ment of the year by a fixed rule of intercalation; and of the various methods which might be employed, no one, perhaps is on the whole more easy of application, or better adapted for the purpose of computation, than the Gregorian now in use. But a system of 31 intercalations in 128 years would be by far the most perfect as regards mathematical accuracy. Its adoption upon our present Gregorian calendar would only require the suppression of the usual bissextile once in every 128 years, and there would be no necessity for any further correction, as the error is so insignificant that it would not amount to a day in 100,000 years.

Of the Lunar Year and Luni-solar Periods.—The lunar year, consisting of twelve lunar months, contains only 354 days; its commencement consequently anticipates that of the solar year by eleven days, and passes through the whole circle of the seasons in about thirty-four lunar years. It is therefore so obviously ill-adapted to the computation of time, that, excepting the modern Jews and Mahometans, almost all nations who have regulated their months by the moon have employed some method of intercalation by means of which the beginning of the year is retained at nearly the same fixed place in the seasons.

In the early ages of Greece the year was regulated entirely by the moon. Solon divided the year into twelve months, consisting alternately of twenty-nine and thirty days, the former of which were called *deficient* months, and the latter *full* months. The lunar year, therefore, contained 354 days, falling short of the exact time of twelve lunations by about 8·8 hours. The first expedient adopted to reconcile the lunar and solar years seems to have been the addition of a month of thirty days to every second year. Two lunar years would thus contain 25 months, or 738 days, while two solar years, of $365\frac{1}{4}$ days each, contain $730\frac{1}{2}$ days. The difference of $7\frac{1}{2}$ days was still too great to escape observation; it was accordingly proposed by Cleostratus of Tenedos, who flourished shortly after the time of Thales, to omit the biennary intercalation every eighth year. In fact, the $7\frac{1}{2}$ days by which two lunar years exceeded two solar years, amounted to thirty days, or a full month, in eight years. By inserting, therefore, three additional months instead of four in every period of eight years, the coincidence between the solar and lunar year would have been exactly restored if the latter had contained only 354 days, inasmuch as the period contains $354 \times 8 + 3 \times 30 = 2922$ days, corresponding with eight solar years of $365\frac{1}{4}$ days each. But the true time of 99 lunations is 2923·528 days, which exceeds the above period by 1·528 days, or thirty-six hours and a few minutes. At the end of two periods, or sixteen years, the excess is three days, and at the end of 160 years, thirty days. It was therefore proposed to employ a period of 160 years, in which one of the intercalary months should be omitted; but as this period was too long to be of any practical use, it was never generally adopted. The common practice was to make occasional corrections as they became necessary, in order to preserve the relation between the octennial period and the state of the heavens; but these corrections being left to the care of incompetent persons, the calendar soon fell into great disorder, and no certain rule was followed till a new division of the year was proposed by Meton and Euctemon, which was immediately adopted in all the states and dependencies of Greece.

The mean motion of the moon in longitude, from the mean equinox, during a Julian year of 365·25 days (according to Hansen's *Tables de la Lune*, London, 1857, pages 15, 16) is, at the present date, $13 \times 360° + 477644''·409$; that of the sun being $360° + 27''·685$. Thus the corresponding relative mean geocentric motion of the moon from the sun is $12 \times 360° + 477616''·724$; and the duration of the mean synodic revolution of the moon, or lunar month, is therefore $\frac{360°}{12 \times 360° + 477616''·724} \times 365·25 = 29·530588$ days, or 29 days, 12 hours, 44 min. 2·8 sec.

The *Metonic Cycle*, which may be regarded as the *chef-d'œuvre* of ancient astronomy, is a period of nineteen solar years, after which the new moons again happen on the same days of the year. In nineteen solar years there are 235 lunations, a number which, on being divided by nineteen, gives twelve lunations for each year, with seven of a remainder, to be distributed among the years of the period. The period of Meton, therefore, consisted of twelve years containing twelve months each, and seven years containing thirteen months each; and these last formed the third, fifth, eighth, eleventh, thirteenth, sixteenth, and nineteenth years of the cycle. As it had now been discovered that the exact length of the lunation is a little more than twenty-nine and a half days, it became necessary to abandon the alternate succession of full and deficient months; and, in order to preserve a more accurate correspondence between the civil month and the lunation, Meton divided the cycle into 125 full months of thirty days, and 110 deficient months of twenty-nine days each. The number of days in the period was therefore 6940. In order to distribute the deficient months through the period in the most equable manner, the whole period may be regarded as consisting of 235 full months of thirty days, or of 7050 days, from which 110 days are to be deducted. This gives one day to be suppressed in sixty-four; so that if we suppose the months to contain each thirty days, and then omit every sixty-fourth day in reckoning from the beginning of the period, those months in which the omission takes place will, of course, be the deficient months.

