following year must be L – 1, retrograding one letter every common year. After x years, therefore, the number of the letter will be $L - x$. But as L can never exceed 7, the number x will always exceed L after the first seven years of the era. In order therefore to render the subtraction possible, L must be increased by some multiple of 7, as $7m$, and the formula then becomes $7m + L - x$. In the year preceding the first of the era, the dominical letter was C; for that year, therefore, we have $L = 3$; consequently for any succeeding year x, $L = 7m + 3 - x$, the years being all supposed to consist of 365 days. But every fourth year is a leap year, and the effect of the intercalation is to throw the dominical letter one place farther back. The above expression must therefore be diminished by the number of units in $\frac{x}{4}$, or by $\left(\frac{x}{4}\right)_w$ (this notation being used to denote the quotient, *in a whole number*, that arises from dividing x by 4). Hence in the Julian calendar the dominical letter is given by the equation $L = 7m + 3 - x - \left(\frac{x}{4}\right)_w$.

This equation gives the dominical letter of any year from the commencement of the era to the reformation. In order to adapt it to the Gregorian calendar, we must first add the 10 days that were left out of the year 1582; in the second place we must add one day for every century that has elapsed since 1600, in consequence of the secular suppression of the intercalary day; and lastly we must deduct the units contained in a fourth of the same number, because every fourth centesimal year is still a leap year. Denoting therefore the number of the century (or the date after the two right-hand digits have been struck out) by c, the value of L must be increased by $10 + (c - 16) - \left(\frac{c-16}{4}\right)_w$. We have then

$$L = 7m + 3 - x - \left(\frac{x}{4}\right)_w + 10 + (c - 16) - \left(\frac{c-16}{4}\right)_w;$$

that is, since $3 + 10 = 13$ or 6 (the 7 days being rejected, as they do not affect the value of L),

$$L = 7m + 6 - x - \left(\frac{x}{4}\right)_w + (c - 16) - \left(\frac{c-16}{4}\right)_w;$$

This formula is perfectly general, and easily calculated.
As an example, let us take the year 1839. In this case,

$$x = 1839, \left(\frac{x}{4}\right)_w = \left(\frac{1839}{4}\right)_w = 459, c = 18, c - 16 = 2,$$

and $\left(\frac{c-16}{4}\right)_w = 0$. Hence

$$L = 7m + 6 - 1839 - 459 + 2 - 0$$
$$L = 7m - 2290 = 7 \times 328 - 2290.$$
$$L = 6 = \text{letter F}.$$

The year therefore begins with Tuesday. It will be remembered that in a leap year there are always two dominical letters, one of which is employed till the 29th of February, and the other till the end of the year. In this case, as the formula supposes the intercalation already made, the resulting letter is that which applies after the 29th of February. Before the intercalation the dominical letter had retrograded one place less. Thus for 1840 the formula gives D; during the first two months, therefore, the dominical letter is E.

In order to investigate a formula for the epact, let us make
E = the true epact of the given year;
J = the Julian epact, that is to say, the number the epact would have been if the Julian year had been still in use, and the lunar cycle had been exact;
S = the correction depending on the solar year;
M = the correction depending on the lunar cycle;
then the equation of the epact will be

$$E = J + S + M;$$

so that E will be known when the numbers J, S, and M are determined.

The epact J depends on the golden number N, and must be determined from the fact that in 1582, the first year of the reformed calendar, N was 6, and J 26. For the following years, then, the golden numbers and epacts are as follows:

$$1583, N = 7, \quad J = 26 + 11 - 30 = 7;$$
$$1584, N = 8, \quad J = 7 + 11 = 18;$$
$$1585, N = 9, \quad J = 18 + 11 = 29;$$
$$1586, N = 10, \quad J = 29 + 11 - 30 = 10;$$

and, therefore, in general $J = \left(\frac{26 + 11(N-6)}{30}\right)_r$. But the numerator of this fraction becomes by reduction $11N - 40$ or $11N - 10$ (the 30 being rejected, as the remainder only is sought) $= N + 10(N - 1)$; therefore, ultimately,

$$J = \left(\frac{N + 10(N-1)}{30}\right)_r.$$

On account of the solar equation S, the epact J must be diminished by unity every centesimal year, excepting always the fourth. After x centuries, therefore, it must be diminished by $x - \left(\frac{x}{4}\right)_w$. Now, as 1600 was a leap year, the first correction of the Julian intercalation took place in 1700; hence, taking c to denote the number of the century as before, the correction becomes $(c - 16) - \left(\frac{c-16}{4}\right)_w$, which must be deducted from J. We have therefore

