Case 4. Warren girder; load on half the top joints, fig. 70.

In designing a Warren girder it is necessary to provide for the advancing load. With a uniformly distributed load the

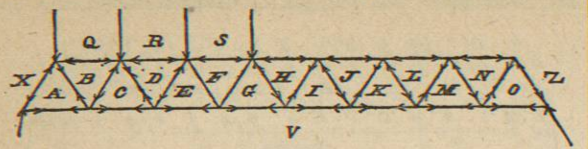

Fig. 70

diagonal bracing consists of alternate struts and ties beginning, if supported at the bottom, with a strut at each end, or with the tie at each end if the truss is hung from the top. At the centre there will be either two ties or two struts in juxtaposition; or, as in the case where there are an even number of loaded joints at the top, the central pair of diagonal members will be unstrained; but an advancing load (neglecting the permanent load) will convert each diagonal member (except the end ones) alternately into a strut and a tie. It is necessary to provide for the maximum extension and maximum compression on each, taking into account the combined effects of the permanent and passing load; the latter has generally sufficient effect to reverse the direction of the stress on the bracing near the centre, but not (in large bridges) towards the ends. Each member of the bracing towards the centre must, therefore, be so designed as to be capable of acting alternately as a strut and as a tie. The maximum stress on the top and bottom members occurs when the bridge is fully loaded.

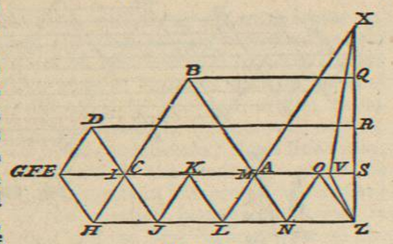

Fig. 70a.

This stress is easily calculated by the method of sections; assume the girder cut by a vertical plane at the joint or pin opposite the member in question, or in other words, by a vertical plane passing through the vertex of the triangle of which the member in question is the base; let d be the perpendicular distance between the member and the pin, t the thrust or tension on the member, and M the bending moment for the section calculated as for a girder loaded at the points corresponding to the joints. Then $M = td$. The stress on the diagonals corresponds to the shearing stress in the solid girder, and may consequently be calculated from the shearing diagrams, examples of which were given in § 19. The loads may be referred to the joints before drawing the diagram, and the continuous curves of figures 18 to 18c will be replaced by lines consisting of a series of steps such as are shown in fig. 18a. We may then proceed to calculate the stress on any diagonal as follows:—Take the shearing stress for the section at the joint where the diagonal in question abuts; if there is a load on this joint, take the shearing stress for a section close to the joint, and on that side

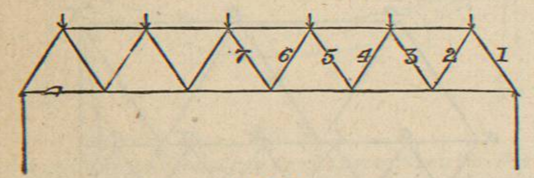

Fig. 71.

of it which is next the diagonal in question; call the shearing force thus found F; let i be the slope of the diagonal or the angle which it makes with the horizon, then the tension or compression on the diagonal is F cosec. i; thus let fig. 71 represent a Warren girder of six equal bays, in which $i = 60°$. Let the load on each top joint be 5 tons, the compressions and tensions on the diagonals are as follows:—

Table XI.

Name of Brace.	F × cosec. i.	Compression.	Tension
1	15×1·1547	17·32	...
2	10×1·1547	...	11·55
3	10×1·1547	11·55	..
4	5×1·1547	...	5·77
5	5×1·1547	5·77	...
6	0×1·1547	...	...

These stresses, given simply as an example, apply to the one special load, and are not to be confounded with the maximum stresses which an advancing load of the same intensity would produce. The stresses on the two halves of the girder are symmetrical.

The arithmetical mode of computation is the simpler where the top and bottom members are parallel and the inclination of the diagonals constant; where these conditions are not fulfilled the method by reciprocal figures is preferable.

