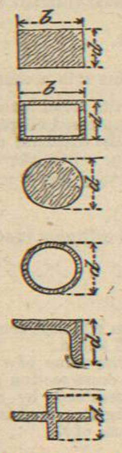
value for any given materials and any constant ratio of l to v. Table XIV. gives the values of B for a series of these ratios, and Table XV. gives the values of n for the most usual cross sections.

TABLE XIV.—*Value of* B (*lbs. and square inches*), *Strength of Struts and Pillars.*

$\frac{l}{d}$	B for Cast Iron.	B for Wrought Iron.	B for Strong Dry Timber.
10	·187	0·033	·25
15	·42	·075	·9
20	·75	·133	1·6
25	1·16	·208	2·5
30	1·69	·3	3·6
35	2·3	·408	4·9
40	3·	·533	6·4
45	4·68	·833	10·

TABLE XV.—*Values of* n, *Strength of Struts and Pillars.*

	n
Square or side d, or rectangle with smallest side d..............	1
Hollow rectangle, thin sides	½
Circle, diameter d	1⅓
Thin ring, external diameter d	⅔
Angle iron, smallest side d...........	2
Cruciform, smallest breadth $=d$......	2



It is of great importance that the connections between the several struts and ties forming a frame should be so designed that the stresses produced may be *axial.* If this is not done the maximum intensity of stress on the strut or tie may greatly exceed those computed on the principles explained in the present paragraph (*vide* § 8).

Mr Unwin in his lectures gives the following empirical rules for the strength of wrought iron struts:—

$$p_1=\frac{fS}{1+\frac{l^2}{ch^2}} \text{ for fixed ends, } \quad . \quad . \quad . \quad . \quad . \quad 6.$$

$$p_1=\frac{fS}{1+\frac{4l^2}{ch^2}} \text{ for round ends, } \quad . \quad . \quad . \quad . \quad 7.$$

where different values are given to f and c according to the different cross sections of the struts.

Rectangular bars	$c=2500$	$f=17$ tons.
Cylindrical bars	$c=3500$	$f=17·5$,,
Angle, T, cross, and charcoal iron	$c=900$	$f=19$,,

The following are Mr Hodgkinson's formulæ for the strength of cast-iron cylindrical pillars, the length of which is not less than thirty times the diameter:—Let P be the load which will produce failure, d the external diameter in *inches*, L the length in *feet*, and A a constant multiplier; then for solid pillars with either round or flat ends—

$$8. \quad . \quad . \quad . \quad . \quad P=Ad^{3·6}\div L^{1·7}.$$

The value of A for rounded ends is 14·9 tons, and for flat ends 44·16 tons. Similarly for hollow pillars of internal diameter d, we have

$$9. \quad . \quad . \quad . \quad . \quad P=A(d^{3·6}-d_i^{3·6})\div L^{1·7},$$

where for rounded ends A is taken as 13 tons, and for flat ends 44·3 tons.

When the length is less than thirty times the diameter, let P be the ultimate load calculated by equations 8 and 9, and let P_i be the load which would crush a short block of the same sectional area S; *i.e.*, let $P_i=49S$; let P_{ii} be the actual ultimate strength, then, according to Professor Hodgkinson's experiments,

$$10. \quad . \quad . \quad . \quad . \quad P_{ii}=\frac{4PP_i}{4P+3P_1}.$$

For rectangular struts of oak and pine, the smallest side being denoted as before by d Hodgkinson gives the formula—

$$11. \quad . \quad . \quad . \quad . \quad . \quad . \quad P=A\frac{d^2}{l^2}S,$$

where A=3,000,000 lbs. The same unit must be employed for d and l, and S must be expressed in square inches. This formula can only give a rough approximation to the truth. In short beams the formula in § 5 would give a smaller strength than equation 11. This smaller value is, then, the true measure of the strength.

