ca.. see, however, that the change would necessarily follow the introduction of clocks and other mechanical methods of measuring time; for, however imperfect these were, the hours they marked would be of the same length in summer and in winter, and the discrepancy between these equal hours and the temporary hours of the sun-dial would soon be too important to be overlooked. Now, we know that a balance clock was put up in the palace of Charles V. of France about the year 1370, and we may reasonably suppose that the new sun-dials came into general use during the 14th and 15th centuries.

Among the earliest of the modern writers on gnomonics must be named Sebastian Munster, a cordelier who published his *Horologiographia* at Basel in 1531. He gives a number of correct rules, but without demonstrations. Among his inventions was a moon-dial,[1] but this does not admit of much accuracy.

During the 17th century dialling was discussed at great length by all writers on astronomy. Clavius devotes a quarto volume of 800 pages entirely to the subject. This was published in 1612, and may be considered to contain all that was known at that time.

In the 18th century clocks and watches began to supersede sun-dials, and these have gradually fallen into disuse except as an additional ornament to a garden, or in remote country districts where the old dial on the church tower still serves as an occasional check on the modern clock by its side. The art of constructing dials may now be looked upon as little more than a mathematical recreation.

General Principles.—The diurnal and the annual motions of the earth are the elementary astronomical facts on which dialling is founded. That the earth turns upon its axis uniformly from west to east in 24 hours, and that it is carried round the sun in one year at a nearly uniform rate, is, we know, the correct way of expressing these facts. But the effect will be precisely the same, and it will suit our purpose better, and make our explanations easier, if we adopt the ideas of the ancients, of which our senses furnish apparent confirmation, and assume the earth to be fixed. Then, the sun and stars revolve round the earth's axis uniformly from east to west once a day,—the sun lagging a little behind the stars, making its day some 4 minutes longer, so that at the end of the year it finds itself again in the same place, having made a complete revolution of the heavens relatively to the stars from west to east.

The fixed axis about which all these bodies revolve daily is a line through the earth's centre; but the radius of the earth is so small, compared with the enormous distance of the sun, that, if we draw a parallel axis through any point of the earth's surface, we may safely look on that as being the axis of the celestial motions. The error in the case of the sun would not, at its maximum, that is, at 6 A.M. and 6 P.M., exceed half a second of time. and at noon would vanish.

An axis so drawn is in the plane of the meridian, and points, as we know, to the pole,—its elevation being equal to the latitude of the place.

The diurnal motion of the stars is strictly uniform, and so would that of the sun be if the daily retardation of about 4 minutes, spoken of above, were always the same. But this is constantly altering, so that the time, as measured by the sun's motion, and also consequently as measured by a sun-dial, does not move on at a strictly uniform pace. This irregularity, which is slight, would be of little consequence in the ordinary affairs of life, but clocks and watches being mechanical measures of time could not, except by extreme complication, be made to follow this irregularity, even if desirable, which is not the case.

The clock is constructed to mark uniform time in such wise that the length of the clock day shall be the average of all the solar days in the year. Four times a year the clock and the sun-dial agree exactly; but the sun-dial, now going a little slower, now a little faster, will be sometimes behind, sometimes before the clock—the greatest accumulated difference being about 16 minutes for a few days in November, but on the average much less. The four days on which the two agree are April 15, June 15, September 1, and December 24.

Clock-time is called *mean time*, that marked by the sun-dial is called *apparent time*, and the difference between them is the *equation of time*. It is given in most calendars and almanacs, frequently under the heading "clock slow," "clock fast." When the time by the sun-dial is known, the equation of time will at once enable us to obtain the corresponding clock time, or *vice versa*.

Atmospheric refraction introduces another error, by altering the apparent position of the sun; but the effect is too small to need consideration in the construction of an instrument which, with the best workmanship, does not after all admit of very great accuracy.

The general principles of dialling will now be readily understood. The problem before us is the following:—A rod, or *style*, as it is called, being firmly fixed in a direction parallel to the earth's axis, we have to find how and where points or lines of reference must be traced on some fixed surface behind the style, so that when the shadow of the style falls on a certain one of these lines we may know that at that moment it is solar noon,—that is, that the plane through the style and through the sun then coincides with the meridian; again, that when the shadow reaches the next line of reference, it is 1 o'clock by solar time, or, which comes to the same thing, that the above plane through the style and through the sun has just turned through the twenty-fourth part of a complete revolution; and so on for the subsequent hours,—the hours before noon being indicated in a similar manner. The style and the surface on which these lines are traced together constitute the dial.

The position of an intended sun-dial having been selected—whether on church tower, south front of farm-stead, or garden wall—the surface must be prepared, if necessary, to receive the hour-lines.

The chief, and in fact the only practical difficulty will be the accurate fixing of the style, for on its accuracy the value of the instrument depends.

It must be in the meridian plane, and must make an angle with the horizon equal to the latitude of the place. The latter condition will offer no difficulty, but the exact determination of the meridian plane which passes through the point where the style is fixed to the surface is not so simple. We shall, further on, show how this may be done; and, in the meantime, we shall assume that we have found the true position, and have firmly fixed the style to the dial and secured it there by cross wires, or by other means. The style itself will be usually a strong metal wire whose thickness may vary with circumstances; and when we speak of the shadow cast by the style it must always be understood that the middle line of the thin band of shade is meant.

The point where the style meets the dial is called the centre of the dial. It is the centre from which all the hour-lines radiate.

The position of the XII o'clock line is the most important to determine accurately, since all the others are usually made to depend on this one. We cannot trace it correctly

[1] In one of the Courts of Queen's College, Cambridge, there is an elaborate sun-dial dating from the end of the 17th or beginning of the 18th century, and around it a series of numbers which make it available as a moon-dial when the moon's age is known.

on the dial until the style has been itself accurately fixed in its proper place, as will be explained hereafter. When that is done the XII o'clock line will be found by the intersection of the dial surface with the vertical plane which contains the style; and the most simple way of drawing it on the dial will be by suspending a plummet from some point of the style whence it may hang freely, and waiting until the shadows of both style and plumb line coincide on the dial. This single shadow will be the XII o'clock line.

In one class of dials, namely, all the vertical ones, the XII o'clock line is simply the vertical line from the centre; it can, therefore, at once be traced on the dial face by using a fine plumb line.

