Parliament, the 11th of Henry VI. 1433), by the fact of his hereditary possession of Arundel Castle only. As hereditary Earl-Marshal, his Grace of Norfolk is the head of the College of Arms. (C. B.)

EARLE, John (1601?–1665), bishop of Worcester and afterwards of Salisbury, was born at York about 1601. He completed his education at Oxford, first entering Christ Church, and taking his degree of B.A. in 1619. He afterwards passed to Merton College, and graduated M.A. in 1624. He was appointed in 1631 proctor of the university, and the same year became chaplain to Philip, earl of Pembroke, then chancellor of the university. He was soon after presented by this nobleman to the rectory of Bishopstone, in Wiltshire, and, having been introduced to the king, Charles I., was appointed chaplain and tutor to Prince Charles. In 1642 Earle took his degree of D.D., and in the following year was elected one of the famous Assembly of Divines at Westminster. But his sympathies with the king and with the Church of England were so strong that he declined to sit. Early in 1643 he was chosen chancellor of the cathedral of Salisbury; but of this preferment he was soon after deprived. After Cromwell's great victory at Worcester, Earle went abroad, and was named clerk of the closet and chaplain to Charles II. He spent a year at Antwerp in the house of Izaak Walton's friend Dr Morley, who became afterwards bishop of Winchester. He next joined the duke of York (James II.) at Paris, returning to England at the Restoration. He was at once appointed dean of Westminster, and in 1661 was one of the commissioners for revising the liturgy. At the end of November 1662 he was consecrated bishop of Worcester, and was translated, ten months later, to the see of Salisbury. During the plague of London Bishop Earle attended the king and queen at Oxford, and there he died, November 17, 1665. Earle's chief title to remembrance is his witty and humorous work entitled *Microcosmography, or a Piece of the World discovered, in Essays and Characters*, which throws light on the manners of the time. First printed in 1628, it became very popular, and ran through eight editions in the lifetime of the author. A new edition with notes and appendix, containing much interesting matter, by Philip Bliss, was published in 1811. The style is quaint and epigrammatic; and the reader is frequently reminded of Thomas Fuller by such passages as this: "A university dunner is a gentlemen follower cheaply purchased, for his own money has hyr'd him." Several reprints of the book have been issued since the author's death; and in 1671 a French translation by J. Dymock appeared with the title of *Le vice ridiculé*. Earle was employed by Charles II. to make the Latin translation of the *Eikon Basilike*, published in 1649.

"Dr Earle," says Lord Clarendon in his *Life*, "was a man of great piety and devotion, a most eloquent and powerful preacher, and of a conversation so pleasant and delightful, so very innocent, and so very facetious, that no man's company was more desired and loved. No man was more negligent in his dress and habit and mien, no man more wary and cultivated in his behaviour and discourse. He was very dear to the Lord Falkland, with whom he spent as much time as he could make his own."

See especially Bliss's edition of the *Microcosmographie*, and Arber's Reprint, London, 1868.

EARLOM, Richard (1742–1822), English mezzotint engraver, was born in London in 1742. His natural faculty for art appears to have been first called into exercise by admiration for the lord mayor's state coach, just decorated by Cipriani. He tried to copy the paintings, and was sent to study under Cipriani. He displayed great skill as a draughtsman, and at the same time acquired without assistance the art of engraving in mezzotint. In 1765 he was employed by Alderman Boydell, then one of the most liberal promoters of the fine arts, to make a series of drawings from the pictures at Houghton Hall; and these he afterwards engraved in mezzotint. His most perfect works as engraver are perhaps the fruit and flower pieces after the Dutch artists Van Os and Van Huysum. Amongst his historical and figure subjects are—Agrippina, after West; Love in Bondage, after Guido Reni; the Royal Academy, the Embassy of Hyderbeck to meet Lord Cornwallis, and a Tiger Hunt, the last three after Zoffany; and Lord Heathfield, after Sir Joshua Reynolds. Earlom also executed a series of 200 facsimiles of the drawings and sketches of Claude Lorraine, which was published in 3 vols. folio, under the title of *Liber Veritatis* (1777–1819). Earlom died in London, October 9, 1822.