$$S = -(c - 16) + \left(\frac{c-16}{4}\right)_w.$$

With regard to the lunar equation M, we have already stated that in the Gregorian calendar the epacts are increased by unity at the end of every period of 300 years seven times successively, and then the increase takes place once at the end of 400 years. This gives eight to be added in a period of twenty-five centuries, and $\frac{8x}{25}$ in x centuries. But $\frac{8x}{25} = \frac{1}{3}\left(x - \frac{x}{25}\right)$. Now, from the manner in which the intercalation is directed to be made (namely, seven times successively at the end of 300 years, and once at the end of 400), it is evident that the fraction $\frac{x}{25}$ must amount to unity when the number of centuries amounts to twenty-four. In like manner, when the number of centuries is $24 + 25 = 49$, we must have $\frac{x}{25} = 2$; when the number of centuries is $24 + 2 \times 25 = 74$, then $\frac{x}{25} = 3$; and, generally, when the number of centuries is $24 + n \times 25$, then $\frac{x}{25} = n + 1$. Now this is a condition which will evidently be expressed in general by the formula $n - \left(\frac{n+1}{25}\right)_w$. Hence the correction of the epact, or the number of days to be intercalated after x centuries reckoned from the commencement of one of the periods of twenty-five centuries, is $\left\{\frac{x - \left(\frac{x+1}{25}\right)_w}{3}\right\}_w$. The last period of twenty-five centuries terminated with 1800; therefore, in any succeeding year, if c be the number of the century, we shall have $x = c - 18$ and $x + 1 = c - 17$. Let $\left(\frac{c-17}{25}\right)_w = a$, then for all years after 1800 the value of M will be given by the formula $\left(\frac{c - 18 - a}{3}\right)_w$; therefore, counting from the beginning of the calendar in 1582,

$$M = \left\{\frac{c - 15 - a}{3}\right\}_w.$$

By the substitution of these values of J, S, and M, the equation of the epact becomes

$$E = \left(\frac{N + 10(N-1)}{30}\right)_r - (c - 16) + \left(\frac{c-16}{4}\right)_w + \left(\frac{c - 15 - a}{3}\right)_w.$$

It may be remarked, that as $a = \left(\frac{c-17}{25}\right)_w$, the value of a will be 0 till $c - 17 = 25$ or $c = 42$; therefore, till the year 4200, a may be neglected in the computation. Had the anticipation of the new moons been taken, as it ought to have been, at one day in 308 years instead of 312½, the lunar equation would have occurred only twelve times in 3700 years, or eleven times successively at the end of 300 years, and then at the end of 400. In strict accuracy, therefore, a ought to have no value till $c - 17 = 37$, or $c = 54$, that is to say, till the year 5400. The above formula for the epact is given by Delambre (*Hist. de l'Astronomie Moderne*, tom. i. p. 9); it may be exhibited under a variety of forms, but the above is perhaps the best adapted for calculation. Another had previously been given by Gauss, but inaccurately, inasmuch as the correction depending on a was omitted.

Having determined the epact of the year, it only remains to find Easter Sunday from the conditions already laid down. Let
P = the number of days from the 21st of March to the 15th of the paschal moon, which is the first day on which Easter Sunday can fall;
p = the number of days from the 21st of March to Easter Sunday.
L = the number of the dominical letter of the year;
l = letter belonging to the day on which the 15th of the moon falls;
then, since Easter is the Sunday following the 14th of the moon, we have

$$p = P + (L - l),$$

which is commonly called the *number of direction.*

The value of L is always given by the formula for the dominical letter, and P and l are easily deduced from the epact, as will appear from the following considerations.

When $P = 1$, the full moon is on the 21st of March, and the new moon on the eighth ($21 - 13 = 8$), therefore the moon's age on the 1st of March (which is the same as on the 1st of January) is twenty-three days; the epact of the year is consequently twenty-three. When $P = 2$ the new moon falls on the ninth, and the epact is consequently twenty-two; and, in general, when P becomes $1 + x$, E becomes $23 - x$, therefore $P + E = 1 + x + 23 - x = 24$, and $P = 24 - E$. In like manner, when $P = 1$, $l = D = 4$; for D is the dominical letter of the calendar belonging to the 22nd of March. But it is evident that when l is increased by unity, that is to say, when the full moon falls a day later, the epact of the year is diminished by unity; therefore, in general, when $l = 4 + x$, $E = 23 - x$, whence $l + E = 27$ and $l = 27 - E$. But P can never be less than 1 nor l less than 4, and in both cases $E = 23$. When, therefore, E is greater than 23, we must add 30 in order that P and l may have positive values in the formula $P = 24 - E$ and $l = 27 - E$. Hence there are two cases.