The XII o'clock line being traced, the easiest and most accurate method of tracing the other hour lines would at the present day when good watches are common, be by marking where the shadow of the style falls when 1, 2, 3, &c., hours have elapsed since noon, and the next morning by the same means the forenoon hour lines could be traced; and in the same manner the hours might be subdivided into halves and quarters, or even into minutes.

But formerly, when watches were not, the tracing of the I, II, III, &c. o'clock lines was done by calculating the angle which each of these lines would make with the XII o'clock line. Now, except in the simple cases of a horizontal dial or of a vertical dial facing a cardinal point, this would require long and intricate calculations, or elaborate geometrical constructions, implying considerable mathematical knowledge, but also introducing increased chances of error. The chief source of error would lie in the uncertainty of the data; for the position of the dial-plane would have to be found before the calculations began,—that is, it would be necessary to know exactly by how many degrees it declined from the south towards the east or west, and by how many degrees it inclined from the vertical. The ancients, with the means at their disposal, could obtain these results only very roughly.

Dials received different names according to their position:—

Horizontal dials, when traced on a horizontal plane;

Vertical dials, when on a vertical plane facing one of the cardinal points;

Vertical declining dials, on a vertical plane not facing a cardinal point;

Inclining dials, when traced on planes neither vertical nor horizontal (these were further distinguished as *reclining* when leaning backwards from an observer, *proclining* when leaning forwards);

Equinoctial dials, when the plane is at right angles to the earth's axis, &c. &c.

We shall limit ourselves to an investigation of the simplest and most usual of these cases, referring the reader, for further details, to the later works given at the end of this article.

Dial Construction.—A very correct view of the problem of dial construction may be obtained as follows:—

Conceive a transparent cylinder (fig. 1) having an axis AB parallel to the axis of the earth. On the surface of the cylinder let equidistant generating lines be traced 15° apart, one of them XII.. XII being in the meridian plane through AB, and the others I...I, II...II, &c., following in the order of the sun's motion.

Then the shadow of the line AB will obviously fall on the line XII...XII at apparent noon, on the line I...I at one hour after noon, on II...II at two hours after noon, and so on. If now the cylinder be cut by any plane MN representing the plane on which the dial is to be traced, the shadow of AB will be intercepted by this plane, and fall on the lines AXII, AI, AII, &c.

The construction of the dial consists in determining the angles made by AI, AII, &c. with AXII; the line AXII itself, being in the vertical plane through AB, may be supposed known.

For the purposes of actual calculation, perhaps a transparent sphere will, with advantage, replace the cylinder, and we shall here apply it to calculate the angles made by the hour line with the XII o'clock line in the two cases of a horizontal dial and of a vertical south dial.

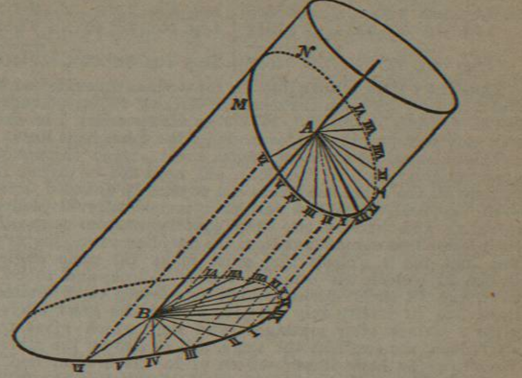

Fig. 1.

Horizontal Dial.—Let PEp (fig. 2), the axis of the supposed transparent sphere, be directed towards the north and south poles of the heavens. Draw the two great circles, HMA, QMa,

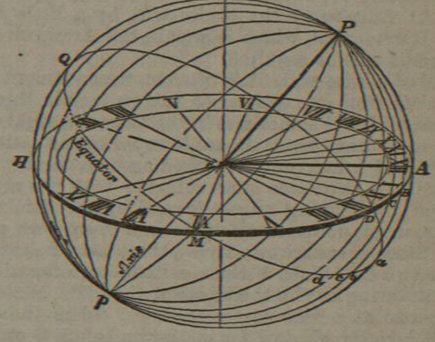

Fig. 2.

the former horizontal, the other perpendicular to the axis Pp, and therefore coinciding with the plane of the equator. Let EZ be vertical, then the circle QZP will be the meridian, and by its intersection A with the horizontal will determine the XII o'clock line EA. Next divide the equatorial circle QMa into 24 equal parts *ab*, *bc*, *cd*, &c. . . . of 15° each, beginning from the meridian Pa, and through the various points of division and the poles draw the great circles Pbp, Pcp, &c. . . . These will exactly correspond to the equidistant generating lines on the cylinder in the previous construction, and the shadow of the style will fall on these circles after successive intervals of 1, 2, 3, &c. hours from noon. If they meet the horizontal in the points B, C, D, &c., then EB, EC, ED, &c. . . . will be the I, II, III, &c., hour lines required; and the problem of the horizontal dial consists in calculating the angles which these lines make with the XII o'clock line EA, whose position is known. The spherical triangles PAB, PAC, &c., enable us to do this readily. They are all right-angled at A, the side PA is the latitude of the place, and the angles APB, APC, &c., are respectively 15°, 30°, &c., then

$$\tan. AB = \tan. 15^\circ \sin. \textit{latitude},$$
$$\tan. AC = \tan. 30^\circ \sin. \textit{latitude},$$
&c., &c.

These determine the sides AB, AC, &c. that is, the angles AEB, AEC, &c., required.

For examples, let us find the angles made by the I o'clock line at the following places—Madras, London, Edinburgh, and Hammerfest (Norway).

Madras (13° 4′ N. lat.)	London (51° 30′ N. lat.)
Log. tan. 15°..........9·42805	Log. tan. 15°..........9·42805
Log. sin. 13° 4′.......9·35427	Log. sin. 51° 30′......9·89354
Log. tan. 3° 28′.......8·78232	Log. tan. 11° 51′.....9·32159
Edinburgh (55° 57′ N. lat.)	**Hammerfest (73° 40′ N. lat.)**
Log. tan. 15°..........9·42805	Log. tan. 15°..........9·42805
Log. sin. 55° 57′......9·91832	Log. sin. 73° 40′......9·98211
Log. tan. 12° 31′.....9·34637	Log. tan. 14° 25′.....9·41016

Thus the I o'clock hour line EB must make an angle on a Madras dial of only 3° 28′ with the meridian EA, 11° 51′ on a London dial, 12° 31′ at Edinburgh, and 14° 25′ at Hammerfest. In the same way may be found the angles made by the other hour lines.

