

cycle contained 6939 days 18 hours, whereas the exact time of 235 lunations, as we have already seen, is $235 \times 29.530588 = 6939$ days 16 hours 31 minutes. The difference, which is 1 hour 29 minutes, amounts to a day in 308 years, so that at the end of this time the new moons occur one day earlier than they are indicated by the golden numbers. During the 1257 years that elapsed between the Council of Nice and the reformation, the error had accumulated to four days, so that the new moons which were marked in the calendar as happening, for example, on the 5th of the month, actually fell on the 1st. It would have been easy to correct this error by placing the golden numbers four lines higher in the new calendar; and the suppression of the ten days had already rendered it necessary to place them ten lines lower, and to carry those which belonged, for example, to the 5th and 6th of the month, to the 15th and 16th. But, supposing this correction to have been made, it would have again become necessary, at the end of 308 years, to advance them one line higher, in consequence of the accumulation of the error of the cycle to a whole day. On the other hand, as the golden numbers were only adapted to the Julian calendar, every omission of the centenary intercalation would require them to be placed one line lower, opposite the 6th, for example, instead of the 5th of the month; so that, generally speaking, the places of the golden numbers would have to be changed every century. On this account Lilius thought fit to reject the golden numbers from the calendar, and supply their place by another set of numbers called *Epacts*, the use of which we shall now proceed to explain.

Epacts.—Epact is a word of Greek origin, employed in the calendar to signify the moon's age at the beginning of the year. The common solar year containing 365 days, and the lunar year only 354 days, the difference is eleven; whence, if a new moon fall on the 1st of January in any year, the moon will be eleven days old on the first day of the following year, and twenty-two days on the first of the third year. The numbers eleven and twenty-two are therefore the epacts of those years respectively. Another addition of eleven gives thirty-three for the epact of the fourth year; but in consequence of the insertion of the intercalary month in each third year of the lunar cycle, this epact is reduced to three. In like manner the epacts of all the following years of the cycle are obtained by successively adding eleven to the epact of the former year, and rejecting thirty as often as the sum exceeds that number. They are therefore connected with the golden numbers by the formula $\left(\frac{11n}{30}\right)$, in which n is any whole number; and for a whole lunar cycle (supposing the first epact to be 11), they are as follows: 11, 22, 3, 14, 25, 6, 17, 28, 9, 20, 1, 12, 23, 4, 15, 26, 7, 18, 29. But the order is interrupted at the end of the cycle; for the epact of the following year, found in the same manner, would be $29 + 11 = 40$ or 10, whereas it ought again to be 11 to correspond with the moon's age and the golden number 1. The reason of this is, that the intercalary month, inserted at the end of the cycle, contains only twenty-nine days instead of thirty; whence, after 11 has been added to the epact of the year corresponding to the golden number 19, we must reject twenty-nine instead of thirty, in order to have the epact of the succeeding year; or, which comes to the same thing, we must add twelve to the epact of the last year of the cycle, and then reject thirty as before.

This method of forming the epacts might have been continued indefinitely if the Julian intercalation had been followed without correction, and the cycle been perfectly exact; but as neither of these suppositions is true, two equations or corrections must be applied, one depending on

the error of the Julian year, which is called the solar equation; the other on the error of the lunar cycle, which is called the lunar equation. The solar equation occurs three times in 400 years, namely, in every secular year which is not a leap year; for in this case the omission of the intercalary day causes the new moons to arrive one day later in all the following months, so that the moon's age at the end of the month is one day less than it would have been if the intercalation had been made, and the epacts must accordingly be all diminished by unity. Thus the epacts 11, 22, 3, 14, &c., become 10, 21, 2, 13, &c. On the other hand, when the time by which the new moons anticipate the lunar cycle amounts to a whole day, which, as we have seen, it does in 308 years, the new moons will arrive one day earlier, and the epacts must consequently be increased by unity. Thus the epacts 11, 22, 3, 14, &c., in consequence of the lunar equation, become 12, 23, 4, 15, &c. In order to preserve the uniformity of the calendar, the epacts are changed only at the commencement of a century; the correction of the error of the lunar cycle is therefore made at the end of 300 years. In the Gregorian calendar this error is assumed to amount to one day in $312\frac{1}{2}$ years, or eight days in 2500 years, an assumption which requires the line of epacts to be changed seven times successively at the end of each period of 300 years, and once at the end of 400 years; and, from the manner in which the epacts were disposed at the reformation, it was found most correct to suppose one of the periods of 2500 years to terminate with the year 1800.