$$\text{When} < 24, \begin{cases} P = 24 - E \\ l = 27 - E, \text{ or } \left(\frac{27 - E}{7}\right)_r, \end{cases}$$

$$\text{When } E > 23, \begin{cases} P = 54 - E \\ l = 57 - E, \text{ or } \left(\frac{57 - E}{7}\right)_r \end{cases}$$

By substituting one or other of these values of P and l, according as the case may be, in the formula $p = P + (L - l)$, we shall have p, or the number of days from the 21st of March to Easter Sunday. It will be remarked, that as $L - l$ cannot either be 0 or negative, we must add 7 to L as often as may be necessary, in order that $L - l$ may be a positive whole number.

By means of the formulæ which we have now given for the dominical letter, the golden number, and the epact, Easter Sunday may be computed for any year after the reformation, without the assistance of any tables whatever. As an example, suppose it were required to compute Easter for the year 1840. By substituting this number in the formula for the dominical letter, we have $x = 1840$, $c - 16 = 2$, $\left(\frac{c-16}{4}\right)_w = 0$, therefore

$$L = 7m + 6 - 1840 - 460 + 2$$
$$= 7m - 2292$$
$$= 7 \times 328 - 2292 = 2296 - 2292 = 4$$
$$L = 4 = \text{letter D} \ldots\ldots\ldots\ldots (1).$$

For the golden number we have $N = \left(\frac{1840 + 1}{19}\right)_r$;
therefore $N = 17 \ldots\ldots\ldots\ldots (2).$

For the epact we have $\left(\frac{N + 10(N-1)}{30}\right)_r = \left(\frac{17 + 160}{30}\right)_r = \left(\frac{177}{30}\right)_r = 27$; likewise $c - 16 = 18 - 16 = 2$, $\frac{c - 15}{3} = 1$, $a = 0$; therefore

$$E = 27 - 2 + 1 = 26 \ldots\ldots\ldots\ldots (3).$$

Now since $E > 23$, we have for P and l,

$$P = 54 - E = 54 - 26 = 28,$$
$$l = \left(\frac{57 - E}{7}\right)_r = \left(\frac{57 - 26}{7}\right)_r = \left(\frac{31}{7}\right)_r = 3;$$

consequently, since $p = P + (L - l)$,

$$p = 28 + (4 - 3) = 29;$$

that is to say, Easter happens twenty-nine days after the 21st of March, or on the 19th April, the same result as was before found from the tables.

The principal church feasts depending on Easter, and the times of their celebration, are as follows:—

Septuagesima Sunday	is	9 weeks	before Easter.
First Sunday in Lent		6 weeks	
Ash Wednesday		46 days	
Rogation Sunday	is	5 weeks	after Easter.
Ascension day or Holy Thursday		39 days	
Pentecost or Whitsunday		7 weeks	
Trinity Sunday		8 weeks	

The Gregorian calendar was introduced into Spain, Portugal, and part of Italy, the same day as at Rome. In France it was received in the same year in the month of December, and by the Catholic states of Germany the year following. In the Protestant states of Germany the Julian calendar was adhered to till the year 1700, when it was decreed by the diet of Ratisbon that the new style and the Gregorian correction of the intercalation should be adopted. Instead, however, of employing the golden numbers and epacts for the determination of Easter and the movable feasts, it was resolved that the equinox and the paschal moon should be found by astronomical computation from the Rudolphine tables. But this method, though at first view it may appear more accurate, was soon found to be attended with numerous inconveniences, and was at length, in 1774, abandoned at the instance of Frederick II. king of Prussia. In Denmark and Sweden the reformed calendar was received about the same time as in the Protestant states of Germany. It is remarkable that Russia still adheres to the Julian reckoning.

