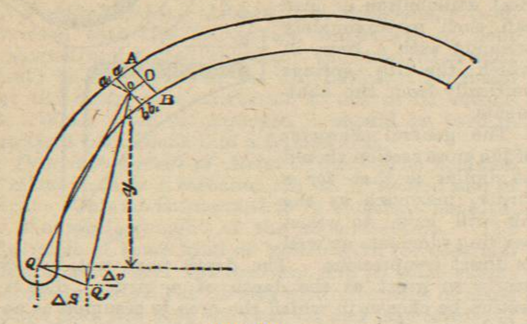


Fig. 58.

top and compress the bottom of the rib, moving the point a to a_1, and the points b to b_1; then, calling p the intensity of the stress at a, we have (equation 4, § 14)—

$$M = \frac{2p_1 I}{d};$$

also we have (§ 24)—

$$aa_1 = \frac{p_1 \Delta L}{E};$$

and therefore—

$$aa_1 = \frac{M\Delta L}{EI}\frac{d}{2};$$

join o and Q and draw the line oQ_1, making the angle QoQ_1 equal to aoa_1; at Q draw $\overline{QQ_1}$ perpendicular to $\overline{o}Q$. Then the effect of the couple M on the length ΔL of the rib, the rest being unstrained, will be to move the point Q to Q_1, and by similar triangles we have—

$$\overline{aa_1} : \overline{QQ_1} = \frac{d}{2} : \overline{oQ};$$

and therefore—

$$\overline{QQ_1} = \frac{M.\Delta L}{E.I} . \overline{oQ}.$$

Then resolving the motion $\overline{QQ_1}$ into horizontal and vertical components Δs and Δv, we have by similar triangles, $\Delta s : QQ_1 = y : oQ$, or, as above—

$$\Delta s = \frac{My.\Delta L}{I.E}.$$

But if, as above stated, the rib does not act as a spring in either direction, the span will remain constant, and the sum of all the changes in span produced by all the successive lengths ΔL will be *nil*, or $\Sigma \Delta s = 0$. Hence, since E is constant, we have

$$2. \quad . \; . \; . \; . \; . \; . \; . \quad \Sigma \frac{My.\Delta L}{I} = 0$$

as a necessary condition for the equilibrium of a loaded rib, hinged at the abutments when these do not yield. This condition must be satisfied whatever be the form or load of the rib, the reasoning by which it was obtained being independent of the form either of the rib or linear arch. When the cross section of the rib is constant, we have—

$$3. \quad . \; . \; . \; . \; . \; . \; . \quad \Sigma My.\Delta L = 0.$$

We shall now proceed to show how the linear arch satisfying this condition can be found for the case of a uniform rib.

Let the line $OO_1O_2O_3O_4$, fig. 59, be the geometrical axis of the rib, and let $OC_1C_2C_3C_4$ be the linear arch required; this arch will, as shown above, cut the geometrical axis at some point, as at O_3.

Let $\overline{C_4C}$ represent the resultant pressure on the rib at any point C_4 in direction and magnitude, fig. 60. If this pressure be resolved into its vertical and horizontal components, the latter C_4H will be equal to the horizontal thrust h (constant throughout the linear arch since the loads are vertical).

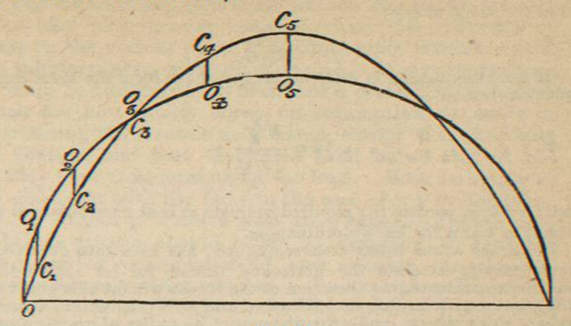


Fig. 59.

This force applied at C_4 will be equivalent to an equal and parallel force, $O_4H_{,,}$, applied at the point, O_4 in the axis, added to a left handed couple, of which the moment is $h\,\overline{O_4C_4}$. This couple

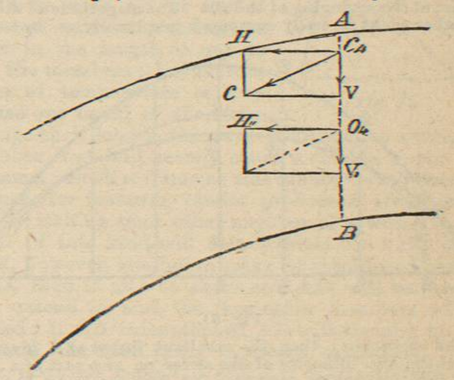


Fig. 60

is, for the point O_4, the couple M* required for equations 1, 2, and 3, the magnitude of which, h being constant, is proportional to the vertical distance OC between the curves at any point. Equation 3 requires that the sum of all the values of My shall be equal to zero, and we now see that this condition results when the sum of all the products of OC into y is equal to zero, or when—

$$4. \quad . \; . \; . \; . \; . \; . \; . \quad \Sigma y . OC = 0;$$

when the cross section is not constant the value of $\frac{y.OC}{I}$ must be substituted for the simple product $y.OC$.

The problem of discovering the actual linear arch which will be called into play with a given rib is now reduced to that of finding the linear arch fulfilling the condition in equation 4. We might proceed tentatively, drawing numerous linear arches, and selecting by trial that which most nearly fulfils the condition, as proposed by Mr Bell, *Proc. I.C.E.*, vol. xxxii., but Professor Fuller of Belfast has shown, *Proc. I.C.E.*, vol. xl., that the ordinates of the required linear arch can at once be calculated from the values of the bending moment at the several sections of a beam of equal span and similarly loaded. Let $og_1g_2g_3$ (fig. 61), be the curve of bending moments which, as was shown in § 30, is one form of linear arch corresponding to the given load, the lengths $\overline{oo_1}$, $\overline{o_1o_2}$, $\overline{o_2c_3}$ &c., being equal and representing ΔL; let y as before be any ordinate of the curve $oo_1o_2o_3$, the axis of the rib; let $\overline{sg}$ be any ordinate of the given curve of bending moments; let $\overline{sc}$ be any ordinate of the required linear arch. Then, since $\overline{oc} = \overline{sc} - y$, we have for the case of uniform cross section and hinged abutments the equation—

$$5. \quad . \; . \; . \; . \; . \quad \Sigma y\,(\overline{sc} - y) = 0, \text{ or } \Sigma y.\overline{sc} = \Sigma y^2;$$

* This couple is sufficient to shift the force from C_4 to O_4, but the resultant of the force at C_4 and the couple would not be tangent to the geometrical axis of the rib. To alter the direction of the force in this manner a vertical component must be added, but this vertical component may be looked upon as a shearing force, which, being vertical, tends neither to extend nor to diminish the span.