The number of days in the period being known, it is easy to ascertain its accuracy both in respect of the solar and lunar motions. The exact length of nineteen solar years is $19 \times 365·2422 = 6939·6018$ days, or 6939 days 14 hours 26·592 minutes; hence the period, which is exactly 6940 days, exceeds nineteen revolutions of the sun by nine and a half hours nearly. On the other hand, the exact time of a synodic revolution of the moon is 29·530588 days; 235 lunations, therefore, contain $235 \times 29·530588 = 6939·68818$ days, or 6939 days 16 hours 31 minutes, so that the period exceeds 235 lunations by only seven and a half hours.

After the Metonic cycle had been in use about a century, a correction was proposed by Calippus. At the end of four cycles, or seventy-six years, the accumulation of the seven and a half hours of difference between the cycle and 235 lunations amounts to thirty hours, or one whole day and six hours. Calippus, therefore, proposed to quadruple the period of Meton, and deduct one day at the end of that time by changing one of the full months into a deficient month. The period of Calippus, therefore, consisted of three Metonic cycles of 6940 days each, and a period of 6939 days; and its error in respect of the moon, consequently, amounted only to six hours, or to one day in 304 years. This period exceeds seventy-six true solar years by fourteen hours and a quarter nearly, but coincides exactly with seventy-six Julian years; and in the time of Calippus the length of the solar year was almost universally supposed to be exactly $365\frac{1}{4}$ days. The Calippic period is frequently referred to as a date by Ptolemy.

ECCLESIASTICAL CALENDAR.—The ecclesiastical calendar, which is adopted in all the Catholic, and most of the Protestant countries of Europe, is luni-solar, being regulated partly by the solar, and partly by the lunar year,—a circumstance which gives rise to the distinction between the movable and immovable feasts. So early as the 2d century of our era, great disputes had arisen among the Christians respecting the proper time of celebrating Easter, which governs all the other movable feasts. The Jews celebrated their passover on the 14th day of the *first month*, that is to say, the lunar month of which the fourteenth day either falls on, or next follows, the day of the vernal equinox.

Most Christian sects agreed that Easter should be celebrated on a Sunday. Others followed the example of the Jews, and adhered to the 14th of the moon; but these, as usually happened to the minority, were accounted heretics, and received the appellation of Quartodecimans. In order to terminate dissensions, which produced both scandal and schism in the church, the Council of Nice, which was held in the year 325, ordained that the celebration of Easter should thenceforth always take place on the Sunday which immediately follows the full moon that happens upon, or next after, the day of the vernal equinox. Should the 14th of the moon, which is regarded as the day of full moon, happen on a Sunday, the celebration of Easter was deferred to the Sunday following, in order to avoid concurrence with the Jews and the above-mentioned heretics. The observance of this rule renders it necessary to reconcile three periods which have no common measure, namely, the week, the lunar month, and the solar year; and as this can only be done approximately, and within certain limits, the determination of Easter is an affair of considerable nicety and complication. It is to be regretted that the reverend fathers who formed the Council of Nice were not advised to abandon the moon altogether, and appoint Easter to be celebrated on the first or second Sunday of April. The ecclesiastical calendar would in that case have possessed all the simplicity and uniformity of the civil calendar, which only requires the adjustment of the civil to the solar year; but they were probably not sufficiently versed in astronomy to be aware of the practical difficulties which their regulation had to encounter.

Dominical Letter.—The first problem which the construction of the calendar presents is to connect the week with the year, or to find the day of the week corresponding to a given day of any year of the era. As the number of days in the week and the number in the year are prime to one another, two successive years cannot begin with the same day; for if a common year begins, for example, with Sunday, the following year will begin with Monday, and if a leap year begins with Sunday, the year following will begin with Tuesday. For the sake of greater generality, the days of the week are denoted by the first seven letters of the alphabet, A, B, C, D, E, F, G, which are placed in the calendar beside the days of the year, so that A stands opposite the first day of January, B opposite the second, and so on to G, which stands opposite the seventh; after which A returns to the eighth, and so on through the 365 days of the year. Now, if one of the days of the week, Sunday for example, is represented by E, Monday will be represented by F, Tuesday by G, Wednesday by A, and so on; and every Sunday through the year will have the same character E, every Monday F, and so with regard to the rest. The letter which denotes Sunday is called the *Dominical Letter*, or the *Sunday Letter;* and when the dominical letter of the year is known, the letters which respectively correspond to the other days of the week become known at the same time.