In the actual design of any girder to suit various combinations of loads, care must be taken to design each member to suit the *maximum* stress which can arise from any combination. The maximum shearing stress is most easily selected by means of the diagrams § 19. Care must be taken to meet both the maximum tension and maximum compression whenever the member is so placed that with some loads it is extended and with some compressed. We have just shown that this case arises in the diagonals near the centre of the girder. When frames are used as continuous girders, it is desirable to make the points of inflection coincide exactly with a joint. This may be done by cutting through or omitting the member opposite the joint. This allows the reactions on each pier to be easily determined by the elementary principles of statics with any load, and without taking into account the form which the beam assumes when deflected. Instead of a long continuous girder, we then have a series of girders, supported at or near the middle of their length by the piers, while a second series hang from the first by pins at determinate points. This arrangement greatly simplifies all the calculations without sensibly diminishing the advantage derived from the use of continuous girders.

§ 55. *Various Forms of Girder.*—The framed girder is sometimes made of the form in fig. 72, which has the

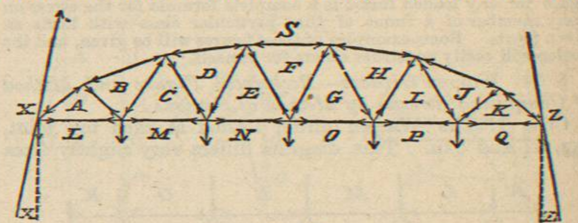

Fig. 72.

advantage of reducing the length of the end diagonals where the stress is heaviest. The reciprocal figure for this truss uniformly loaded on the bottom joints is shown in fig. 72a. This girder is sometimes called a bowstring girder, though this name more properly belongs to an obsolete form with no diagonals. Z and X are the spaces between the end loads and the reactions which, for clearness, are shown as pulling up; but the same reciprocal would result if the reaction were shown pushing up, and the letters X and Z were placed as dotted. This form has the advantage of reducing the compression on the diagonal struts to a comparatively small amount. A great part of the shearing stress is taken by the curved upper boom; deeper girders can be profitably used of the bowstring than of the Warren type. The long and expensive struts in the latter form more than counterbalance

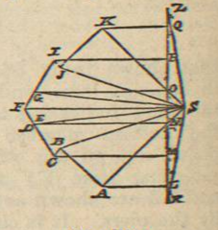

Fig. 72a.

the saving to be obtained by increasing the depth beyond about $\frac{1}{12}$th of the span.

The diagonal bracing in the Warren and bowstring girders is sometimes arranged as in fig. 73; the girder is

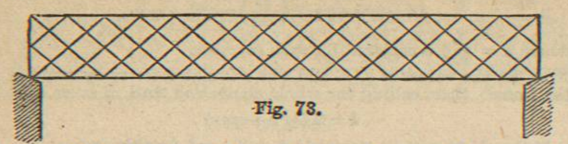

Fig. 73.

then called a lattice-girder. It is to be treated as a series of superposed Warren girders, each bearing its share of the load. This form has a slight advantage inasmuch as the diagonal struts are stiffened by being pinned to the ties where they cross.

Fig. 74 shows a very common and useful form of girder

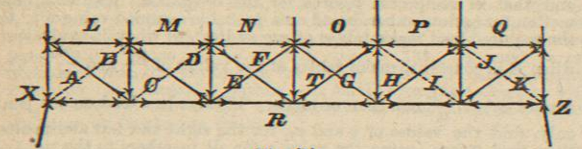

Fig. 74.

where the uprights between the top and bottom members are able to sustain both tension and compression, while the diagonals are only "semi-members," being flexible rods or bars. The reciprocal for this design is shown in fig. 74a with two joints more heavily loaded than the others. The members in fig. 74 which have no arrows on them are idle or unused, with this particular distribution of load. When the permanent load is considerable as compared with the passing load, the end or two end diagonals shown dotted will not come into action with any distribution of load, and may therefore be omitted. This form is much used when wood is employed for the compression members. It has in every case the advantage that the struts in the bracing are shorter than in the Warren girder. The lettering is arranged so that only those spaces have letters which are divided by members actually in use under this particular load.

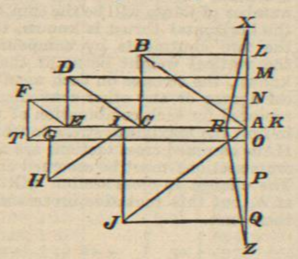

Fig. 74a.