EAR-RING, an ornament worn pendent from the ear, and generally suspended by means of a ring or hook passing through the pendulous lobe of the ear. The general usage appears to have been to have ear-rings worn in pairs, the two ornaments in all respects resembling each other; in ancient times, or sometimes more recently among Oriental races, a single ear-ring has sometimes been worn. The use of this kind of ornament, which constantly was of great value and sometimes was made of large size, dates from the remotest historical antiquity, the earliest mention of ear-rings occurring in the book of Genesis. It appears probable that the ear-rings of Jacob's family, which he buried with his strange idols at Bethel, were regarded as amulets or talismans, such unquestionably being the estimation in which some ornaments of this class have been held from a very early period, as they still are held in the East. Among all the Oriental races of whom we have any accurate knowledge, the Hebrews and Egyptians excepted, ear-rings always have been in general use by both sexes; while in the West, as well as by the Hebrews and Egyptians, as a general rule they have been considered exclusively female ornaments. By the Greeks and Romans also ear-rings were worn only by women; and the prevalence of this fashion among the races of classic antiquity is illustrated in a singular manner by the ears of the famous statue of the Venus de' Medici being bored, evidently for the reception of pendent jewels. Ear-rings invariably occupy important positions among the various remains of ancient and mediæval goldsmiths' work that from time to time have rewarded the researches of archæological inquirers. And these early relics, with rare exceptions objects of great beauty and delicacy, never fail to exemplify the artistic styles of their periods, as they were prevalent among the races by whom each individual jewel was produced. Ear-rings of costly materials and elaborate workmanship have been brought to light in considerable numbers in the Troad and in Peloponnesus by Dr Schliemann; jewels of the same class, of exquisite beauty, and of workmanship that is truly wonderful, have been rescued from the sepulchres of ancient Etruria and Greece by Signor Castellani; other ear-rings of gold of characteristic forms have come down to our own times from the ancient Egyptians; we know well what styles of ear-rings were worn by the Romans of the empire and by the early Scandinavians; and recent researches among the burial places of our Anglo-Saxon predecessors in the occupancy of this island have led to the discovery of jewels in considerable numbers, which among their varieties include ear-rings executed in a style that proves the Anglo-Saxons to have made no inconsiderable advance in the arts of civilization. These same ornaments, which never have fallen into disuse, enjoy at the present day a very high degree of favour; like all other modern jewels, however, the ear-rings of our own times as works of arts can claim no historical attributes, because they consist as well of reproductions from all past ages and of every race as of fanciful productions that certainly can be assigned to no style of art whatever.

EARTH, Figure of the. The determination of the figure of the earth is a problem of the highest importance in astronomy, inasmuch as the diameter of the earth is the unit to which all celestial distances must be referred. Reasoning, doubtless, from the uniform level appearance of the horizon in any situation in which a spectator can be placed—the variations in altitude of the circumpolar stars as one travels towards the north or south, the disappearance of a ship standing out to sea, and perhaps other phenomena—the earliest astronomers universally regarded this earth as a sphere, and they endeavoured to ascertain its dimensions. Aristotle relates that the mathematicians had found the circumference to be 400,000 stadia. But Eratosthenes appears to have been the first who entertained an accurate idea of the principles on which the determination of the figure of the earth really depends, and attempted to reduce them to practice. His results were very inaccurate, but his method is the same as that which is followed at the present day—depending, in fact, on the comparison of a line measured on the earth's surface with the corresponding arc of the heavens. He observed that at Syene in Upper Egypt, on the day of the summer solstice, the sun was exactly vertical, whilst at Alexandria at the same season of the year its zenith distance was 7° 12′, or one-fiftieth of the circumference of a circle. He assumed that these places were on the same meridian; and, reckoning their distance apart as 5000 stadia, he inferred that the circumference of the earth was 250,000 stadia. A similar attempt was made by Posidonius, who adopted a method which differed from that of Eratosthenes only in using a star instead of the sun. He obtained 240,000 stadia for the circumference. But it is impossible to form any correct opinions as to the degree of accuracy attained in these measures, as the length of the stadium is unknown. Ptolemy in his *Geography* assigns the length of the degree as 500 stadia.