The years in which the solar equation occurs, counting from the reformation, are 1700, 1800, 1900, 2100, 2200, 2300, 2500, &c. Those in which the lunar equation occurs are 1800, 2100, 2400, 2700, 3000, 3300, 3600, 3900, after which, 4300, 4600, and so on. When the solar equation occurs, the epacts are diminished by unity; when the lunar equation occurs, the epacts are augmented by unity; and when both equations occur together, as in 1800, 2100, 2700, &c., they compensate each other, and the epacts are not changed.

In consequence of the solar and lunar equations, it is evident that the epact, or moon's age at the beginning of the year, must, in the course of centuries, have all different values from one to thirty inclusive, corresponding to the days in a full lunar month. Hence, for the construction of a perpetual calendar, there must be thirty different sets or lines of epacts. These are exhibited in the subjoined table (Table III.) called the *Extended Table of Epacts*, which is constructed in the following manner. The series of golden numbers is written in a line at the top of the table, and under each golden number is a column of thirty epacts, arranged in the order of the natural numbers, beginning at the bottom and proceeding to the top of the column. The first column, under the golden number 1, contains the epacts, 1, 2, 3, 4, &c., to 30 or 0. The second column, corresponding to the following year in the lunar cycle, must have all its epacts augmented by 11; the lowest number, therefore, in the column is 12, then 13, 14, 15, and so on. The third column, corresponding to the golden number 3, has for its first epact $12 + 11 = 23$; and in the same manner all the nineteen columns of the table are formed. Each of the thirty lines of epacts is designated by a letter of the alphabet, which serves as its index or argument. The order of the letters, like that of the numbers, is from the bottom of the column upwards.

In the tables of the church calendar the epacts are usually printed in Roman numerals, excepting the last, which is designated by an asterisk (*), used as an indefinite symbol to denote 30 or 0, and 25, which in the last eight columns is expressed in Arabic characters, for a reason that will immediately be explained. In the table

here given, this distinction is made by means of an accent placed over the last figure.

At the reformation the epacts were given by the line D. The year 1600 was a leap year; the intercalation accordingly took place as usual, and there was no interruption in the order of the epacts; the line D was employed till 1700. In that year the omission of the intercalary day rendered it necessary to diminish the epacts by unity, or to pass to the line C. In 1800 the solar equation again occurred, in consequence of which it was necessary to descend one line to have the epacts diminished by unity; but in this year the lunar equation also occurred, the anticipation of the new moons having amounted to a day; the new moons accordingly happened a day earlier, which rendered it necessary to take the epacts in the next higher line. There was, consequently, no alteration; the two equations destroyed each other. The line of epacts

belonging to the present century is therefore C. In 1900 the solar equation occurs, after which the line is B. The year 2000 is a leap year, and there is no alteration. In 2100 the equations again occur together and destroy each other, so that the line B will serve three centuries, from 1900 to 2200. From that year to 2300 the line will be A. In this manner the line of epacts belonging to any given century is easily found, and the method of proceeding is obvious. When the solar equation occurs alone, the line of epacts is changed to the next lower in the table; when the lunar equation occurs alone, the line is changed to the next higher; when both equations occur together, no change takes place. In order that it may be perceived at once to what centuries the different lines of epacts respectively belong, they have been placed in a column on the left hand side of the following table.