VI. COMPOUND STRUCTURES.

§ 58. Many bridges have been built with superstructures such that the stresses on the several parts or members cannot be computed by the rules hitherto given. These superstructures are generally constructed by superposing two or more types so as to form a compound structure capable of acting at once say as an arch and as a girder. These bridges may be called compound bridges. The designs are usually unworthy of imitation. Mr Robert Stevenson's original design for the Britannia Bridge, in which the great girder would have been partly supported by chains, is an example of this type of structure in which the two parts are clearly visible. Many wooden American bridges are trusses which almost defy analysis, the designs being, however, obviously suggested by an attempt to combine at least two of the three main types of bridges. No advantage whatever is gained by a combination of this kind; on the contrary great disadvantage is almost sure to follow its adoption, namely, that it will be impossible that each part of the structure should, under all circumstances, carry that portion of the load which the designer entrusted to it. For suppose a bridge constructed partly as a girder and partly as a suspension bridge, the girder being very stiff and deep and the chain perfectly flexible with considerable dip. Let the chain and girder be each fit to carry half the passing load. It is perfectly conceivable that the deflections of the two should be so different that the girder would, under the actual load, break before the chain was sensibly strained, or the difference in the relative dip of the chain and depth of the girder might be such as to cause the former to give way first. Even if the two were so designed that at a given temperature each should take the designed share of the load, a change of temperature would entirely alter the proportion borne by the two parts of the structure. A few forms are free from this defect, and these will now be described.

Fig. 85.

§ 59. *Fink Truss.*—This truss (fig. 85) has been much employed in America. The upright struts, numbered 1 to 7, divide the span into eight equal parts. If a weight w rests on the top of each strut the whole truss may be looked upon as made up of seven distinct and independent trusses superposed on one another; strut 4 is used seven times, and is compressed with a total force of $4w$. Struts 2 and 6 are used three times, and each compressed with a total force of $2w$. Struts 1, 3, 5, and 7 are used once, and each compressed with a force w. The stresses on the inclined ties are at once got from the compression of the strut by the resolution of forces; and the stress on the upper member or boom is the sum of all the pulls on the ties resolved horizontally;

the boom is uniformly strained over its whole length. This truss would not be so light as a Warren girder if both were made of the same depth, and if the end struts in the Warren girder did not require to be much stiffened in consequence of their great length. The Fink truss is, however, generally made in practice more cheaply than the Warren girder, because the depth of a girder practically depends on the greatest length of strut which is admissible, and for equal lengths of strut the Fink truss gives a deeper beam than the Warren girder.

Fig. 86.

§ 60. *Bollman Truss* (fig. 86).—This truss is the result of superposing seven simple frames consisting of a top member, a strut, and two ties. The stresses are easily computed. It is one form of the old false suspension bridge already alluded to (§ 34), with the difference that the top member replaces the horizontal resistance at the points of support. The defect of this truss is that two ties supporting any strut except the central one are of unequal length; expansion or extension, consequently, affects these unequally. It is inferior to the Fink truss.

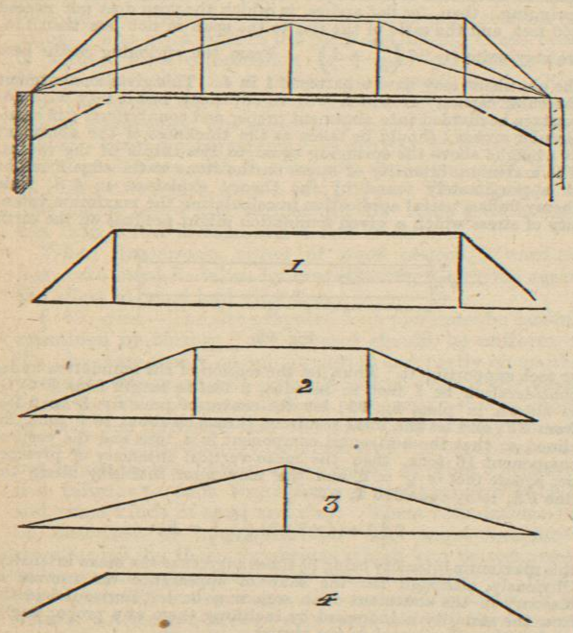


Fig. 87.