The calculations of these angles must extend throughout one quadrant from noon to VI o'clock, but need not be carried further, because all the other hour-lines can at once be deduced from these.—In the first place the dial is symmetrically divided by the meridian, and therefore two times equidistant from noon will have their hour lines equidistant from the meridian; thus the XI o'clock line and the I o'clock line must make the same angles with it, the XI o'clock the came as the II o'clock, and so on. And next, the 24 great circles, which were drawn to determine these lines, are in reality only 12; for clearly the great circle which gives I o'clock after midnight, and that which gives I o'clock after noon, are one and the same, and so also for the other hours. Therefore the hour lines between VI in the evening and VI the next morning are the prolongations of the remaining twelve.

Let us now remove the imaginary sphere with all its circles, and retain only the style EP and the plane HMA with the lines traced on it, and we shall have the horizontal dial.

On the longest day in London the sun rises a little after 4 o'clock, and sets a little before 8 o'clock; there is therefore no necessity for extending a London dial beyond those hours. At Edinburgh the limits will be a little longer, while at Hammerfest, which is within the Arctic circle, the whole circuit will be required.

Instead of a wire style it is often more convenient to use a metal plate from one quarter to half an inch in thickness. This plate, which is sometimes in the form of a right-angled triangle, must have an acute angle equal to the latitude of the place, and, when properly fixed in a vertical position on the dial, its two faces must coincide with the meridian plane, and the sloping edges formed by the thickness of the plate must point to the pole and form two parallel styles. Since there are two styles, there must be two dials, or rather two half dials, because a little consideration will show that, owing to the thickness of the plate, these styles will only one at a time cast a shadow. Thus the eastern edge will give the shadow for all hours before 6 o'clock in the morning. From 6 o'clock until noon the western edge will be used. At noon, it will change again to the eastern edge until 6 o'clock in the evening, and finally the western edge for the remaining hours of daylight.

The centres of the two dials will be at the points where the styles meet the dial face; but, in drawing the hour-lines, we must be careful to draw only those lines for which the corresponding style is able to give a shadow as explained above. The dial will thus have the appearance of a single dial plate, and there will be no confusion (see fig. 3).

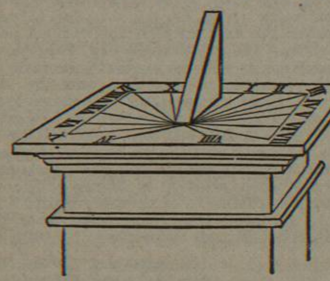

Fig. 3.

The line of demarcation between the shadow and the light will be better defined than when a wire style is used; but the indications by this double dial will always be one minute too fast in the morning and one minute too slow in the afternoon. This is owing to the magnitude of the sun, whose angular breadth is half a degree. The well-defined shadows are given, not by the centre of the sun, as we should require them, but by the forward limb in the morning and by the backward one in the afternoon; and the sun takes just about a minute to advance through a space equal to its half-breadth.

Dials of this description are frequently met with in the country. Placed on an ornamental pedestal some 4 feet high, they form a pleasing and useful addition to a lawn or to a garden terrace. The dial plate is of metal as well as the vertical piece upon it, and they may be purchased ready for placing on the pedestal,—the dial with all the hour-lines traced on it, and the style-plate firmly fastened in its proper position, if not even cast in the same piece with the dial-plate.

When placing it on the pedestal care must be taken that the dial be perfectly horizontal and accurately oriented. The levelling will be done with a spirit-level, and the orientation will be best effected either in the forenoon or in the afternoon, by turning the dial-plate till the time given by the shadow (making the *one* minute correction mentioned above) agrees with a good watch whose error on solar time is known. It is, however, important to bear in mind that a dial, so built up beforehand, will have the angle at the base equal to the latitude of some selected place, such as London, and the hour-lines will be drawn in directions calculated for the same latitude. Such a dial can therefore not be used near Edinburgh or Glasgow, although it would, without appreciable error, be adapted to any place whose latitude did not differ more than 20 or 30 miles from that of London, and it would be safe to employ it in Essex, Kent, or Wiltshire.

If a series of such dials were constructed, differing by 30 miles in latitude, then an intending purchaser could select one adapted to a place whose latitude was within 15 miles of his own, and the error of time would never exceed a small fraction of a minute. The following table will enable us to check the accuracy of the hour-lines and of the angle of the style,—all angles on the dial being readily measured with an ordinary protractor. It extends from 50° lat. to 59½° lat., and therefore includes the whole of Great Britain and Ireland :—

Lat.	XI. A.M. I. P.M.	X. A.M. II. P.M.	IX. A.M. III. P.M.	VIII. A.M. IV. P.M.	VII. A.M. V. P.M.	VI. A.M. VI. P.M.
50° 0′	11° 36′	23° 51′	37° 27′	53° 0′	70° 43′	90 0
50 30	11 41	24 1	37 39	53 12	70 51	90 0
51 0	11 46	24 10	37 51	53 23	70 59	90 0
51 30	11 51	24 19	38 3	53 35	71 6	90 0
52 0	11 55	24 28	38 14	53 46	71 13	90 0
52 30	12 0	24 37	38 25	53 57	71 20	90 0
53 0	12 5	24 45	38 37	54 8	71 27	90 0
53 30	12 9	24 54	38 48	54 19	71 34	90 0
54 0	12 14	25 2	38 58	54 29	71 40	90 0
54 30	12 18	25 10	39 9	54 39	71 47	90 0
55 0	12 23′	25 19	39 19	54 49	71 53	90 0
55 30	12 27	25 27	39 30	54 59	71 59	90 0
56 0	12 31	25 35	39 40	55 9	72 5	90 0
56 30	12 36	25 43	39 50	55 18	72 11	90 0
57 0	12 40	25 50	39 59	55 27	72 17	90 0
57 30	12 44	25 58	40 9	55 36	72 22	90 0
58 0	12 48	26 5	40 18	55 45	72 28	90 0
58 30	12 52	26 13	40 27	55 54	72 33	90 0
59 0	12 56	26 20	40 36	56 2	72 39	90 0
59 30	13 0	26 27	45 45	56 11	72 44	90 0