Figs. 75 and 75a show a modification of this truss for small spans, and its reciprocal when loaded on one joint;

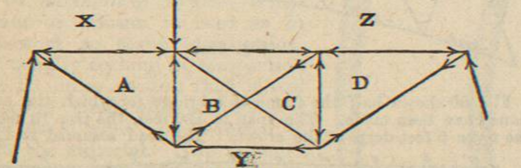

Fig. 75.

the lettering here also only suits the one distribution of load, and the idle members have no arrows on them.

§ 56. *Framed Suspension Bridges and Arches.*—These frames, like the girders, consist of top and bottom members, braced together by ties and struts. The bridge is a suspension bridge, if the frame is supported by inclined forces pulling outwards from the bridge as in fig. 76, and an arch is supported by forces pushing inwards as in fig. 77. The reciprocals of these two forms with the joints at the platform uniformly loaded are annexed. These reciprocals are drawn on the hypothesis that the direction of the thrust or pull is known; and this has been chosen in this case so as to reduce the stress on NG to zero, as would necessarily be the case if NG were omitted or cut. When this is not the case the direction of the thrust or pull at the abutments must be found in the manner explained below.

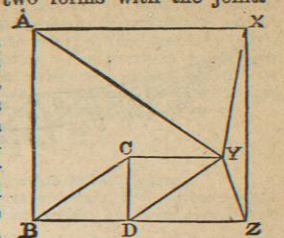

Fig. 75a.

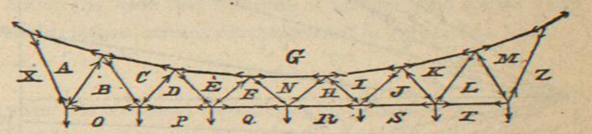

Fig. 76.

It is easy to design the bridge so that both the top and bottom members of the suspension bridge remain in tension, and both those of the arch in compression under all distributions of load. This would allow wire rope to be used for both members of the suspension bridge, and cast-iron or steel for both members of the arch. The stresses on the bracing are very uniform and small as compared with those on the diagonals of a girder.

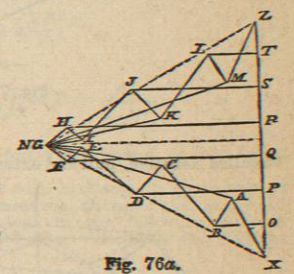

Fig. 76a.

Fig. 78 shows a slight modification of the design for a

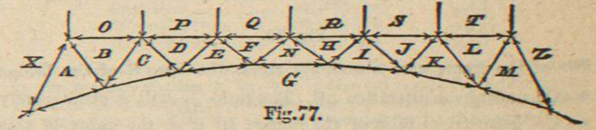

Fig. 77.

suspension bridge, very suitable for spans so large that the end struts in the preceding form would be inconveniently long. The reciprocal annexed is drawn for the case in which a double load is placed on half the bridge. The same design is suitable for an arch.

The resultant thrust or tension at the supports of framed suspension bridges or arches can only be found by a method analogous to that already explained for the solid metal rib. This method was first given by Prof. Clerk Maxwell.

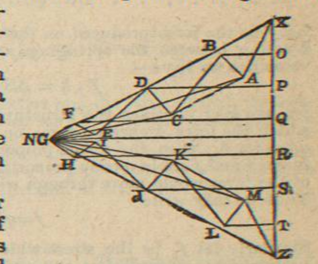

Fig. 77a.

Consider any member A, fig. 79, of length L and cross-section a, made of a material having the modulus of elasticity E; under the action of a stress F, the length L will be altered by an amount—

$$\Delta L = F \frac{L}{Ea}.$$

Appropriate signs must be given to the arithmetical values of the force and the alteration of length; thus thrust and compression may