The Arabs, who were not inattentive to astronomy, did not overlook the question of the earth's magnitude. The caliph Almamoum, 814 A.D., having fixed on a spot in the plains of Mesopotamia, despatched one company of astronomers northwards and another southwards, measuring the journey by rods, until each found the altitude of the pole to have changed one degree. But the result of this measurement does not appear to have been very satisfactory. From this time the subject seems to have attracted no attention until about 1500, when Fernel, a Frenchman, measured a distance in the direction of the meridian near Paris by counting the number of revolutions of the wheel of his carriage as he travelled. His astronomical observations were made with a triangle used as a quadrant, and his resulting length of a degree was by a happy chance very near the truth.

The next geodesist, Willebrord Snell, took an immense step in the right direction by substituting a chain of triangles for actual linear measurement. The account of this operation was published at Leyden in 1617. He measured his base line on the frozen surface of the meadows near Leyden, and measured the angles of his triangles, which lay between Alkmaar and Bergen-op-Zoom, with a quadrant and semicircles. He took the precaution of comparing his standard with that of the French, so that his result was expressed in toises (the length of the toise is about 6·39 English feet). The work was recomputed and reobserved by Muschenbroek in 1729.

In 1637 an Englishman, Richard Norwood, published his own determination of the figure of the earth in a volume entitled *The Seaman's Practice, contayning a Fundamentall Probleme in Navigation experimentally verified, namely, touching the Compasse of the Earth and Sea and the quantity of a Degree in our English Measures.* It appears that he observed on the 11th June 1633 the sun's meridian altitude in London as 62° 1′, and on June 6, 1635, his meridian altitude in York as 59° 33′. He measured the distance between these places along the public road partly with a chain and partly by pacing. By this means, through compensation of errors, he arrived at 367,176 feet for the degree—a very fair result.

The application of the telescope to circular instruments was the next important step in the science of measurement. Picard was the first who in 1669, with the telescope, using such precautions as the nature of the operation requires, measured an arc of meridian. He measured with wooden rods a base line of 5663 toises, and a second or base of verification of 3902 toises; his triangulation extended from Malvoisine, near Paris, to Sourdon, near Amiens. The angles of the triangles were measured with a quadrant furnished with a telescope having cross wires in its focus. The difference of latitude of the terminal stations was determined by observations made with a sector on a star in Cassiopeia, giving 1° 22′ 55″ for the amplitude. The terrestrial measurement gave 78,850 toises, whence he inferred for the length of the degree 57,060 toises.

Hitherto geodetic observations had been confined to the determination of the magnitude of the earth considered as a sphere, but a discovery made by Richer turned the attention of mathematicians to its deviation from a spherical form. This astronomer, having been sent by the Academy of Sciences of Paris to the island of Cayenne, in South America, for the purpose of determining the amount of terrestrial refraction and other astronomical objects, observed that his clock, which had been regulated at Paris to beat seconds, lost about two minutes and a half daily at Cayenne, and that in order to bring it to measure mean solar time it was necessary to shorten the pendulum by more than a line. This fact, which appeared exceedingly curious, and was scarcely credited till it had been confirmed by the subsequent observations of Varin and Deshayes on the coasts of Africa and America, was first explained in the third book of Newton's *Principia*, who showed that it could only be referred to a diminution of gravity arising either from a protuberance of the equatorial parts of the earth and consequent increase of the distance from the centre or from the counteracting effect of the centrifugal force. About the same time, 1673, appeared the work of Huyghens entitled *De Horologio Oscillatorio*, in which for the first time were found correct notions on the subject of centrifugal force. It does not, however, appear that they were applied to the theoretical investigation of the figure of the earth before the publication of Newton's *Principia*. In 1690 Huyghens, following up the subject, published his treatise entitled *De Causa Gravitatis*, which contains an investigation of the figure of the earth on the supposition that the attraction of every particle is towards the centre.