Vertical South Dial.—Let us take again our imaginary transparent sphere QZPA (fig. 4), whose axis PEp is parallel to the earth's axis. Let Z be the zenith, and consequently, the great circle QZP the meridian. Through E, the centre of the sphere, draw a vertical plane facing south. This will cut the sphere in the great circle ZMA which, being vertical, will pass through the zenith, and, facing south, will be at right angles to the meridian. Let QM*a* be the equatorial circle, obtained by drawing a plane through E at right angles to the axis PEp. The lower portion Ep of the axis will be the style, the vertical line EA in

the meridian plane will be the XII o'clock line, and the horizontal line EM[1] will be the VI o'clock line. Now, as in the previous problem, divide the equatorial circle into 24 equal arcs of 15° each, beginning at *a*, viz., *ab*, *bc*, &c.,—each quadrant aM, MQ, &c., containing six,—then through each point of division

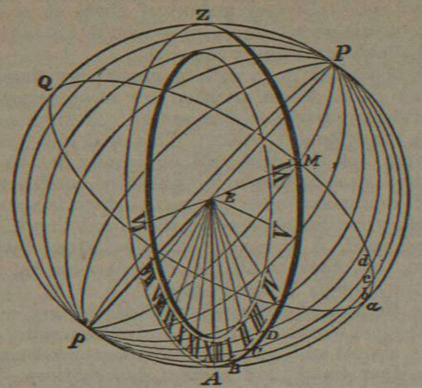

Fig. 4.

and through the axis Pp draw a plane cutting the sphere in 24 equidistant great circles. As the sun revolves round the axis the shadow of the axis will successively fall on these circles at intervals of one hour, and if these circles cross the vertical circle ZMA in the points A, B, C, &c., the shadow of the lower portion Ep of the axis will fall on the lines EA, EB, EC, &c., which will therefore be the required hour lines on the vertical dial, Ep being the style.

There is no necessity for going beyond the VI o'clock hour-line on each side of noon; for, in the winter months the sun sets earlier than 6 o'clock, and in the summer months it passes behind the plane of the dial before that time, and is no longer available.

It remains to show how the angles AEB, AEC, &c., may be calculated.

The spherical triangles pAB, pAC, &c., will give us a simple rule. These triangles are all right-angled at A, the side pA, equal to ZP, is the co-latitude of the place, that is, the difference between the latitude and 90°; and the successive angles ApB, ApC, &c. are 15°, 30°, &c., respectively. Then

tan. AB = tan. 15° sin. *co-latitude;*

or more simply,

tan. AB = tan. 15° cos. *latitude,*
tan. AC = tan. 30° cos. *latitude,*
&c., &c.

and the arcs AB, AC so found are the measure of the angles AEB, AEC, &c., required.

We shall, as examples, calculate the I o'clock hour angle AEB for each of the four places we had already taken in the horizontal dial.

Madras (13° 4′ N. lat.)	London (51° 30′ N. lat.)
Log. tan. 15°..........9·42805	Log. tan. 15°..........9·42805
Log. cos. 13° 4′.......9·98861	Log. cos. 51° 30′......9·79415
Log. tan. 14° 38′.....9·41666	Log. tan. 9° 28′.......9·22220
Edinburgh (55° 57′ N. lat.)	**Hammerfest (73° 40′ N. lat.)**
Log. tan. 15°..........9·42805	Log. tan. 15°..........9·42805
Log. cos. 55° 57′......9·74812	Log. cos. 73° 40′......9·44905
Log. tan. 8° 32′.......9·17617	Log. tan. 4° 19′.......8·87710

In this case the angles diminish as the latitudes increase, the opposite result to that of the horizontal dial.

Inclining, Reclining, &c., Dials.—We shall not enter into the calculation of these cases. Our imaginary sphere being, as before supposed, constructed with its centre at the centre of the dial, and all the hour-circles traced upon it, the intersection of these hour-circles with the plane of the dial will determine the hour-lines just as in the previous cases; but the triangles will no longer be right-angled, and the simplicity of the calculation will be lost, the chances of error being greatly increased by the difficulty of drawing the dial-plane in its true position on the sphere, since that true position will have to be found from observations which can be only roughly performed.

In all these cases, and in cases where the dial surface is not a plane, and the hour-lines, consequently, are not straight lines, the only safe practical way is to mark rapidly on the dial a few points (one is sufficient when the dial face is plane) of the shadow at the moment when a good watch shows that the hour has arrived, and afterwards connect these points with the centre by a continuous line. Of course the style must have been accurately fixed in its true position before we begin.

Equatorial Dial.—The name equatorial dial is given to one whose plane is at right angles to the style, and therefore parallel to the equator. It is the simplest of all dials. A circle (fig. 5) divided into 24 equal arcs is placed at right angles to the style, and hour divisions are marked upon it. Then if care be taken that the style point accurately to the pole, and that the noon division coincide with the meridian plane, the shadow of the style will fall on the other divisions, each at its proper time. The divisions must be marked on both sides of the dial, because the sun will shine on opposite sides in the summer and in the winter months, changing at each equinox.

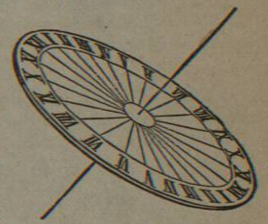

Fig. 5.

To find the Meridian Plane.—We have, so far, assumed the meridian plane to be accurately known; we shall proceed to describe some of the methods by which it may be found.

The mariner's compass may be employed as a first rough approximation. It is well known that the needle of the compass, when free to move horizontally, oscillates upon its pivot and settles in a direction termed the magnetic meridian. This does not coincide with the true north and south line, but the difference between them is generally known with tolerable accuracy, and is called the variation of the compass. The variation differs widely at different parts of the surface of the earth, being now about 20° W. in London, 7° W. in New York, and 17° E. in San Francisco. Nor is the variation at any place stationary, though the change is slow. We said that *now* the variation in London is about 20° W.; in 1837 it was about 24° W.; and there is even a small daily oscillation which takes place about the mean position, but too small to need notice here.