§ 61. *Schaffhausen Truss.*—The famous wooden bridge of Schaffhausen (fig. 87, see also § 76) is in its main parts a compound bridge, composed by superposing a series of simple frames of the type shown in 1, 2, 3, and 4. Nos. 1 and 2 are imperfect frames, *i.e.*, if the joints were flexible they would collapse in consequence of the want of the diagonals across the centre parallelogram. The stiffness of the joints supplies this want.

§ 62. *Dredge's Suspension Bridge.*—This bridge differs from the usual suspension bridge in having the suspending rods inclined, and in the use of a lower member, which may be a compression member transmitting a thrust to the piers, as in 88a, or a tension member, as in 88b, with a

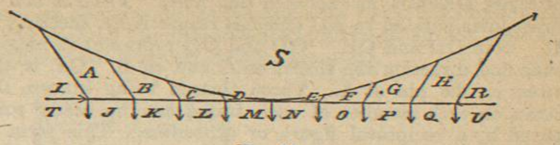


Fig. 88a.

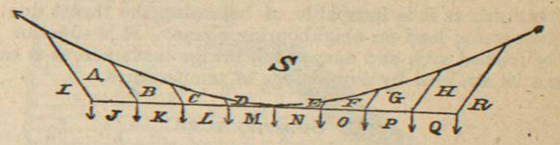


Fig. 88b.

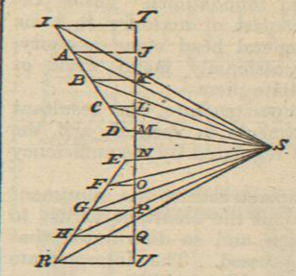


Fig. 88c.

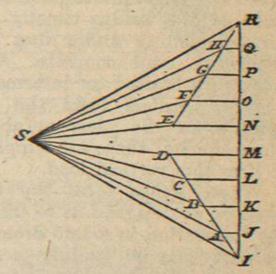


Fig. 88d.

maximum tension at the centre. Fig. 88c and fig. 88d show the reciprocal figures corresponding to those two cases. This bridge is somewhat stiffer than the ordinary suspension bridge, but is far inferior to the complete framed bridge.

§ 63. *Arch or Suspension Bridge, hinged at Abutments and Centre.*—Figs. 89 and 90 show two designs of considerable merit, consisting of two frames (the shape of which might vary considerably) hinged together at A, and

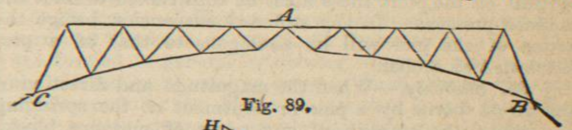


Fig. 89.

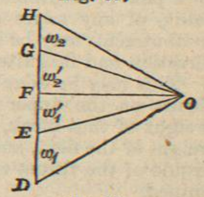


Fig. 89a.

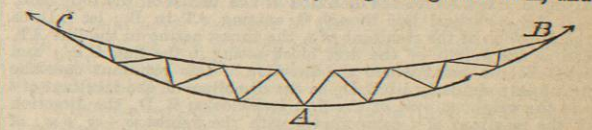


Fig. 90.

supported on hinges at B and C. The direction of the sustaining forces at A and B is to be determined as for a simple pair of beams hinged at A, B, and C. Find w_1 and w_1', the proportions of the load on AC borne by C and A respectively; and w_2 and w_2' the proportions of the load on AB borne by B and A respectively. Then draw the line of loads DEFGH as sketched (fig. 89*a*). From E and G draw lines parallel to AC and AB respectively, meeting in O. Join OD and OH. OD and OH represent in magnitude and direction the thrust on C and on B. OF is the stress on the pin at A. These thrusts being known, the stress on each member of each frame can be easily computed by a reciprocal figure or otherwise. This form is only inferior to the true framed arch or suspension bridge inasmuch as it is incapable of balancing the thrust due to the passing load on neighbouring spans. It is superior to the framed arch and suspension bridge inasmuch as it cannot be strained by any change of temperature.

VII SUBSTRUCTURE.