In Great Britain the alteration of the style was for a long time successfully opposed by popular prejudice. The inconvenience, however, of using a different date from that employed by the greater part of Europe, in matters of history and chronology, began to be generally felt; and at length, in 1751, an Act of Parliament was passed for the adoption of the new style in all public and legal transactions. The difference of the two styles, which then amounted to eleven days, was removed by ordering the day following the 2d of September of the year 1752 to be accounted the 14th of that month; and in order to preserve uniformity in future, the Gregorian rule of intercalation respecting the secular years was adopted. At the same time, the commencement of the legal year was changed from the 25th of April to the 1st of January. In Scotland, the new style was adopted from the beginning of 1600, according to an Act of the privy council in December 1599. This fact is of importance with reference to the date of legal deeds executed in Scotland between that period and 1751, when the change was effected in England. With respect to the movable feasts, Easter is determined by the rule laid down by the Council of Nice; but instead of employing the new moons and epacts, the golden numbers are prefixed to the days of the *full* moons. In those years in which the line of epacts is changed in the Gregorian calendar, the golden numbers are removed to different days, and of course a new table is required whenever the solar or lunar equation occurs. The golden numbers have been placed so that Easter may fall on the same day as in the Gregorian calendar. The calendar of the church of England is therefore from century to century the same in form as the old Roman calendar, excepting that the golden numbers indicate the full moons instead of the new moons.

HEBREW CALENDAR.—In the construction of the Jewish calendar numerous details require attention. The calendar is dated from the Creation, which is considered to have taken place 3760 years and 3 months before the commencement of the Christian era. The year is luni-solar, and, according as it is ordinary or embolismic, consists of twelve or thirteen lunar months, each of which has 29 or 30 days. Thus the duration of the ordinary year is 354 days, and that of the embolismic is 384 days. In either case, it is sometimes made a day more, and sometimes a day less, in order that certain festivals may fall on proper days of the week for their due observance. The distribution of the embolismic years, in each cycle of 19 years, is determined according to the following rule:—

The number of the Hebrew year (Y) which has its commencement in a Gregorian year (x) is obtained by the addition of 3761 years; that is, $Y = x + 3761$. Divide the Hebrew year by 19; then the quotient is the number of the last completed cycle, and the remainder is the year of the current cycle. If the remainder be 3, 6, 8, 11, 14, 17, or 19 (0), the year is embolismic; if any other number, it is ordinary. Or, otherwise, if we find the remainder

$$R = \left(\frac{2Y + 1}{19}\right)_r$$

the year is embolismic when $R < 7$

The calendar is constructed on the assumptions that the mean lunation is 29 days 12 hours 44 min. $3\frac{1}{3}$ sec., and that the year commences on, or immediately after, the new moon following the autumnal equinox. The mean solar year is also assumed to be 365 days 5 hours 55 min. $25\frac{25}{57}$ sec., so that a cycle of nineteen of such years, containing 6939 days 16 hours 33 min. $3\frac{1}{3}$ sec., is the exact measure of 235 of the assumed lunations. The year 5606 was the first of a cycle, and the mean new moon, appertaining to the 1st of Tisri for that year, was 1845, October 1, 15 hours 42 min. $43\frac{1}{3}$ sec., as computed by Lindo, and adopting the civil mode of reckoning from the previous midnight. The times of all future new moons may consequently be deduced by successively adding 29 days 12 hours 44 min. $3\frac{1}{3}$ sec. to this date.

To compute the times of the new moons which determine the commencement of successive years, it must be observed that in passing from an ordinary year the new moon of the following year is deduced by subtracting the interval that twelve lunations fall short of the corresponding Gregorian year of 365 or 366 days; and that, in passing from an embolismic year, it is to be found by adding the excess of thirteen lunations over the Gregorian year. Thus to deduce the new moon of Tisri, for the year immediately following any given year (Y), when Y is

$$\begin{cases} \text{ordinary, subtract} & \binom{10}{11} \text{ days 15 hours 11 min. 20 sec.,} \\ \text{embolismic, add} & \binom{18}{17} \text{ days 21 hours 32 min. } 43\tfrac{1}{3} \text{ sec.,} \end{cases}$$

the second-mentioned number of days being used, in each case, whenever the following or new Gregorian year is bissextile.

Hence, knowing which of the years are embolismic, from their ordinal position in the cycle, according to the rule before stated, the times of the commencement of successive years may be thus carried on indefinitely without any difficulty. But some slight adjustments will occasionally be needed for the reasons before assigned, viz., to avoid certain festivals falling on incompatible days of the week. Whenever the computed conjunction falls on a Sunday, Wednesday, or Friday, the new year is in such case to be fixed on the day after. It will also be requisite to attend to the following conditions:—

If the computed new moon be after 18 hours, the following day is to be taken, and if that happen to be Sunday, Wednesday, or Friday, it must be further postponed one day. If, for an ordinary year, the new moon falls on a Tuesday, as late as 9 hours 11 min. 20 sec., it is not to be observed thereon; and as it may not be held on a Wednesday, it is in such case to be postponed to Thursday. If, for a year immediately following an embolismic year, the computed new moon is on Monday, as late as 15 hours 30 min. 52 sec., the new year is to be fixed on Tuesday.