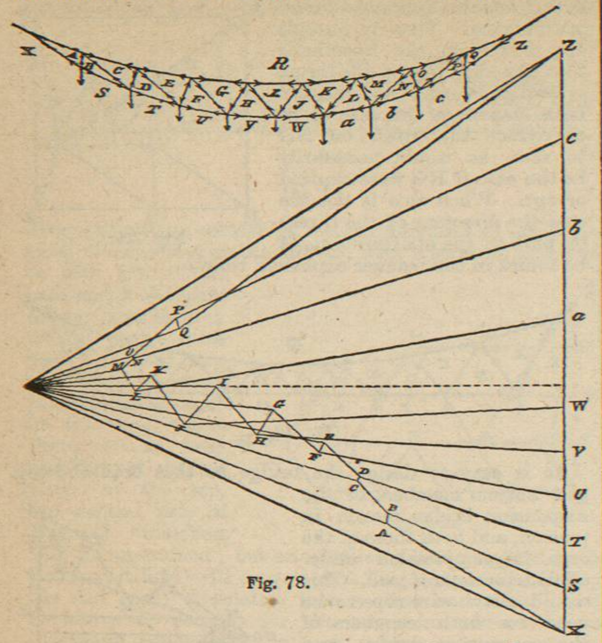

Fig. 78.

be called negative, pull and extension positive. If every other mem-

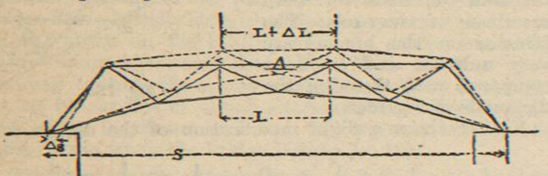

Fig. 79.

ber of the frame were absolutely rigid, the span S of the bridge would undergo an alteration ΔS. The ratio $\frac{\Delta S}{\Delta L}$ will depend merely on the geometrical form of the frame; let q be the value of this constant ratio, so that—

1\. . . . . $\Delta S = q\Delta L = qF\frac{L}{E.a}$.

Let f be the force produced on the member A by a horizontal force h acting between the springings; then by the principle of virtual velocities we have—

$$f : h = \Delta S : \Delta L,$$

for we may consider the structure merely as a kind of lever, by which the force h exerts a force on A, the fulcrum being the joint opposite A; then the above proportion expresses the fact that the forces h and f are inversely proportional to the spaces which the ends of the lever would move through when a small displacement occurs. Thus we may write

$$f = qh.$$

Similarly let f_1 be the stress which would be produced on A by a vertical force v, applied at one springing, while the other springing was held rigidly so that the whole frame could not turn; the ratio between f_1 and v is constant, depending merely on the form of the frame, so that we may write—

$$f_1 = pv,$$

where p is a constant, to be found in the same manner as q; p may be defined as equal to the whole stress produced on the given member by a unit *vertical* force at the springing, and q as the whole stress produced on the same member by a unit *horizontal* force at the springing, the frame being held rigidly at the opposite abutment.

Then any thrust t at the springing having the vertical and horizontal components v and h will produce a stress F on A, equal to the sum of the stresses of f and f_1, or—

$$F = qh + pv.$$

Substituting this value of F in equation 1 we have—

2\. . . . $\Delta S = (q^2h + pqv)\left(\frac{L}{Ea}\right) = eq^2h + epqv,$

where $e = \frac{L}{Ea}$, a constant for each member.

Now, let the values of q, p, and e be calculated for every member of the frame: then, calling the whole elongation $\Sigma\Delta S = k$, we have—

$$k = \Sigma\,(eq^2h + epqv).$$

If the abutments do not yield $k = 0$, and for this case—

3\. . . . . $h = \frac{\Sigma epqv}{\Sigma eq^2}$.

v is to be found as for a girder similarly loaded, and t, the required thrust or tension, is the resultant of h and v. The calculation is best made as follows:—Construct tables of the values of d and q for each member of the frame; the method of sections or moments will answer best for the top and bottom members, and that of reciprocal figures for the diagonals; assume a cross section for each member, based on a probable assumed value of t; for the required load make tables of epq and eq^2, or what is equivalent, when E is constant, make tables of the values of $\frac{pq^2}{a}$ and $\frac{q^2L}{a}$. The sum of eq^2 or $\frac{q^2L}{a}$ can then be made. If there be a load on one joint only, find the values of v and v_1, for the right and left abutments, then find $\Sigma epqv$, using the value v for all members to the right of the load, and v_1 for all members to the left of the load; equation 3 will now give the value of h for this single load.