§ 64. *Preliminary.*—The substructure of a bridge comprises the piers, abutments, and foundations. These portions of the bridge usually consist of masonry in some form, including under that general head stone masonry, brickwork, and concrete. Occasionally metal work or woodwork is used for intermediate piers.

When girders form the superstructure, the resultant pressure on the piers or abutments is vertical, and the dimensions of these are simply regulated by the sufficiency to bear this vertical load.

When arches form the superstructure, the abutment must be so designed as to transmit the resultant thrust to the foundation in a safe direction, and so distributed that no part may be unduly compressed. The intermediate piers should also have considerable stability, so as to counterbalance the thrust arising when one arch is loaded while the other is free from load.

For suspension bridges the abutment forming the anchorage must be so designed as to be thoroughly stable under the greatest pull which the chains can exert. The piers require to be carried above the platform, and their design must be modified according to the type of suspension bridge adopted. When the resultant pressure is not vertical on the piers these must be constructed to meet the inclined pressure. In any stiffened suspension bridge the action of the pier will be analogous to that of a pier between two arches.

§ 65. *Stability.*—When the magnitude and direction of the thrust borne by a pier or abutment at the springing are known, the stability of any series of masonry blocks forming the pier or abutment may be studied by drawing lines showing the direction and magnitude of the resultant force on each joint. This may be done as for the voussoirs of an arch. The thrust on the upper block may be compounded with the weight of that block, the resultant compounded with the weight of the next, and so forth, until the direction and magnitude of the thrust on the rock or earth foundation is determined.

A better method of making the drawing is shown in fig. 91; find the centre of gravity $C_{,}$ of block 1, the centre of gravity $C_{,,}$ of blocks 1 and 2 treated as a single mass, similarly $C_{,,,}$ for blocks 1, 2, and 3 Let AT be the direction of the thrust on the top block, and $C_{,}B_{,}$ a vertical line though $C_{,}$ cutting AT in $B_{,}$; let $B_{,}D_{,}$ be the direction of the resultant of t, the thrust acting in the line AT, and $w_{,}$ the weight of the first block acting in the line $C_{,}B_{,}$; and let $D_{,}$ be the point where the direction of this resultant cuts the first joint; similarly let $B_{,,}D_{,,}$ be the direction of the resultant of t and the weight $w_{,}+w_{,,}$ of the first two blocks; $B_{,,,}D_{,,,}$ the direction of the resultant of t compounded with the weight $w_{,}+w_{,,}+w_{,,,}$ of the three first blocks, &c., &c. This method of proceeding gives the direction and magnitude of each force and centre of pressure D independently of the values obtained for the preceding joints. For stability the line BD must not make a greater angle with the normal to the joint than the angle of repose; and the point D must nowhere fall beyond the edge of the joint; for strength and safety the point D might be required to fall within the middle two-thirds of the joint, or within the middle three-quarters. The theory by which the joints furthest from the centre of pressure would open when the centre of pressure leaves the

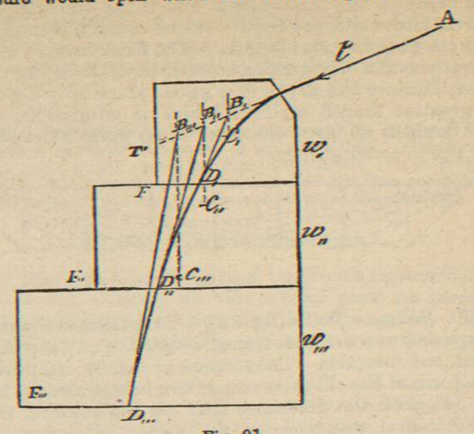


Fig. 91.