After the dates of commencement of the successive Hebrew years are finally adjusted, conformably with the foregoing directions, an estimation of the consecutive intervals, by taking the differences, will show the duration and character of the years that respectively intervene. According to the number of days thus found to be comprised in the different years, the days of the several months are distributed as in Table VI.

The signs + and – are respectively annexed to Hesvan and Kislev to indicate that the former of these months may sometimes require to have one day more, and the latter sometimes one day less, than the number of days shown in the table,—the result, in every case, being at once determined by the total number of days that the year may happen to contain. An ordinary year may comprise 353, 354, or 355 days; and an embolismic year 383, 384, or 385 days. In these cases respectively the year is said to be imperfect, common, or perfect. The intercalary month, Veadar, is introduced in embolismic years in order that Passover, the 15th day of Nisan, may be kept at its proper season, which is the full moon of the vernal equinox, or that which takes place after the sun has entered the sign Aries. It always precedes the following new year by 163 days, or 23 weeks and 2 days; and Pentecost always precedes the new year by 113 days, or 16 weeks and 1 day.

TABLE VI.—*Hebrew Months.*

Hebrew Month	Ordinary Year	Embolismic Year
Tisri	30	30
Hesvan	29+	29+
Kislev	30–	30–
Tebet	29	29
Sebat	30	30
Adar	29	30
(Veadar)	(...)	(29)
Nisan	30	30
Yiar	29	29
Sivan	30	30
Tamuz	29	29
Ab	30	30
Elul	29	29
Total	354	384

The Gregorian epact being the age of the moon of Tebet at the beginning of the Gregorian year, it represents the day of Tebet which corresponds to January 1; and thus the approximate date of Tisri 1, the commencement of the Hebrew year, may be otherwise deduced by subtracting the epact from

$$\left.\begin{matrix}\text{Sept. 24}\\ \text{Oct. 24}\end{matrix}\right\} \text{after an} \left\{\begin{matrix}\text{ordinary}\\ \text{embolismic}\end{matrix}\right\} \text{Hebrew year.}$$

The result so obtained would in general be more accurate than the Jewish calculation, from which it may differ a day, as fractions of a day do not enter alike in these computations. Such difference may also in part be accounted for by the fact that the assumed duration of the solar year is 6 min. $39\frac{25}{57}$ sec. in excess of the true astronomical value, which will cause the dates of commencement of future Jewish years, so calculated, to advance forward from the equinox a day in error in 216 years. The lunations are estimated with much greater precision.

The following table is extracted from Woolhouse's *Measures, Weights, and Moneys of all Nations*:—

TABLE VII.—*Hebrew Years.*

	Jewish Year.	Number of Days.	Commencement (1st of Tisri).			Jewish Year.	Number of Days.	Commencement (1st of Tisri).	
296 Cycle.	5606	354	Thur.	2 Oct. 1845	297 Cycle.	5625	355	Sat.	1 Oct. 1864
	07	355	Mon.	21 Sept. 1846		26	354	Thur.	21 Sept. 1865
	08	383	Sat.	11 Sept. 1847		27	385	Mon.	10 Sept. 1866
	09	354	Thur.	28 Sept. 1848		28	353	Mon.	30 Sept. 1867
	10	355	Mon.	17 Sept. 1849		29	354	Thur.	17 Sept. 1868
	11	385	Sat.	7 Sept. 1850		30	385	Mon.	6 Sept. 1869
	12	353	Sat.	27 Sept. 1851		31	355	Mon.	26 Sept. 1870
	13	384	Tues.	14 Sept. 1852		32	383	Sat.	16 Sept. 1871
	14	355	Mon.	3 Oct. 1853		33	354	Thur.	3 Oct. 1872
	15	355	Sat.	23 Sept. 1854		34	355	Mon.	22 Sept. 1873
	16	383	Thur.	13 Sept. 1855		35	383	Sat.	12 Sept. 1874
	17	354	Tues.	30 Sept. 1856		36	355	Thur.	30 Sept. 1875
	18	355	Sat.	19 Sept. 1857		37	354	Tues.	19 Sept. 1876
	19	385	Thur.	9 Sept. 1858		38	385	Sat.	8 Sept. 1877
	20	354	Thur.	29 Sept. 1859		39	355	Sat.	28 Sept. 1878
	21	353	Mon.	17 Sept. 1860		40	354	Thur.	18 Sept. 1879
	22	385	Thur.	5 Sept. 1861		41	383	Mon.	6 Sept. 1880
	23	354	Thur.	25 Sept. 1862		42	355	Sat.	24 Sept. 1881
	24	383	Mon.	14 Sept. 1863		43	383	Thur.	14 Sept. 1882