The process of finding $\Sigma epqv$ must be repeated for each joint which is loaded, and the whole horizontal thrust due to the load on any number of joints will be the sum of the separate values of h. When the horizontal thrust is known, the thrusts t and t_1 are obtained at the two abutments by compounding the horizontal thrust with the vertical weight borne at that abutment. When t and t_1 are known, the stresses on each member are to be computed by reciprocal figures or any other convenient method. The process must be repeated for each combination of passing and permanent load, so as to find the maximum stress to which any member can be subjected. If the assumed cross sections are not suitable for these stresses, fresh cross sections must be assumed and the whole calculation repeated. The change in cross section will cause some change in the values of h, but this tentative process need seldom be gone through more than once.

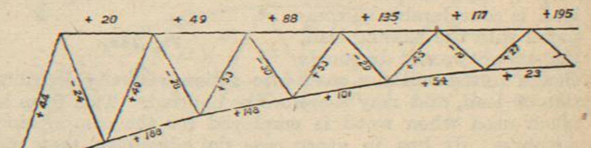

Fig. 80.

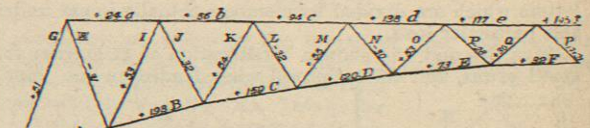

Fig. 80*a*.

Fig. 80 shows half the frame of a bridge for which the calculations have been made. The span is 120 feet, the rise 12 feet, and the truss 5 feet deep at the crown. The load assumed is 10 tons

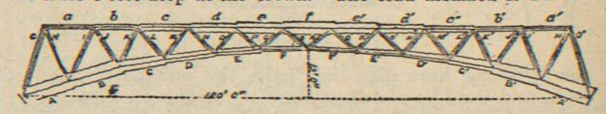

Fig. 81.

permanently on each top joint, and 10 tons of passing load. On fig. 80 are marked the stresses when the bridge is wholly covered with the passing load. On fig. 80*a* are marked the maximum stresses with any distribution of load. Figs. 81 and 82 show graphi-

cally the amount of metal required in this bridge and in a girder of the same span and 12 feet deep; the breadth given in the diagram to each member is proportional to the cross section required. The quantity of metal required for the girder exceeds that for the arch in the ratio of about 175 to 100; a similar calculation for a bowstring

Fig. 82.

lattice girder 17 feet deep at the centre gives the ratio between the weights of metal required as 100 for the arch and 155 for the bowstring. Even these ratios understate the great advantage to be derived from the braced arch or suspension bridge for large spans, since they assume that the loads to be carried are the same, whereas the permanent load is in large spans much less for the lighter construction of bridge. The following table shows the probable weight in tons of several different types of trusses, assuming that the maximum intensity of stress on the metal in girders and arches is everywhere 4 tons per square inch, that the passing load to be carried is 1·4 tons per foot run, and that the practical weight is 25 per cent. in excess of the minimum possible weight if no metal were wasted, vide *Trans. R.S.S.A.*, vol. viii. p. 135. It must be remembered that the abutments for arches will in all cases be more expensive than those for girders; in small spans this expense will often outweigh the saving which could be effected in the superstructure by employing the arch.

TABLE XII.—*Weights of Trusses of different types.*

Span.	Weight of wrought iron Girder in tons.		Weight of wrought iron Bowstring, in tons.		Weight of wrought iron Suspension Bridge, and wrought iron or cast iron Arch, in tons.		Weight of wooden Arch, in tons.		Weight of Suspension Bridge, in tons. Strain 8 tons per square inch.	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
Feet.	Total.	Per ft. run.	Total.	Per foot run.	Total.	Per ft. run.	Total.	Per ft. run.	Total.	Per ft. run.
200	83	·415	70	·35	41	·205	57·1	·285	19	0·095
300	222	·74	180	·60	100	·33	142	·473	45	0·15
400	475	1·19	375	·94	194	·485	285	·712	83	0·207
500	940	1·88	700	1·4	330	·65	510	1·02	134	0·268
600	2,220	3·7	1260	2·1	530	·883	870	1·45	200	0·333
700	3,900	5·57	2290	3·27	790	1·13	1440	2·06	280	0·400
800	12,800	16·	4450	5·56	1180	1·475			384	0·48
1000					2490	2·49			655	0·655
1200					5600	4·67			1060	0·885
1400									1580	1·128
1600									2360	1·475
1800									3410	1·89
2000									4950	2·475

The framed arch is a very suitable form for wooden bridges. The ties are few and subject to insignificant strains. In designing a series of arches supported on piers care must be taken to provide for the thrust from loaded to unloaded arches across the pier at the springing.