With all these elements of uncertainty, it is obvious that the compass can only give a rough approximation to the position of the meridian, but it will serve to fix the style so that only a small further alteration will be necessary when a more perfect determination has been made.

A very simple practical method is the following:—

Place a table (fig. 6), or other plane surface, in such a position that it may receive the sun's rays both in the morning and in the afternoon. Then carefully level the surface by means of a spirit-level. This must be done very accurately, and the table in that position made perfectly secure, so that there be no danger of its shifting during the day.

Next, suspend a plummet SH from a point S, which must be rigidly fixed. The extremity H, where the plummet just meets the surface, should be somewhere near the

[1] EM is obviously horizontal, since M is the intersection of two great circles ZM, QM, each at right angles to the vertical plane QZP.

middle of one end of the table. With H for centre, describe any number of concentric arcs of circles, AB, CD, EF, &c.

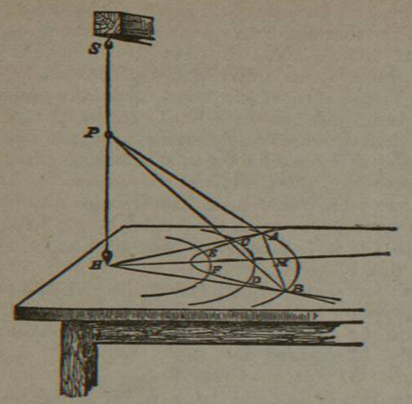

Fig. 6.

A bead P, kept in its place by friction, is threaded on the plummet line at some convenient height above H.

Every thing being thus prepared, let us follow the shadow of the bead P as it moves along the surface of the table during the day. It will be found to describe a curve ACE . . . . FDB, approaching the point H as the sun advances towards noon, and receding from it afterwards. (The curve is a conic section—an hyperbola in these regions.) At the moment when it crosses the arc AB, mark the point A; AP is then the direction of the sun, and, as AH is horizontal, the angle PAH is the altitude of the sun. In the afternoon mark the point B where it crosses the same arc; then the angle PBH is the altitude. But the right-angled triangles PHA, PHB are obviously equal; and the sun has therefore the same altitudes at those two instants, the one before, the other after noon. It follows that, *if the sun has not changed its declination* during the interval, the two positions will be symmetrically placed one on each side of the meridian. Therefore, drawing the chord AB, and bisecting it in M, HM will be the meridian line.

Each of the other concentric arcs, CD, EF, &c., will furnish its meridian line. Of course these should all coincide, but if not, the mean of the positions thus found must be taken.

The proviso mentioned above, that the sun has not changed its declination, is scarcely ever realized; but the change is slight, and may be neglected, except perhaps about the time of the equinoxes, at the end of March and at the end of September. Throughout the remainder of the year the change of declination is so slow that we may safely neglect it. The most favourable times are at the end of June and at the end of December, when the sun's declination is almost stationary. If the line HM be produced both ways to the edges of the table, then the two points on the ground vertically below those on the edges may be found by a plummet, and, if permanent marks be made there, the meridian plane, which is the vertical plane passing through these two points, will have its position perfectly secured.

To place the Style of a Dial in its True Position.—Before giving any other method of finding the meridian plane, we shall complete the construction of the dial, by showing how the style may now be accurately placed in its true position. The angle which the style makes with a hanging plumb-line, being the co-latitude of the place, is known, and the north and south direction is also roughly given by the mariner's compass. The style may therefore be already adjusted approximately—correctly, indeed, as to its inclination—but probably requiring a little horizontal motion east or west. Suspend a fine plumb-line from some point of the style, then the style will be properly adjusted if, at the very instant of noon, its shadow falls exactly on the plumb-line,—or, which is the same thing, if both shadows coincide on the dial.

This instant of noon will be given very simply by the meridian plane, whose position we have secured by the two permanent marks on the ground. Stretch a cord from the one mark to the other. This will not generally be horizontal, but the cord will be wholly in the meridian plane, and that is the only necessary condition. Next, suspend a plummet over the mark which is nearer to the sun, and, when the shadow of the plumb-line falls on the stretched cord, it is noon. A signal from the observer there to the observer at the dial enables the latter to adjust the style as directed above.

Other Methods of finding the Meridian Plane.—We have dwelt at some length on these practical operations because they are simple and tolerably accurate, and because they want neither watch, nor sextant, nor telescope—nothing more, in fact, than the careful observation of shadow lines.

The polar star may also be employed for finding the meridian plane without other apparatus than plumb-lines. This star is now only about 1° 21′ from the pole; if therefore a plumb-line be suspended at a few feet from the observer, and if he shift his position till the star is exactly hidden by the line, then the plane through his eye and the plumb-line will never be far from the meridian plane. Twice in the course of the 24 hours the planes would be strictly coincident. This would be when the star crosses the meridian above the pole, and again when it crosses it below. If we wished to employ the method of determining the meridian, the times of the stars crossing would have to be calculated from the data in the *Nautical Almanac*, and a watch would be necessary to know when the instant arrived. The watch need not, however, be very accurate, because the motion of the star is so slow that an error of ten minutes in the time would not give an error of one-eighth of a degree in the azimuth.

The following accidental circumstance enables us to dispense with both calculation and watch. The right ascension of the star *η Ursæ Majoris*, that star in the tail of the Great Bear which is farthest from the "pointers," happens to differ by a little more than 12 hours from the right ascension of the polar star. The great circle which joins the two stars passes therefore close to the pole. When the polar star, at a distance of about $1\frac{1}{3}$° from the pole, is crossing the meridian above the pole, the star *η Ursæ Majoris*, whose polar distance is about 40°, has not yet reached the meridian below the pole.

When *η Ursæ Majoris* reaches the meridian, which will be within half an hour later, the polar star will have left the meridian; but its slow motion will have carried it only a very little distance away. Now at some instant between these two times—much nearer the latter than the former—the great circle joining the two stars will be exactly vertical; and at this instant, which the observer determines by seeing that the plumb-line hides the two stars simultaneously, neither of the stars is strictly in the meridian; but the deviation from it is so small that it may be neglected, and the plane through the eye and the plumb-line taken for meridian plane.

In all these cases it will be convenient, instead of fixing the plane by means of the eye and one fixed plummet, to have a second plummet at a short distance in front of the eye; this second plummet, being suspended so as to allow of lateral shifting, must be moved so as always to be

between the eye and the fixed plummet. The meridian plane will be secured by placing two permanent marks on the ground, one under each plummet.