TABLE VII.—*Hebrew Years (continued).*

	Jewish Year.	Number of Days	Commencement (1st of Tisri).			Jewish Year.	Number of Days.	Commencement (1st of Tisri).	
298 Cycle.	5644	354	Tues.	2 Oct. 1883	302 Cycle.	5720	355	Sat.	3 Oct. 1959
	45	355	Sat.	20 Sept. 1884		21	354	Thur.	22 Sept. 1960
	46	385	Thur.	10 Sept. 1885		22	383	Mon.	11 Sept. 1961
	47	354	Thur.	30 Sept. 1886		23	355	Sat.	29 Sept. 1962
	48	353	Mon.	19 Sept. 1887		24	354	Thur.	19 Sept. 1963
	49	385	Thur.	6 Sept. 1888		25	385	Mon.	7 Sept. 1964
	50	354	Thur.	26 Sept. 1889		26	353	Mon.	27 Sept. 1965
	51	383	Mon.	15 Sept. 1890		27	385	Thur.	15 Sept. 1966
	52	355	Sat.	3 Oct. 1891		28	354	Thur.	5 Oct. 1967
	53	354	Thur.	22 Sept. 1892		29	355	Mon.	23 Sept. 1968
	54	385	Mon.	11 Sept. 1893		30	383	Sat.	13 Sept. 1969
	55	353	Mon.	1 Oct. 1894		31	354	Thur.	1 Oct. 1970
	56	355	Thur.	19 Sept. 1895		32	355	Mon.	20 Sept. 1971
	57	384	Tues.	8 Sept. 1896		33	383	Sat.	9 Sept. 1972
	58	355	Mon.	27 Sept. 1897		34	355	Thur.	27 Sept. 1973
	59	353	Sat.	17 Sept. 1898		35	354	Tues.	17 Sept. 1974
	60	384	Tues.	5 Sept. 1899		36	385	Sat.	6 Sept. 1975
	61	355	Mon.	24 Sept. 1900		37	353	Sat.	25 Sept. 1976
	62	383	Sat.	14 Sept. 1901		38	384	Tues.	13 Sept. 1977
299 Cycle.	5663	355	Thur.	2 Oct. 1902	303 Cycle.	5739	355	Mon.	2 Oct. 1978
	64	354	Tues.	22 Sept. 1903		40	355	Sat.	22 Sept. 1979
	65	385	Sat.	10 Sept. 1904		41	383	Thur.	11 Sept. 1980
	66	355	Sat.	30 Sept. 1905		42	354	Tues.	29 Sept. 1981
	67	354	Thur.	20 Sept. 1906		43	355	Sat.	18 Sept. 1982
	68	383	Mon.	9 Sept. 1907		44	385	Thur.	8 Sept. 1983
	69	355	Sat.	26 Sept. 1908		45	354	Thur.	27 Sept. 1984
	70	383	Thur.	16 Sept. 1909		46	383	Mon.	16 Sept. 1985
	71	354	Tues.	4 Oct. 1910		47	355	Sat.	4 Oct. 1986
	72	355	Sat.	23 Sept. 1911		48	354	Thur.	24 Sept. 1987
	73	385	Thur.	12 Sept. 1912		49	383	Mon.	12 Sept. 1988
	74	354	Thur.	2 Oct. 1913		50	355	Sat.	30 Sept. 1989
	75	353	Mon.	21 Sept. 1914		51	354	Thur.	20 Sept. 1990
	76	385	Thur.	9 Sept. 1915		52	385	Mon.	9 Sept. 1991
	77	354	Thur.	28 Sept. 1916		53	353	Mon.	28 Sept. 1992
	78	355	Mon.	17 Sept. 1917		54	355	Thur.	16 Sept. 1993
	79	383	Sat.	7 Sept. 1918		55	384	Tues.	6 Sept. 1994
	80	354	Thur.	25 Sept. 1919		56	355	Mon.	25 Sept. 1995