Solar Cycle.—In the Julian calendar the dominical letters are readily found by means of a short cycle, in which they recur in the same order without interruption. The number of years in the intercalary period being four, and the days of the week being seven, their product is $4 \times 7 = 28$; twenty-eight years is therefore a period which includes all the possible combinations of the days of the week with the commencement of the year. This period is called the *Solar Cycle*, or the *Cycle of the Sun*, and restores the first day of the year to the same day of the week. At the end of the cycle the dominical letters return again in the same order on the same days of the month; hence a table of dominical letters, constructed for twenty-eight years, will serve to show the dominical letter of any given year from the commencement of the era to the reformation. The cycle, though probably not invented before the time of the Council of Nice, is regarded as having commenced nine years before the era, so that the year *one* was the tenth of the solar cycle. To find the year of the cycle, we have therefore the following rule:—*Add nine to the date, divide the sum by twenty-eight; the quotient is the number of cycles elapsed, and the remainder is the year of the cycle.* Should there be no remainder, the proposed year is the twenty-eighth or last of the cycle. This rule is conveniently expressed by the formula $\left(\frac{x+9}{28}\right)_r$, in which x denotes the date, and the symbol r denotes that the remainder, which arises from the division of $x+9$ by 28, is the number required. Thus, for 1840, we have $\frac{1840+9}{28} = 66\frac{1}{28}$; therefore $\left(\frac{1840+9}{28}\right)_r = 1$, and the year 1840 is the first of the solar cycle. In order to make use of the solar cycle in finding the dominical letter, it is necessary to know that the first year of the Christian era began with Saturday. The dominical letter of that year, which was the tenth of the cycle, was consequently B. The following year, or the 11th of the cycle, the letter was A; then G. The fourth year was bissextile, and the dominical letters were F, E; the following year D, and so on. In this manner it is easy to find the dominical letter belonging to each of the twenty-eight years of the cycle. But at the end of a century the order is interrupted in the Gregorian calendar by the secular suppression of the leap year; hence the cycle can only be employed during a century. In the reformed calendar the intercalary period is four hundred years, which number being multiplied by seven, gives two thousand eight hundred years as the interval in which the coincidence is restored between the days of the year and the days of the week. This long period, however, may be reduced to four hundred years; for since the dominical letter goes back five places every four years, its variation in four hundred years, in the Julian calendar, was five hundred places, which is equivalent to only three places (for five hundred divided by seven leaves three); but the Gregorian calendar suppresses exactly three intercalations in four hundred years, so that after four hundred years the dominical letters must again return in the same order.

Hence the following table of dominical letters for four hundred years will serve to show the dominical letter of any year in the Gregorian calendar for ever. It contains four columns of letters, each column serving for a century. In order to find the column from which the letter in any given case is to be taken, strike off the two last figures of the date, divide the preceding figures by four, and the remainder will indicate the column. The symbol X, employed in the formula at the top of the column, denotes the number of centuries, that is, the figures remaining after the last two have been struck off. For example, required the dominical letter of the year 1839? In this case $X = 18$, therefore $\left(\frac{X}{4}\right)_r = 2$; and in the second column of letters, opposite 39, in the table we find F, which is the letter of the proposed year.

It deserves to be remarked, that as the dominical letter of the first year of the era was B, the first column of the following table will give the dominical letter of every year from the commencement of the era to the reformation. For this purpose divide the date by 28, and the letter opposite the remainder, in the first column of figures, is the dominical letter of the year. For example, supposing the date to be 1148. On dividing by 28, the remainder is 0, or 28; and opposite 28, in the first column of letters, we find D, C, the dominical letters of the year 1148.