This method, by means of the two stars, is only available for the upper transit of *Polaris*; for, at the lower transit, the other star *η Ursæ Majoris* would pass close to or beyond the zenith, and the observation could not be made. Also the stars will not be visible when the upper transit takes place in the day-time, so that one-half of the year is lost to this method.

Neither could it be employed in lower latitudes than 40° N., for there the star would be below the horizon at its lower transit;—we may even say not lower than 45° N., for the star must be at least 5° above the horizon before it becomes distinctly visible.

There are other pairs of stars which could be similarly employed, but none so convenient as these two, on account of *Polaris* with its very slow motion being one of the pair.

To place the Style in its True Position without previous determination of the Meridian Plane.—The various methods given above for finding the meridian plane have for ultimate object the determination of the plane, not on its own account, but as an element for fixing the instant of noon, whereby the style may be properly placed.

We shall dispense, therefore, with all this preliminary work if we determine noon by astronomical observation. For this we shall want a good watch, or pocket chronometer, and a sextant or other instrument for taking altitudes. The local time at any moment may be determined in a variety of ways by observation of the celestial bodies. The simplest and most practically useful methods will be found described and investigated in any good educational work on astronomy.

For our present purpose a single altitude of the sun taken in the forenoon will be most suitable. At some time in the morning, when the sun is high enough to be free from the mists and uncertain refractions of the horizon—but to insure accuracy, while the rate of increase of the altitude is still tolerably rapid, and, therefore, not later than 10 o'clock—take an altitude of the sun, an assistant, at the same moment, marking the time shown by the watch. The altitude so observed being properly corrected for refraction, parallax, &c., will, together with the latitude of the place, and the sun's declination, taken from the *Nautical Almanac*, enable us to calculate the time. This will be the solar or apparent time, that is, the very time we require; and we must carefully abstain from applying the equation of time. Comparing the time so found with the time shown by the watch, we see at once by how much the watch is fast or slow of solar time, we know, therefore, exactly what time the watch must mark when solar noon arrives, and waiting for that instant we can fix the style in its proper position as explained before.

We can dispense with the sextant and with all calculation and observation if, by means of the pocket chronometer, we bring the time from some observatory where the work is done; and, allowing for the change of longitude, and also for the equation of time, if the time we have brought is clock time, we shall have the exact instant of solar noon as in the previous case.

In remote country districts a dial will always be of use to check and even to correct the village clock, and the description and directions here given will, we think, enable any ingenious artisan to construct one.

In former times the fancy of dialists seems to have run riot in devising elaborate surfaces on which the dial was to be traced. Sometimes the shadow was received on a cone, sometimes on a cylinder, or on a sphere, or on a combination of these. A universal dial was constructed of a figure in the shape of a cross, another universal dial showed the hours by a globe and by several gnomons. These universal dials required adjusting before use, and for this a mariner's compass and a spirit-level were necessary. But it would be tedious and useless to enumerate the various forms designed, and, as a rule, the more complex the less accurate.

Another class of useless dials consisted of those with variable centres. They were drawn on fixed horizontal planes, and each day the style had to be shifted to a new position. Instead of hour-*lines* they had hour-*points*; and the style, instead of being parallel to the axis of the earth, might make any chosen angle with the horizon. There was no practical advantage in their use, but rather the reverse; and they can only be considered as furnishing material for new mathematical problems.

Portable Dials.—The dials so far described have been fixed dials, for even the fanciful ones to which reference was just now made were to be fixed before using. There were, however, other dials, made generally of a small size, so as to be carried in the pocket; and these, so long as the sun shone, roughly answered the purpose of a watch.

The description of the portable dial has generally been mixed up with that of the fixed dial, as if it had been merely a special case, and the same principle had been the basis of both; whereas there are essential points of difference between them, besides those which are at once apparent.

In the fixed dial the result depends on the *uniform* angular motion of the sun round the fixed style; and a small error in the assumed position of the sun, whether due to the imperfection of the instrument, or to some small neglected correction, has only a trifling effect on the time. This is owing to the angular displacement of the sun being so rapid—a quarter of a degree every minute—that for the ordinary affairs of life greater accuracy is not required, as a displacement of a quarter of a degree, or at any rate of one degree, can be readily seen by nearly every person. But with a portable dial this is no longer the case. The uniform angular motion is not now available, because we have no determined fixed plane to which we may refer it. In the new position, to which the observer has gone, the zenith is the only point of the heavens he can at once practically find; and the basis for the determination of the time is the constantly but *very irregularly* varying zenith distance of the sun.

At sea the observation of the altitude of a celestial body is the only method available for finding local time; but the perfection which has been attained in the construction of the sextant (chiefly by the introduction of telescopes) enables the sailor to reckon on an accuracy of seconds instead of minutes. Certain precautions have, however, to be taken. The observations must not be made within a couple of hours of noon, on account of the slow rate of change at that time, nor too near the horizon, on account of the uncertain refractions there; and the same restrictions must be observed in using a portable dial.

To compare roughly the value (as to accuracy) of the fixed and the portable dials, let us take a mean position in Great Britain, say 54° lat., and a mean declination when the sun is in the equator. It will rise at 6 o'clock, and at noon have an altitude of 36°,—that is, the portable dial will indicate an average change of one-tenth of a degree in each minute, or two and half times slower than the fixed dial. The vertical motion of the sun increases, however, nearer the horizon, but even there it will be only one-eighth of a degree each minute, or half the rate of the fixed dial, which goes on at nearly the same speed throughout the day.

Portable dials are also much more restricted in the range of latitude for which they are available, and they should not be used more than 4 or 5 miles north or south of the place for which they were constructed.

We shall briefly describe two portable dials which were in actual use.

Dial on a Cylinder.—A hollow cylinder of metal (fig. 7), 4 or 5 inches high, and about an inch in diameter, has a lid which admits of tolerably easy rotation. A hole in the lid receives the style, shaped somewhat like a bayonet; and the straight part of the style, which, on account of the two bends, is lower than the lid, projects horizontally out from the cylinder to a distance of 1 or $1\frac{1}{2}$ inches. When not in use the style would be taken out and placed inside the cylinder.