TABLE III.—*Extended Table of Epacts.*

Years.	Index.	GOLDEN NUMBERS.																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1700	C	*	11	22	3	14	25	6	17	28	9	20	*	11	22	3	14	25	6	17
1800	B	29	10	21	2	13	24	5	16	27	8	19	*	11	22	3	14	25	6	17
1900	A	28	9	20	1	12	23	4	15	26	7	18	29	10	21	2	13	24	5	16
2000	u	27	8	19	*	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15
2100	t	26	7	18	29	10	21	2	13	24	5	16	27	8	19	*	11	22	3	14
2200	s	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29	10	21	2	13
2300	r	24	5	16	27	8	19	*	11	22	3	14	25	6	17	28	9	20	1	12
2400	q	23	4	15	26	7	18	29	10	21	2	13	24	5	16	27	8	19	*	11
2500	p	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29	10
2600	o	21	2	13	24	5	16	27	8	19	*	11	22	3	14	25	6	17	28	9
2700	n	20	1	12	23	4	15	26	7	18	29	10	21	2	13	24	5	16	27	8
2800	m	19	*	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7
2900	l	18	29	10	21	2	13	24	5	16	27	8	19	*	11	22	3	14	25	6
3000	k	17	28	9	20	1	12	23	4	15	26	7	18	29	10	21	2	13	24	5
3100	j	16	27	8	19	*	11	22	3	14	25	6	17	28	9	20	1	12	23	4
3200	i	15	26	7	18	29	10	21	2	13	24	5	16	27	8	19	*	11	22	3
3300	h	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29	10	21	2
3400	g	13	24	5	16	27	8	19	*	11	22	3	14	25	6	17	28	9	20	1
3500	f	12	23	4	15	26	7	18	29	10	21	2	13	24	5	16	27	8	19	*
3600	e	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29
3700	d	10	21	2	13	24	5	16	27	8	19	*	11	22	3	14	25	6	17	28
3800	c	9	20	1	12	23	4	15	26	7	18	29	10	21	2	13	24	5	16	27
3900	b	8	19	*	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26
4000	a	7	18	29	10	21	2	13	24	5	16	27	8	19	*	11	22	3	14	25
4100	Z	6	17	28	9	20	1	12	23	4	15	26	7	18	29	10	21	2	13	24
4200	Y	5	16	27	8	19	*	11	22	3	14	25	6	17	28	9	20	1	12	23
4300	X	4	15	26	7	18	29	10	21	2	13	24	5	16	27	8	19	*	11	22
4400	W	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29	10	21
4500	V	2	13	24	5	16	27	8	19	*	11	22	3	14	25	6	17	28	9	20
4600	U	1	12	23	4	15	26	7	18	29	10	21	2	13	24	5	16	27	8	19

The use of the epacts is to show the days of the new moons, and consequently the moon's age on any day of the year. For this purpose they are placed in the calendar (Table IV.) along with the days of the month and dominical letters, in a retrograde order, so that the asterisk stands beside the 1st of January, 29 beside the 2nd, 28 beside the 3rd, and so on to 1, which corresponds to the 30th. After this comes the asterisk, which corresponds to the 31st of January, then 29, which belongs to the 1st of February, and so on to the end of the year. The reason of this distribution is evident. If the last lunation of any year ends, for example, on the 2nd of December, the new moon falls on the 3rd; and the moon's age on the 31st, or at the end of the year, is twenty-nine days. The epact of the following year is therefore twenty-nine. Now that lunation having commenced on the 3rd of December, and consisting of thirty days, will end on the 1st of January. The 2nd of January is therefore the day of the new moon, which is

indicated by the epact twenty-nine. In like manner, if the new moon fell on the 4th of December, the epact of the following year would be twenty-eight, which, to indicate the day of next new moon, must correspond to the 3rd of January.

When the epact of the year is known, the days on which the new moons occur throughout the whole year are shown by Table IV., which is called the *Gregorian Calendar of Epacts*. For example, the golden number of the year 1832, is $\left(\frac{1832+1}{19}\right)_r = 9$, and the epact, as found in Table III., is twenty-eight. This epact occurs at the 3rd of January, the 2nd of February, the 3rd of March, the 2nd of April, the 1st of May, &c.; and these days are consequently the days of the ecclesiastical new moons in 1832. The astronomical new moons generally take place one or two days, sometimes even three days, earlier than those of the calendar.

TABLE IV.—Gregorian Calendar.

Table with 12 columns for months (January to December) and 31 rows for days. Each cell contains a letter (A-L) representing the dominical letter for that day. Some cells are empty, indicating leap years or specific calendar adjustments.