	81	385	Mon.	13 Sept. 1920		57	383	Sat.	14 Sept. 1996
300 Cycle.	5682	355	Mon.	3 Oct. 1921	304 Cycle.	5758	354	Thur.	2 Oct. 1997
	83	353	Sat.	23 Sept. 1922		59	355	Mon.	21 Sept. 1998
	84	384	Tues.	11 Sept. 1923		60	385	Sat.	11 Sept. 1999
	85	355	Mon.	29 Sept. 1924		61	353	Sat.	30 Sept. 2000
	86	355	Sat.	19 Sept. 1925		62	354	Tues.	18 Sept. 2001
	87	383	Thur.	9 Sept. 1926		63	385	Sat.	7 Sept. 2002
	88	354	Tues.	27 Sept. 1927		64	355	Sat.	27 Sept. 2003
	89	385	Sat.	15 Sept. 1928		65	383	Thur.	16 Sept. 2004
	90	353	Sat.	5 Oct. 1929		66	354	Tues.	4 Oct. 2005
	91	354	Tues.	23 Sept. 1930		67	355	Sat.	23 Sept. 2006
	92	385	Sat.	12 Sept. 1931		68	383	Thur.	13 Sept. 2007
	93	355	Sat.	1 Oct. 1932		69	354	Tues.	30 Sept. 2008
	94	354	Thur.	21 Sept. 1933		70	355	Sat.	19 Sept. 2009
	95	383	Mon.	10 Sept. 1934		71	385	Thur.	9 Sept. 2010
	96	355	Sat.	28 Sept. 1935		72	354	Thur.	29 Sept. 2011
	97	354	Thur.	17 Sept. 1936		73	353	Mon.	17 Sept. 2012
	98	385	Mon.	6 Sept. 1937		74	385	Thur.	5 Sept. 2013
	99	353	Mon.	26 Sept. 1938		75	354	Thur.	25 Sept. 2014
	5700	385	Thur.	14 Sept. 1939		76	385	Mon.	14 Sept. 2015
301 Cycle.	5701	354	Thur.	3 Oct. 1940	305 Cycle.	5777	353	Mon.	3 Oct. 2016
	02	355	Mon.	22 Sept. 1941		78	354	Thur.	21 Sept. 2017
	03	383	Sat.	12 Sept. 1942		79	385	Mon.	10 Sept. 2018
	04	354	Thur.	30 Sept. 1943		80	355	Mon.	30 Sept. 2019
	05	355	Mon.	18 Sept. 1944		81	353	Sat.	19 Sept. 2020
	06	383	Sat.	8 Sept. 1945		82	384	Tues.	7 Sept. 2021
	07	354	Thur.	26 Sept. 1946		83	355	Mon.	26 Sept. 2022
	08	385	Mon.	15 Sept. 1947		84	383	Sat.	16 Sept. 2023
	09	355	Mon.	4 Oct. 1948		85	355	Thur.	3 Oct. 2024
	10	353	Sat.	24 Sept. 1949		86	354	Tues.	23 Sept. 2025
	11	384	Tues.	12 Sept. 1950		87	385	Sat.	12 Sept. 2026
	12	355	Mon.	1 Oct. 1951		88	355	Sat.	2 Oct. 2027
	13	355	Sat.	20 Sept. 1952		89	354	Thur.	21 Sept. 2028
	14	383	Thur.	10 Sept. 1953		90	383	Mon.	10 Sept. 2029
	15	354	Tues.	28 Sept. 1954		91	355	Sat.	28 Sept. 2030
	16	355	Sat.	17 Sept. 1955		92	354	Thur.	18 Sept. 2031
	17	385	Thur.	6 Sept. 1956		93	383	Mon.	6 Sept. 2032
	18	354	Thur.	26 Sept. 1957		94	355	Sat.	24 Sept. 2033
	19	383	Mon.	15 Sept. 1958		95	385	Thur.	14 Sept. 2034