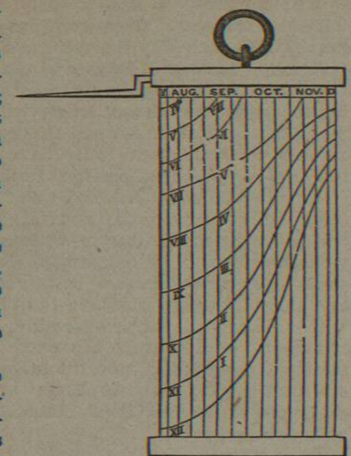

Fig 7.

A horizontal circle is traced on the cylinder opposite the projecting style, and this circle is divided into 36 approximately equidistant intervals.[1] These intervals represent spaces of time, and to each division is assigned a date, so that each month has three dates marked as follows:—January 10, 20, 31; February 10, 20, 28; March 10, 20, 31; April 10, 20, 30, and so on,—always the 10th, the 20th, and the last day of each month.

Through each point of division a vertical line parallel to the axis of the cylinder is drawn from top to bottom. Now it will be readily understood that if, upon one of these days, the lid be turned so as to bring the style exactly opposite the date, and if the dial be then placed on a horizontal table so as to receive sun-light, and turned round *bodily* until the shadow of the style falls exactly on the vertical line below it, the shadow will terminate at some definite point of this line, the position of which point will depend on the length of the style—that is, the distance of its end from the surface of the cylinder—and on the altitude of the sun at that instant. Suppose that the observations are continued all day, the cylinder being very gradually turned so that the style may always face the sun, and suppose that marks are made on the vertical line to show the extremity of the shadow at each exact hour from sunrise to sunset—these times being taken from a good fixed sun dial,—then it is obvious that the next year, on the *same date*, the sun's declination being about the same, and the observer in about the same latitude, the marks made the previous year will serve to tell the time all that day.

What we have said above was merely to make the principle of the instrument clear, for it is evident that this mode of marking, which would require a whole year's sunshine and hourly observation, cannot be the method employed.

The positions of the marks are, in fact, obtained by calculation. Corresponding to a given date, the declination of the sun is taken from the almanac, and this, together with the latitude of the place and the length of the style, will constitute the necessary data for computing the length of the shadow, that is, the distance of the mark below the style for each successive hour.

We have assumed above that the declination of the sun is the same at the same date in different years. This is not quite correct, but, if the dates be taken for the second year after leap year, the results will be sufficiently approximate. The actual calculations will offer no difficulty.

When all the hour marks have been placed opposite to their respective dates, then a continuous curve, joining the corresponding hour-points, will serve to find the time for a day intermediate to those set down, the lid being turned till the style occupy a proper position between the two divisions. The horizontality of the surface on which the instrument rests is a very necessary condition, especially in summer, when, the shadow of the style being long, the extreme end will shift rapidly for a small deviation from the vertical, and render the reading uncertain. The dial can also be used by holding it up by a small ring in the top of the lid, and probably the verticality is better ensured in that way.

Portable Dial on a Card.—This neat and very ingenious dial is attributed by Ozanam to a Jesuit Father, *De Saint Rigaud*, and probably dates from the early part of the 17th century. Ozanam says that it was sometimes called the *capuchin*, from some fancied resemblance to a cowl thrown back.

Construction.—Draw a straight line ACB parallel to the top of the card (fig. 8) and another DCE at right angles to it; with C as

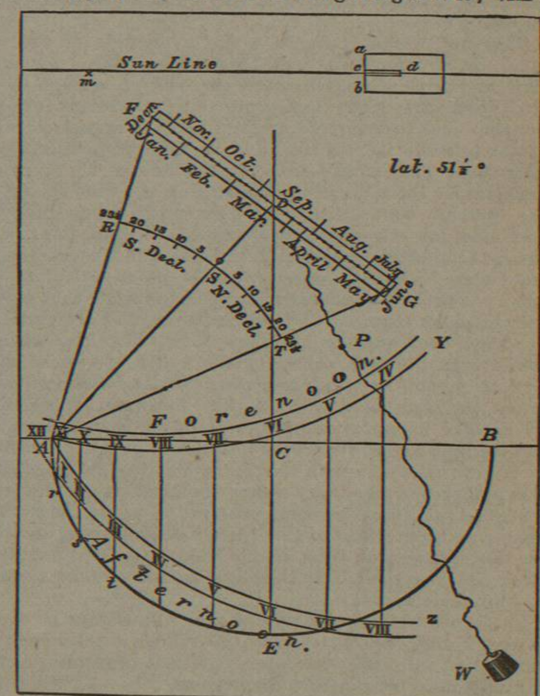

Fig. 8.

centre, and any convenient radius CA, describe the semi-circle AEB below the horizontal. Divide the whole arc AEB into 12 equal parts at the points *r*, *s*, *t*, &c., and through these points draw perpendiculars to the diameter ACB, these lines will be the hour lines, viz., the line through *r* will be the XI ..I line; the line through *s* the X...II line, and so on; the hour line of noon will be the point A itself; by subdivision of the small arcs *Ar*, *rs*, *st*, &c., we may draw the hour lines corresponding to halves and quarters but this only where it can be done without confusion.

[1] Strict equality is not necessary, as the observations made are on the vertical line through each division-point, without reference to the others. It is not even requisite that the divisions should go completely and exactly round the cylinder, although they were always so drawn, and both these conditions were insisted upon in the directions for the construction.

Draw ASD making with AC an angle equal to the latitude of the place, and let it meet EC in D, through which point draw FDG at right angles to AD.

With centre A, and any convenient radius AS, describe an arc of circle RST, and graduate this arc by marking degree divisions on it, extending from 0° at S to $23\frac{1}{2}$° on each side at R and T. Next determine the points on the straight line FDG where radii drawn from A to the degree divisions on the arc would cross it, and carefully mark these crossings.

The divisions of RST are to correspond to the sun's declination, south declinations on RS and north declinations on ST. In the other hemisphere of the earth this would be reversed; the north declinations would be on the upper half.