TABLE I.—*Dominical Letters.*

Years of the Century.	$\left(\frac{x}{4}\right)_r=1$	$\left(\frac{x}{4}\right)_r=2$	$\left(\frac{x}{4}\right)_r=3$	$\left(\frac{x}{4}\right)_r=0$
0	C	E	G	B, A
1 29 57 85	B	D	F	G
2 30 58 86	A	C	E	F
3 31 59 87	G	B	D	E
4 32 60 88	F, E	A, G	C, B	D, C
5 33 61 89	D	F	A	B
6 34 62 90	C	E	G	A
7 35 63 91	B	D	F	G
8 36 64 92	A, G	C, B	E, D	F, E
9 37 65 93	F	A	C	D
10 38 66 94	E	G	B	C
11 39 67 95	D	F	A	B
12 40 68 96	C, B	E, D	G, F	A, G
13 41 69 97	A	C	E	F
14 42 70 98	G	B	D	E
15 43 71 99	F	A	C	D
16 44 72	E, D	G, F	B, A	C, B
17 45 73	C	E	G	A
18 46 74	B	D	F	G
19 47 75	A	C	E	F
20 48 76	G, F	B, A	D, C	E, D
21 49 77	E	G	B	C
22 50 78	D	F	A	B
23 51 79	C	E	G	A
24 52 80	B, A	D, C	F, E	G, F
25 53 81	G	B	D	E
26 54 82	F	A	C	D
27 55 83	E	G	B	C
28 56 84	D, C	F, E	A, G	B, A

TABLE II.—*The Day of the Week.*

Month.	Dominical Letter.						
Jan. Oct.	A	B	C	D	E	F	G
Feb. Mar. Nov.	D	E	F	G	A	B	C
April July	G	A	B	C	D	E	F
May	B	C	D	E	F	G	A
June	E	F	G	A	B	C	D
August	C	D	E	F	G	A	B
Sept. Dec.	F	G	A	B	C	D	E
1 8 15 22 29	Sun.	Sat.	Frid.	Thur.	Wed.	Tues.	Mon.
2 9 16 23 30	Mon.	Sun.	Sat.	Frid.	Thur.	Wed.	Tues.
3 10 17 24 31	Tues.	Mon.	Sun.	Sat.	Frid.	Thur.	Wed.
4 11 18 25	Wed.	Tues.	Mon.	Sun.	Sat.	Frid.	Thur.
5 12 19 26	Thur.	Wed.	Tues.	Mon.	Sun.	Sat.	Frid.
6 13 20 27	Frid.	Thur.	Wed.	Tues.	Mon.	Sun.	Sat.
7 14 21 28	Sat.	Frid.	Thur.	Wed.	Tues.	Mon.	Sun.

Lunar Cycle and Golden Number.—In connecting the lunar month with the solar year, the framers of the ecclesiastical calendar adopted the period of Meton, or lunar cycle, which they supposed to be exact. A different arrangement has, however, been followed with respect to the distribution of the months. The lunations are supposed to consist of twenty-nine and thirty days alternately, or the lunar year of 354 days; and in order to make up nineteen solar years, six embolismic or intercalary months, of thirty days each, are introduced in the course of the cycle, and one of twenty-nine days is added at the end. This gives $19 \times 354 + 6 \times 30 + 29 = 6935$ days, to be distributed among 235 lunar months. But every leap year one day must be added to the lunar month in which the 29th of February is included. Now if leap year happens on the first, second, or third year of the period, there will be five leap years in the period, but only four when the first leap year falls on the fourth. In the former case the number of days in the period becomes 6940 and in the latter 6939. The mean length of the cycle is therefore $6939\frac{3}{4}$ days, agreeing exactly with nineteen Julian years.

By means of the lunar cycle the new moons of the calendar were indicated before the reformation. As the cycle restores these phenomena to the same days of the civil month, they will fall on the same days in any two years which occupy the same place in the cycle; consequently a table of the moon's phases for 19 years will serve for any year whatever when we know its number in the cycle. This number is called the *Golden Number*, either because it was so termed by the Greeks, or because it was usual to mark it with red letters in the calendar. The Golden Numbers were introduced into the calendar about the year 530, but disposed as they would have been if they had been inserted at the time of the Council of Nice. The cycle is supposed to commence with the year in which the new moon falls on the 1st of January, which took place the year preceding the commencement of our era. Hence, to find the Golden Number N, for any year x, we have $N = \left(\frac{x+1}{19}\right)_r$, which gives the following rule: *Add* 1 *to the date, divide the sum by* 19; *the quotient is the number of cycles elapsed, and the remainder is the Golden Number.* When the remainder is 0, the proposed year is of course the last or 19th of the cycle. It ought to be remarked that the new moons, determined in this manner, may differ from the astronomical new moons sometimes as much as two days. The reason is, that the sum of the solar and lunar inequalities, which are compensated in the whole period, may amount in certain cases to 10°, and thereby cause the new moon to arrive on the second day before or after its mean time.