There are some artifices employed in the construction of this table, to which it is necessary to pay attention. The thirty epacts correspond to the thirty days of a full lunar month; but the lunar months consist of twenty-nine and thirty days alternately, therefore in six months of the year the thirty epacts must correspond only to twenty-nine days. For this reason the epacts twenty-five and twenty-four are placed together, so as to belong only to one day in the months of February, April, June, August, September, and November, and in the same months another 25', distinguished by an accent, or by being printed in a different character, is placed beside 26, and belongs to the same day. The reason for doubling the 25 was to prevent the new moons from being indicated in the calendar as happening twice on the same day in the course of the lunar cycle, a thing which actually cannot take place. For example, if we observe the line B in Table III., we shall see that it contains both the epacts twenty-four and twenty-five, so that if these correspond to the same day of the month, two new moons would be indicated as happening on that day within nineteen years. Now the three epacts 24, 25, 26, can never occur in the same line; therefore in those lines in which 24 and 25 occur, the 25 is accented, and placed in the calendar beside 26. When 25 and 26 occur in the same line of epacts, the 25 is not accented, and in the calendar stands beside 24. The lines of epacts in which 24 and 25 both occur, are those which are marked by one of the eight letters b, e, l, n, r, B, E, N, in all of which 25' stands in a column corresponding to a golden number higher than 11. There are also eight lines in which 25 and 26 occur, namely, c, f, i, p, s, C, F, P. In the other 14 lines, 25 either does not occur at all, or it occurs in a line in which neither 24 nor 26 is found. From this it appears, that if

the golden number of the year exceeds 11, the epact 25, in six months of the year, must correspond to the same day in the calendar as 26; but if the golden number does not exceed 11, that epact must correspond to the same day as 24. Hence the reason for distinguishing 25 and 25'. In using the calendar, if the epact of the year is 25, and the golden number not above 11, take 25; but if the golden number exceeds 11, take 25'.

Another peculiarity requires explanation. The epact 19' (also distinguished by an accent or different character) is placed in the same line with 20 at the 31st of December. It is, however, only used in those years in which the epact 19 concurs with the golden number 19. When the golden number is 19, that is to say, in the last year of the lunar cycle, the supplementary month contains only 29 days. Hence, if in that year the epact should be 19, a new moon would fall on the 2nd of December, and the lunation would terminate on the 30th, so that the next new moon would arrive on the 31st. The epact of the year, therefore, or 19, must stand beside that day, whereas, according to the regular order, the epact corresponding to the 31st of December is 20; and this is the reason for the distinction.

As an example of the use of the preceding tables, suppose it were required to determine the moon's age on the 10th of April 1832. In 1832 the golden number is (1832+1)/19 = 9, and the line of epacts belonging to the century is C. In Table III. under 9, and in the line C, we find the epact 28. In the calendar, Table IV. look for April, and the epact 28 is found opposite the second day. The 2nd of April is therefore the first day of the moon, and the 10th is consequently the ninth day of the moon.

TABLE V.—Perpetual Table, showing Easter.

Table with columns for Epact and Dominical Letter (A-G). It shows the correspondence between the epact and the dominical letter for each day of the month, including leap year adjustments.

Again, suppose it were required to find the moon's age on the 2nd of December in the year 1916. In this case the golden number is (1916+1)/19 = 17, and in Table III., opposite to 1900, the line of epacts is B. Under 17, in line B, the epact is 25'. In the calendar this epact first occurs before the 2nd of December at the 26th of November. The 26th of November is consequently the first day of the moon, and the 2nd of December is therefore the seventh day.

Easter.—The next, and indeed the principal use of the calendar, is to find Easter, which, according to the regulation of the Council of Nice, must be determined from the following conditions:—1st, Easter must be celebrated on a Sunday; 2nd, this Sunday must follow the 14th day of the paschal moon, so that if the 14th of the paschal moon falls on a Sunday, then Easter must be celebrated on the Sunday following; 3rd, the paschal moon is that of which the 14th day falls on or next follows the day of the vernal equinox; 4th, the equinox is fixed invariably in the calendar on the 21st of March. Sometimes a misunderstanding has arisen from not observing that this regulation is to be construed according to the tabular full moon as determined from the epact, and not by the true full moon, which, in general, occurs one or two days earlier.