middle third cannot apply to such a structure as a masonry abutment, all parts of which are bonded together. Professor G. Fuller calculates the thickness of abutments by the following empirical rule, deduced from many practical examples:—Let c = half the span in feet; d = versed sine in feet; t = thickness of abutment at springing: then, for flat arches, in which the span does not exceed 150 feet, and the ratio of the rise to the span is not less than 116, we may write $t = \cdot 17\left(\frac{c^2}{d} + d\right)$. From the springing to the base the abutment may have a batter of 1 in 4. This gives an abutment the cubic capacity of which will be sufficient, but it may with advantage be divided into abutment proper and counterforts; in semicircular arches t should be taken as the thickness of the abutment at a height above the springing equal to two-thirds of the radius. The maximum intensity of stress on the stone at the edge F might be approximately found by the theory explained in § 8. This theory finds a useful application in calculating the maximum intensity of stress which a given foundation might produce on the earth

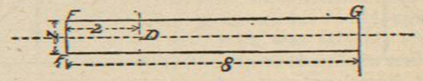


Fig. 92.

or rock supporting it. Thus, let the section of the foundation under consideration be 1 foot in breadth, 8 feet in length from F to G, as shown in plan, fig. 92; let the centre of pressure D be 2 feet from FF, and let the total resultant thrust be about 16·5 tons, inclined so that the horizontal component is 4 tons and the vertical component 16 tons, then the mean vertical intensity of pressure per square foot is $\frac{16}{8} = 2$, and the maximum intensity along the line FF, is by equation 2, § 8—

$$2\left\{1+(4\times2\times8)\frac{12}{512}\right\} = 5,$$

this maximum intensity being $2\frac{1}{2}$ times as great as the mean intensity. Obviously, although for the sake of appearance the courses of masonry in the abutment of an arch may be left horizontal in the face, the stability is increased by inclining them at a proper angle, so that they lie normal to the thrust.

The stability of abutments may be tested by taking moments round points in the joints selected as the points beyond which the thrust must not come. The same methods apply to the anchorages of suspension bridges, and to intermediate piers, which are intended to take a given horizontal thrust. When metal or wooden piers are adopted their weight will generally be insignificant, and such as may be neglected in calculating their stability. Metal-work piers or wooden piers usually consist of wrought or cast-iron frame work, and the stress on each part of the frame, as well as the resultant stress on the foundations where each upright member reaches it, is

easily calculated by the method of reciprocal figures or otherwise.

Occasionally metal piers are continuous metal structures, such as cast-iron cylinders. The maximum intensity of stress can then be calculated by resolving the thrust on the upper part of the pier into a horizontal and a vertical component, calculating the bending moment produced on each horizontal cross section by the horizontal component, and adding the intensity of stress caused by this bending moment to the mean intensity caused by direct compression. The manner in which any metal-work pier is held by its foundation against a bending moment will require special consideration; the resultant pressure should always fall well within the base.

§ 66. *Practice.*—In the design of the usual masonry bridge the thickness of the pier is generally determined by practical considerations. In small arches the pier is made thick enough to allow the two rings of the two abutting arches to spring from the pier without interfering with one another, a clearance of about half a brick being often allowed between the two rings. In larger arches the piers will generally be found to vary in thickness from $\frac{1}{8}$th to $\frac{1}{10}$th of the span, with a slight batter (*i.e.*, with walls spreading outward towards the base). In very old bridges piers are sometimes found equal in width to the opening of the arch. In large bridges, or with very high piers, care must be taken that the pressure per square foot on the masonry or foundation does not exceed a safe value. The brickwork in the piers of Charing Cross bridge is subject to a compression of 9 tons per square foot; four or five tons is a much more usual load. Eight tons per square foot may be considered a maximum for rubble stone-work, and perhaps 20 tons for the best dressed ashlar. Strong concrete may be trusted with 3 tons; firm rock foundations with 9 tons, soft sandstone with 2 tons, and firm earth with from 1 to $1\frac{1}{2}$ tons. The depth of the first course below the surface (on dry land) should not be less than 3 feet in sand and 4 feet in clay.

When framework, either of wood or iron, is used as a pier, care must be taken by cross-bracing to provide against the effect of wind and vibration.