§ 57. *Strength of Struts.*—When a strut or column is used as in framework to resist compression, it is usually so long in comparison with its cross section that it will bend and yield with a much less stress than would be required to crush the material. The strength of a strut of this kind can be approximately computed according to the following theory:—

Fig. 83.

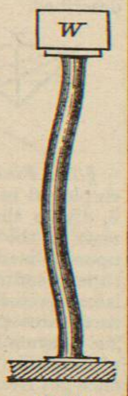

Fig. 84.

Let a strut with a cross section S be pinned or hinged at both ends, or let it have round ends, so that if it yields under compression it bends as in fig. 83. Let the cross section have two axes of symmetry, and let the section be such that the column will bend in the plane of one axis; let the depth of cross section measured in this direction be called d, and the breadth measured at right angles to the depth d be called b. Let the maximum or breaking load be called P, and the maximum deflection of the longitudinal axis of the strut from its unbent position be called v, this quantity being analogous to the deflection in a beam. The moment tending to produce flexure at the cross section where v is measured will be Pv. This moment must, as in the case of a girder, be equal to the moment of the elastic forces, which we already know to be—

$$u = \frac{2p_1 I}{d},$$

where p_1 and I have the same signification as for girders.

(I must be taken about the unstrained axis, or in other words about the axis running in the direction which has been called that of the breadth.) Hence we have—

$$Pv = \frac{2p_1 I}{d};\ \text{or}\ p_1 = \frac{Pvd}{2I};$$

but we also know that for beams of uniform cross section under similar internal stresses v is proportional to $\frac{l^2}{I}$; hence we have—

1\. . . . . $p_1 = a\frac{Pl^2}{I}$,

where a is a constant depending on the material only. Let $p_0 = \frac{P}{S}$ be the mean intensity of stress which would be produced if the load compressed all parts of the cross section equally, and let f_c as before be the ultimate strength of the material per unit of cross section, then, when the beam is on the point of yielding, we must have $f_c = v_0 + p_1$, or—

$$f_c = \frac{P}{S} + a\frac{Pl^2}{I},$$

and calling r the radius of gyration—

2\. . . . . $f_c = \frac{P}{S}\left(1 + a\frac{l^2}{r^2}\right)$, or $P = \frac{f_cS}{1 + \frac{al^2}{r^2}}$

from which f_c can be computed in terms of P and a or P in terms of f_c and a.

The value of a does not change with a change in the design of the cross section; the values for cast iron, wrought iron, and wood determined by experiment are given in the following table, S being measured in square inches, P in lbs. The table also contains values of f_c.

TABLE XIII.—*Constants for the Strength of Struts and Pillars.*

	f_c.	a.
Cast-iron	80,000 lbs.	$\frac{1}{6400}$
Wrought iron	36,000 lbs.	$\frac{1}{36000}$
Dry timber	7,200 lbs.	$\frac{1}{3000}$

When the direction of both ends of the column is fixed it must bend in the manner shown in fig. 84; if the curvature be uniform this would have the effect of reducing the deflection v to one-fourth of the amount caused by the same stresses when the ends are hinged, giving—

$$p_1 = \frac{aPl^2}{4I},$$

and

3\. . . . $f_c = \frac{P}{S}\left(1 + \frac{al^2}{4r^2}\right)$, or $P = \frac{f_cS}{1 + \frac{al^2}{4r^2}}$.

When one end is fixed in direction and the other hinged, the ultimate load which the strut can bear may be taken as a mean between the strength of two pillars of the same length and cross section, one having both ends fixed in direction and the other having both ends hinged; for similar cross sections r^2 is proportional to d^2, the square of the depth (measured in the direction of the shorter side or axis), thus the ratio $\frac{l^2}{r^2} \propto \frac{l^2}{d^2}$.

Let $n = \frac{d^2}{12r^2}$, and let $B = 3\frac{al^2}{d^2}$. Then from equation 2 we have for hinged ends—

4\. . . . . $P = \frac{fS}{1 + 4nB}$,

and from equation 3 for fixed ends—

5\. . . . . $P = \frac{fS}{1 + nB}$.

n has a constant value for all similar cross sections, and B a constant