Now, taking a second year after leap year (because the declinations of that year are about the mean of each set of four years), find the days of the month when the sun has these different declinations, and place these dates, or so many of them as can be shown without confusion, opposite the corresponding marks on FDG. Draw the *sun-line* at the top of the card parallel to the line ACB; and, near the extremity, to the right, draw any small figure intended to form, as it were, a door of which *a b* shall be the hinge. Care must be taken that this hinge is exactly at right angles to the *sun-line*. Make a fine open slit *c d* right through the card and extending from the hinge to a short distance on the door,—the centre line of this slit coinciding accurately with the *sun-line*. Now, cut the door completely through the card; except, of course, along the hinge, which, when the card is thick, should be partly cut through at the back, to facilitate the opening. Cut the card right through along the line FDG, and pass a thread carrying a little plummet W and a *very* small bead P; the bead having sufficient friction with the thread to retain any position when acted on only by its own weight, but sliding easily along the thread when moved by the hand. At the back of the card the thread terminates in a knot to hinder it from being drawn through; or better, because giving more friction and a better hold, it passes through the centre of a small disc of card—a fraction of an inch in diameter—and, by a knot, is made fast at the back of the disc.

To complete the construction,—with the centres F and G, and radii FA and GA, draw the two arcs AY and AZ which will limit the hour lines; for in an observation the bead will always be found between them. The forenoon and afternoon hours may then be marked as indicated in the figure. The dial does not of itself discriminate between forenoon and afternoon; but extraneous circumstances, as, for instance, whether the sun is rising or falling, will settle that point, except when close to noon, where it will always be uncertain.

To *rectify* the dial (using the old expression, which means to prepare the dial for an observation),—open the small door, by turning it about its hinge, till it stands well out in front. Next, set the thread in the line FG opposite the day of the month, and stretching it over the point A, slide the bead P along till it exactly coincide with A.

To find the hour of the day,—hold the dial in a vertical position in such a way that its plane may pass through the sun. The verticality is ensured by seeing that the bead rests against the card without pressing. Now gradually tilt the dial (without altering its vertical plane), until the central line of sunshine, passing through the open slit of the door, just falls along the sun-line. The hour line against which the bead P then rests indicates the time.

The *sun-line* drawn above has always, so far as we know, been used as a *shadow-line*. The upper edge of the rectangular door was the prolongation of the line, and, the door being opened, the dial was gradually tilted until the shadow cast by the upper edge exactly coincided with it. But this shadow tilts the card one-quarter of a degree more than the sun-line, because it is given by that portion of the sun which just appears above the edge, that is, by the upper limb of the sun, which is one-quarter of a degree higher than the centre. Now, even at some distance from noon, the sun will sometimes take a considerable time to rise one-quarter of a degree, and by so much time will the indication of the dial be in error.

The central line of light which comes through the open slit will be free from this error, because it is given by light from the centre of the sun.

The card-dial deserves to be looked upon as something more than a mere toy. Its ingenuity and scientific accuracy give it an educational value which is not to be measured by the roughness of the results obtained, and the following demonstration of its correctness will, it is hoped, usefully close what we have to say on this subject.

Demonstration.—Let H (fig. 9) be the point of suspension of the plummet at the time of observation, so that the angle DAH is the north declination of the sun,—P, the bead, resting against the hour-line VX. Join CX, then the angle ACX is the hour angle from noon given by the bead, and we have to prove that this hour-angle is the correct one corresponding to a north latitude DAC, a north declination AH and an altitude equal to the angle which the

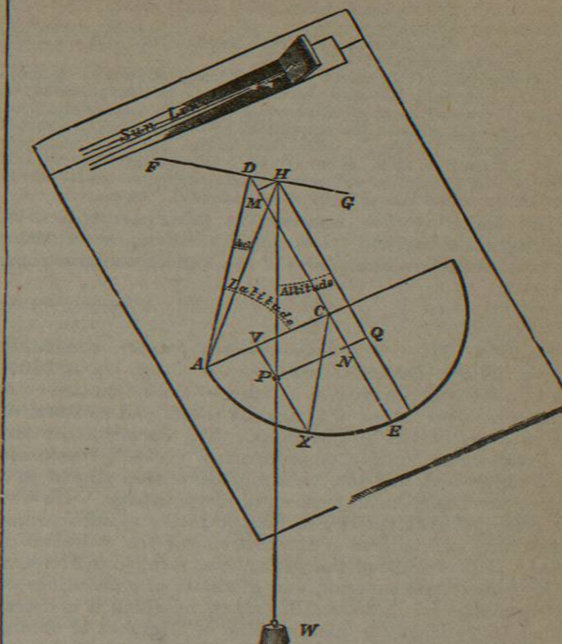

Fig. 9.

sun-line, or its parallel AC, makes with the horizontal. The angle PHQ will be equal to the altitude, if HQ be drawn parallel to DC, for the pair of lines HQ, HP will be respectively at right angles to the sun-line and the horizontal.

Draw PQ and HM parallel to AC, and let them meet DCE in M and N respectively.

Let HP and its equal HA be represented by a. Then the following values will be readily deduced from the figure:—

$$AD = a \cos. decl.,\ DH = a \sin. decl.,\ PQ = a \sin. alt.$$
$$CX = AC = AD \cos. lat. = a \cos. decl. \cos. lat.$$
$$PN = CV = CX \cos. ACX = a \cos. decl. \cos. lat. \cos. ACX.$$
$$NQ = MH = DH \sin. MDH = a \sin. decl. \sin. lat.$$
($\because$ the angle MDH = DAC = latitude).

And, since $PQ = NQ + PN$,

we have, by simple substitution,

$$a \sin. alt. = a \sin. decl. \sin. lat. + a \cos. decl. \cos. lat. \cos ACX;$$

or, dividing by a throughout.

$$\sin. alt. = \sin. decl. \sin. lat. + \cos. decl. \cos. lat. \cos. ACX \ldots (A)$$

which equation determines the hour angle ACX shewn by the bead.

To determine the hour-angle of the sun at the same moment, let

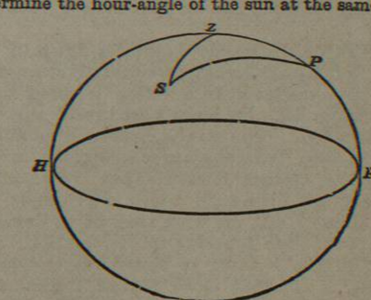

Fig. 10.

fig. 10 represent the celestial sphere, HR the horizon, P the pole and Z the zenith, and S the sun.