§ 67. *Site.*—The site proposed for a pier must be carefully examined by borings; the ground should be uniform, for if a pier rests partly on one formation and partly on another, unequal settlement will certainly occur, even if the weaker formation be such as would have been amply strong enough to bear the pressure had the pier been wholly founded upon it. Solid rock may be considered the best foundation, but where rock is broken up by cracks or other inequalities it is inferior to such formations as uniform gravel, chalk, and some kinds of sand and clay. These foundations may be described as incompressible. The worst foundations are afforded by those formations which can be compressed or squeezed out sideways by the imposition of weight. Muddy earth, certain clays, and certain sands are of this nature. Alternate beds of stone and slippery clay are very treacherous. The foundation should be dressed level so that the masonry may everywhere start from the same height, and therefore settle equally. Unavoidable inequalities are better filled up by concrete than by masonry.

For foundations in water it is very important that the ground should not be such as can be scoured away by the current or wash of the water; many bridges have failed by the undermining of the piers due to this cause. Special precautions, to be presently described, must be taken against the effects of the scour if the soil itself is not of a sufficiently resisting nature. The piers must be so placed and formed that the obstruction to the flow of water may be as small as possible and the effect which the piers will have in altering the level of the stream above and below the bridge must be considered. Data as to the maximum flood waters to be provided for must be examined; and provision must in some climates be made against ice by suitable cut-waters or fenders, an example of which is given in fig. 93, showing a pier of the Victoria Bridge, Montreal.

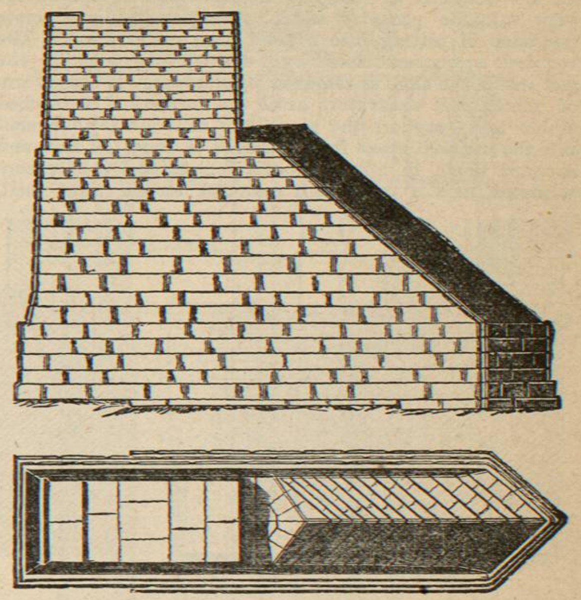


FIG. 93.—Side Elevation and Plan of Pier.

§ 68. *Mode of Founding in Water.*—The chief difficulty met with by an engineer about to erect a large bridge over a deep stream is to secure a sound foundation for the piers. The following are some of the principal methods of building piers in or under water:—

Cofferdams.—Cofferdams are embankments or dams which surround the site so as to exclude the water from it. They are formed in general by driving two rows of piles round the site so as to enclose between them a water-tight wall of clay puddle; in depths of less than 3 or 4 feet, where there is little current, a simple clay dam may be used. In greater depths, the timber walls consist of guide piles at intervals, with some form of sheet piling between them; in extreme depths the timber walls may be composed of stout piles driven in side by side all round. The dam must be sufficiently strong to bear the pressure of the water against the outside when the space enclosed has been pumped dry. Rankine states that the common rule for the thickness of a cofferdam is to make it equal to the height above ground if the height does not exceed 10 feet, and for greater heights, to add to 10 feet one-third of the excess of the height above 10 feet. The "Cours de Ponts" at the school of the Ponts et Chaussées, states that a cofferdam need never be made of greater thickness than from 4 to 6 feet, as the interior can always be sufficiently stayed inside. This method of founding is now seldom practised; it is costly and causes great obstruction in the stream.

Caissons.—Some foundations have been constructed as follows:—A level or nearly level bed was prepared in the stream by digging or by driving piles and sawing off the heads at a uniform depth; a huge timber box, called a caisson, was then filled with masonry, and sunk on the foundation thus roughly prepared. This method is now abandoned. It was peculiarly liable to danger from the scour of the stream. The name caisson is also sometimes