Dionysian Period.—The cycle of the sun brings back the days of the month to the same day of the week; the lunar cycle restores the new moons to the same day of the month; therefore $28 \times 19 = 532$ years, includes all the variations in respect of the new moons and the dominical letters, and is consequently a period after which the new moons again occur on the same day of the month and the same day of the week. This is called the *Dionysian* or *Great Paschal Period*, from its having been employed by Dionysius Exiguus, familiarly styled "Denys the Little," in determining Easter Sunday. It was, however, first proposed by Victorius of Aquitain, who had been appointed by Pope Hilary to revise and correct the church calendar. Hence it is also called the *Victorian Period*. It continued in use till the Gregorian reformation.

Cycle of Indiction.—Besides the solar and lunar cycles, there is a third of 15 years, called the cycle of indiction, frequently employed in the computations of chronologists. This period is not astronomical, like the two former, but has reference to certain judicial acts which took place at stated epochs under the Greek emperors. Its commencement is referred to the 1st of January of the year 313 of the common era. By extending it backwards, it will be found that the first of the era was the fourth of the cycle of indiction. The number of any year in this cycle will therefore be given by the formula $\left(\frac{x+3}{15}\right)_r$, that is to say, *add* 3 *to the date, divide the sum by* 15, *and the remainder is the year of the indiction.* When the remainder is 0, the proposed year is the fifteenth of the cycle.

Julian Period.—The Julian period, proposed by the celebrated Joseph Scaliger as an universal measure of chronology, is formed by taking the continued product of the three cycles of the sun, of the moon, and of the indiction, and is consequently $28 \times 19 \times 15 = 7980$

years. In the course of this long period no two years can be expressed by the same numbers in all the three cycles. Hence, when the number of any proposed year in each of the cycles is known, its number in the Julian period can be determined by the resolution of a very simple problem of the indeterminate analysis. It is unnecessary, however, in the present case to exhibit the general solution of the problem, because when the number in the period corresponding to any one year in the common era has been ascertained, it is easy to establish the correspondence for all other years, without having again recourse to the direct solution of the problem. We shall therefore find the number of the Julian period corresponding to the first of our era.

We have already seen that the year 1 of the era had 10 for its number in the solar cycle, 2 in the lunar cycle, and 4 in the cycle of indiction; the question is therefore to find a number such, that when it is divided by the three numbers 28, 19, and 15 respectively, the three quotients shall be 10, 2, and 4.

Let x, y, and z be the three quotients of the divisions; the number sought will then be expressed by $28\,x + 10$, by $19\,y + 2$, or by $15\,z + 4$. Hence the two equations

$$28\,x + 10 = 19\,y + 2 = 15\,z + 4.$$

To resolve the equation $28\,x + 10 = 19\,y + 2$, or $y = x + \frac{9\,x + 8}{19}$, let $m = \frac{9\,x+8}{19}$, we have then $x = 2\,m + \frac{m-8}{9}$.

Let $\frac{m-3}{9} = m'$; then $m = 9\,m' + 8$; hence

$$x = 18\,m' + 16 + m' = 19\,m' + 16 \ldots \quad \ldots\ldots (1).$$

Again, since $28\,x + 10 = 15\,z + 4$, we have

$$15\,z = 28\,x + 6, \text{ or } z = 2x - \frac{2x-6}{15}.$$

Let $\frac{2\,x-6}{15} = n$; then $2\,x = 15\,n + 6$, and $x = 7\,n + 3 + \frac{n}{2}$.

Let $\frac{n}{} = n'$; then $n' = 2\,n'$; consequently

$$x = 14\,n' + 3 + n' = 15\,n' + 3 \ldots\ldots\ldots\ldots\ldots (2).$$

Equating the above two values of x, we have

$$15\,n' + 3 = 19\,m' + 16;\ \text{whence } n' = m' + \frac{4\,m' + 13}{15}.$$

Let $\frac{4\,m'+13}{15} = p$; we have then

$$4\,m' = 15\,p - 13, \text{ and } m' = 4\,p - \frac{p+13}{4}.$$

Let $\frac{p+13}{4} = p'$; then $p = 4\,p' - 13$;

$$\text{whence } m' = 16\,p' - 52 - p' = 15\,p' - 52.$$

Now in this equation p' may be any number whatever, provided $15\,p'$ exceed 52. The smallest value of p' (which is the one here wanted) is therefore 4; for $15 \times 4 = 60$. Assuming therefore $p' = 4$, we have $m' = 60 - 52 = 8$; and consequently, since $x = 19\,m' + 16$, $x = 19 \times 8 + 16 = 168$. The number required is consequently $28 \times 168 + 10 = 4714$.