TABLE VII.—*Hebrew Years (continued).*

	Jewish Year.	Number of Days.	Commencement (1st of Tisri).			Jewish Year.	Number of Days.	Commencement (1st of Tisri).	
306 Cycle.	5796	354	Thur.	4 Oct. 2035	307 Cycle.	5815	355	Sat.	3 Oct. 2054
	97	353	Mon.	22 Sept. 2036		16	354	Thur.	23 Sept. 2055
	98	385	Thur.	10 Sept. 2037		17	383	Mon.	11 Sept. 2056
	99	354	Thur.	30 Sept. 2038		18	355	Sat.	29 Sept. 2057
	5800	355	Mon.	19 Sept. 2039		19	354	Thur.	19 Sept. 2058
	01	383	Sat.	8 Sept. 2040		20	383	Mon.	8 Sept. 2059
	02	354	Thur.	26 Sept. 2041		21	355	Sat.	25 Sept. 2060
	03	385	Mon.	15 Sept. 2042		22	385	Thur.	15 Sept. 2061
	04	353	Mon.	5 Oct. 2043		23	354	Thur.	5 Oct. 2062
	05	355	Thur.	22 Sept. 2044		24	353	Mon.	24 Sept. 2063
	06	384	Tues.	12 Sept. 2045		25	385	Thur.	11 Sept. 2064
	07	355	Mon.	1 Oct. 2045		26	354	Thur.	1 Oct. 2065
	08	353	Sat.	21 Sept. 2047		27	355	Mon.	20 Sept. 2066
	09	384	Tues.	8 Sept. 2048		28	383	Sat.	10 Sept. 2067
	10	355	Mon.	27 Sept. 2049		29	354	Thur.	27 Sept. 2068
	11	355	Sat.	17 Sept. 2050		30	355	Mon.	16 Sept. 2069
	12	383	Thur.	7 Sept. 2051		31	383	Sat.	6 Sept. 2070
	13	354	Tues.	24 Sept. 2052		32	355	Thur.	24 Sept. 2071
	14	385	Sat.	13 Sept. 2053		33	384	Tues.	13 Sept. 2072

MAHOMETAN CALENDAR.—The Mahometan era, or era of the Hegira, employed in Turkey, Persia, Arabia, &c., is dated from the flight of Mahomet from Mecca to Medina, which was in the night of Thursday the 15th of July 622 A.D., and it commenced on the day following. The years of the Hegira are purely lunar, and always consist of twelve lunar months, commencing with the approximate new moon, without any intercalation to keep them to the same season with respect to the sun, so that they retrograde through all the seasons in about $32\frac{1}{2}$ years. They are also partitioned into cycles of 30 years, 19 of which are common years of 354 days each, and the other 11 are intercalary years having an additional day appended to the last month. The mean length of the year is therefore $354\frac{11}{30}$ days, or 354 days 8 hours 48 min., which divided by 12 gives $29\frac{191}{360}$ days, or 29 days 12 hours 44 min., as the time of a mean lunation, and this differs from the astronomical mean lunation by only 2·8 seconds. This small error will only amount to a day in about 2400 years.

To find if a year is intercalary or common, divide it by 30; the quotient will be the number of completed cycles and the remainder will be the year of the current cycle; if this last be one of the numbers 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, 29, the year is intercalary and consists of 355 days; if it be any other number, the year is ordinary.

Or if Y denote the number of the Mahometan year, and

$$R = \left(\frac{11\,Y + 14}{30}\right)_r,$$

the year is intercalary when $R < 11$.

Also the number of intercalary years from the year 1 up to the year Y inclusive $= \left(\frac{11\,Y + 14}{30}\right)_w$; and the same up to the year $Y - 1 = \left(\frac{11\,Y + 3}{30}\right)_w$.

To find the day of the week on which any year of the Hegira begins, we observe that the year 1 began on a Friday, and that after every common year of 354 days, or 50 weeks and 4 days, the day of the week must necessarily become postponed 4 days, besides the additional day of each intercalary year.

Hence if $w = 1$ indicate Sun.	2 Mon.	3 Tues.	4 Wed.	5 Thur.	6 Frid.	7 Sat.

the day of the week on which the year Y commences will be

$$w = 2 + 4\left(\frac{Y}{7}\right)_r + \left(\frac{11\,Y + 3}{30}\right)_w \text{ (rejecting sevens).}$$

$$\text{But, } 30\left(\frac{11\,Y + 3}{30}\right)_w + \left(\frac{11\,Y + 3}{30}\right)_r = 11\,Y + 3$$

$$\text{gives } 120\left(\frac{11\,Y + 3}{30}\right)_w = 12 + 44\,Y - 4\left(\frac{11\,Y + 3}{30}\right)_r,$$

$$\text{or } \left(\frac{11\,Y + 3}{30}\right)_w = 5 + 2\,Y + 3\left(\frac{11\,Y + 3}{30}\right)_r \text{ (rejecting sevens).}$$