From these conditions it follows that the paschal full moon, or the 14th of the paschal moon, cannot happen before the 21st of March, and that Easter in consequence cannot happen before the 22nd of March. If the 14th of the moon falls on the 21st, the new moon must fall on the 8th; for 21 - 13 = 8; and the paschal new moon cannot happen before the 8th; for suppose the new moon to fall on the 7th, then the full moon would arrive on the 20th, or the day before the equinox. The following moon would be the paschal moon. But the fourteenth of this moon falls at the latest on the 18th of April, or 29 days after the 20th of March; for by reason of the double epact that occurs at the 4th and 5th of April, this lunation has only 29 days. Now, if in this case the 18th of April is Sunday, then Easter must be celebrated on the following Sunday, or the 25th of April. Hence Easter Sunday cannot happen earlier than the 22nd of March, or later than the 25th of April.

Hence we derive the following rule for finding Easter Sunday from the tables:—1st, Find the golden number, and, from Table III., the epact of the proposed year. 2nd, Find in the calendar (Table IV.) the first day after the 7th of March which corresponds to the epact of the year; this will be the first day of the paschal moon. 3rd, Reckon thirteen days after that of the first of the moon, the following will be the 14th of the moon, or the day of the full paschal moon. 4th, Find from Table I. the dominical letter of the year, and observe in the calendar the first day, after the fourteenth of the moon, which corresponds to the dominical letter; this will be Easter Sunday.

Example.—Required the day on which Easter Sunday falls in the year 1840? 1st, For this year the golden number is (1840+1)/19 = 17, and the epact (Table III. line C) is 26. 2nd, After the 7th of March the epact 26 first occurs in Table III. at the 4th of April, which, therefore, is the day of the new moon. 3rd, Since the new moon falls on the 4th, the full moon is on the 17th (4 + 13 = 17). 4th, The dominical letters of 1840 are E, D (Table I.), of which D must be taken, as E belongs only to January and February. After the 17th of April D first occurs in the calendar (Table IV.) at the 19th. Therefore, in 1840, Easter Sunday falls on the 19th of April. The operation is in all cases much facilitated by means of the following table:—

Such is the very complicated and artificial, though highly ingenious method, invented by Lilius, for the determination of Easter and the other movable feasts. Its principal, though perhaps least obvious advantage, consists in its being entirely independent of astronomical tables, or indeed of any celestial phenomena whatever; so that all chances of disagreement arising from the inevitable errors of tables, or the uncertainty of observation, are avoided, and Easter determined without the possibility of mistake. But this advantage is only procured by the sacrifice of some accuracy; for notwithstanding the cumbersome apparatus employed, the conditions of the problem are not always exactly satisfied, nor is it possible that they can be always satisfied by any similar method of proceeding. The equinox is fixed on the 21st of March, though the sun enters Aries generally on the 20th of that month, sometimes even on the 19th. It is accordingly quite possible that a full moon may arrive after the true equinox, and yet precede the 21st of March. This, therefore, would not be the paschal moon of the calendar, though it undoubtedly ought to be so, if the intention of the Council of Nice were rigidly followed. The new moons indicated by the epacts also differ from the astronomical new moons, and even from the mean new moons, in general by one or two days. In imitation of the Jews, who counted the time of the new moon, not from the moment of the actual phase, but from the time the moon first became visible after the conjunction, the fourteenth day of the moon is regarded as the full moon; but the moon is in opposition generally on the 16th day; therefore, when the new moons of the calendar nearly concur with the true new moons, the full moons are considerably in error. The epacts are also placed so as to indicate the full moons generally one or two days after the true full moons; but this was done purposely, to avoid the chance of concurring with the Jewish passover, which the framers of the calendar seem to have considered a greater evil than that of celebrating Easter a week too late.

We will now show in what manner this whole apparatus of methods and tables may be dispensed with, and the Gregorian calendar reduced to a few simple formulæ of easy computation.

And, first, to find the dominical letter. Let L denote the number of the dominical letter of any given year of the era. Then, since every year which is not a leap year ends with the same day as that with which it began, the dominical letter of the