Having found the number 4714 for the first of the era, the correspondence of the years of the era and of the period is as follows:—

Era,	1,	2,	3, ...	x,
Period,	4714,	4715,	4716, ...	$4713 + x$;

from which it is evident, that if we take P to represent the year of the Julian period, and x the corresponding year of the Christian era, we shall have

$$P = 4713 + x, \text{ and } x = P - 4713.$$

With regard to the numeration of the years previous to the commencement of the era, the practice is not uniform. Chronologists, in general, reckon the year preceding the first of the era -1, the next preceding -2, and so on. In this case

Era,	-1,	-2,	-3, ...	$-x$,
Period,	4713,	4712,	4711, ...	$4714 - x$;

whence

$$P = 4714 - x, \text{ and } x = 4714 - P.$$

But astronomers, in order to preserve the uniformity of computation, make the series of years proceed without interruption, and reckon the year preceding the first of the era 0. Thus

Era,	0,	-1,	-2, ...	$-x$,
Period,	4713,	4712,	4711, ...	$4713 - x$;

therefore, in this case

$$P = 4713 - x, \text{ and } x = 4713 - P.$$

Reformation of the Calendar.—The ancient church calendar was founded on two suppositions, both erroneous, namely, that the year contains $365\frac{1}{4}$ days, and that 235 lunations are exactly equal to nineteen solar years. It could not therefore long continue to preserve its correspondence with the seasons, or to indicate the days of the new moons with the same accuracy. About the year 730 the venerable Bede had already perceived the anticipation of the equinoxes, and remarked that these phenomena then took place about three days earlier than at the time of the Council of Nice. Five centuries after the time of Bede, the divergence of the true equinox from the 21st of March, which now amounted to seven or eight days, was pointed out by John of Sacrobosco, in a work published under the title *De Anni Ratione;* and by Roger Bacon, in a treatise *De Reformatione Calendarii*, which, though never published, was transmitted to the Pope. These works were probably little regarded at the time; but as the errors of the calendar went on increasing, and the true length of the year, in consequence of the progress of astronomy, became better known, the project of a reformation was again revived in the 15th century; and in 1474 Pope Sextus IV. invited Regiomontanus, the most celebrated astronomer of the age, to Rome, to superintend the reconstruction of the calendar. The premature death of Regiomontanus caused the design to be suspended for the time; but in the following century numerous memoirs appeared on the subject, among the authors of which were Stöffler, Albert Pighius, John Schöner, Lucas Gauricus, and other mathematicians of celebrity. At length Pope Gregory XIII. perceiving that the measure was likely to confer a great eclat on his pontificate, undertook the long-desired reformation; and having found the Governments of the principal Catholic states ready to adopt his views, he issued a brief in the month of March 1582, in which he abolished the use of the ancient calendar, and substituted that which has since been received in almost all Christian countries under the name of the *Gregorian Calendar* or *New Style*. The author of the system adopted by Gregory was Aloysius Lilius, or Luigi Lilio Ghiraldi, a learned astronomer and physician of Naples, who died, however, before its introduction; but the individual who most contributed to give the ecclesiastical calendar its present form, and who was charged with all the calculations necessary for its verification, was Clavius, by whom it was completely developed and explained in a great folio treatise of 800 pages, published in 1603, the title of which is given at the end of this article.

It has already been mentioned that the error of the Julian year was corrected in the Gregorian calendar by the suppression of three intercalations in 400 years. In order to restore the commencement of the year to the same place in the seasons that it had occupied at the time of the Council of Nice, Gregory directed the day following the feast of St Francis, that is to say the 5th of October, to be reckoned the 15th of that month. By this regulation the vernal equinox which then happened on the 11th of March was restored to the 21st. From 1582 to 1700 the difference between the old and new style continued to be ten days; but 1700 being a leap year in the Julian calendar, and a common year in the Gregorian, the difference of the styles during the 18th century was eleven days. The year 1800 was also common in the new calendar, and, consequently, the difference in the present century is twelve days. From 1900 to 2100 inclusive it will be thirteen days.

The restoration of the equinox to its former place in the year, and the correction of the intercalary period, were attended with no difficulty; but Lilius had also to adapt the lunar year to the new rule of intercalation. The lunar