Between 1684 and 1718 J. and D. Cassini, starting from Picard's base, carried a triangulation northwards from Paris to Dunkirk and southwards from Paris to Collioure. They measured a base of 7246 toises near Perpignan, and a somewhat shorter base near Dunkirk; and from the northern portion of the arc, which had an amplitude of 2° 12′ 9″, obtained for the length of a degree 56,960 toises; while from the southern portion, of which the amplitude was 6° 18′ 57″, they obtained 57,097 toises. The immediate inference from this was that, the degree diminishing with increasing latitude, the earth must be a prolate spheroid. This conclusion was totally opposed to the theoretical investigations of Newton and Huyghens, and created a great sensation among the scientific men of the day. The question was far too important to be allowed to remain unsettled, and accordingly the Academy of Sciences of Paris determined to apply a decisive test by the measurement of arcs at a great distance from each other. For this purpose some of the most distinguished members of their body undertook the measurement of two meridian arcs—one in the neighbourhood of the equator, the other in a high latitude; and so arose the celebrated expeditions of the French Academicians. In May 1735, MM. Godin, Bouguer, and De la Condamine, under the auspices of Louis XV., proceeded to Peru, where, assisted by two Spanish officers, after ten years of laborious exertion they measured an arc of 3° 7′ intersected by the equator. The second party consisted of Maupertuis, Clairaut, Camus, Lemonnier, and Outhier, who reached the Gulf of Bothnia in July 1736; they were in some respects more fortunate than the first party, inasmuch as they completed the measurement of an arc near the polar circle of 57′ amplitude and returned to Europe within sixteen months from the date of their departure.

The measurement of Bouguer and De la Condamine was executed with great care, and on account of the locality, as well as the manner in which all the details were conducted, it has always been regarded as a most valuable determination. The southern limit was at a place called Tarqui, the northern at Cotchesqui. A base of 6272 toises was measured in the vicinity of Quito, near the northern extremity of the arc, and a second base of 5260 toises near the southern extremity. The mountainous nature of the country made the work very laborious, in some instances the difference of heights of two neighbouring stations exceeding a mile. The difficulties with which the observers had to contend were increased by the opposition of the more ignorant of the inhabitants, and they were at times in danger of losing their lives. They had also much trouble with their instruments, those with which they were to determine the latitudes proving untrustworthy. But their energy and ingenuity were equal to the occasion, and they succeeded by simultaneous observations of the same star at the two extremities of the arc in obtaining very fair results. The whole length of the arc amounted to 176,945 toises, while the difference of latitudes was 3° 7′ 3″. In consequence of a misunderstanding that arose between De la Condamine and Bouguer, their operations were conducted separately, and each wrote a full and interesting account of the operation. Bouguer's book was published in 1749; that of De la Condamine in 1751. The toise used in this measure was ever after regarded as *the* standard toise, and is always referred to as the *Toise of Peru*.

The party of Maupertuis, though their work was quickly despatched, had also to contend with great difficulties. They were disappointed in not being able to make use of the small islands in the Gulf of Bothnia for the trigonometrical stations, and were forced to penetrate into the forests of Lapland. They commenced operations at Tornea, a city situated on the mainland near the extremity of the gulf. From this, the southern extremity of their arc, they carried a chain of triangles northward to the mountain Kittis, which they selected as the northern terminus. In the prosecution of this work they suffered greatly from cold and the bites of flies and gnats. The latitudes were determined by observations with a sector (made by Graham) of the zenith distance of α and δ Draconis. The base line was measured on the frozen surface of the river Tornea about the middle of the arc; two parties measured it separately, and they differed by about 4 inches. The result of the whole was that the difference of latitudes of the terminal stations was 57′ 29″·6, and the length of the arc 55,023 toises. In this expedition, as well as in that to Peru, observations were made with a pendulum to determine the force of gravity; and these observations coincided with the geodetical results in proving that the earth was an oblate and not prolate spheroid.

In 1740 was published in the Paris *Mémoires* an account, by Cassini de Thury, of a remeasurement by himself and Lacaille of the meridian of Paris. With a view to determine more accurately the variation of the degree along the meridian, they divided the distance from Dunkirk to Collioure into four partial arcs of about two degrees each, by observing the latitude at five stations. The anomalous results previously obtained by J. and D. Cassini were not confirmed, but on the contrary the length of the degree derived from these partial arcs showed on the whole an increase with increasing latitude. In continuation of their labours, Cassini and Lacaille further measured an arc of parallel across the mouth of the Rhone. The difference of time of the extremities was determined by the observers at either end noting the instant of a signal given by flashing gunpowder at a point near the middle of the arc.

While at the Cape of Good Hope in 1752, engaged in various astronomical observations, Lacaille measured an arc of meridian of 1° 13′ 17″, which gave him for the length of the degree 57,037 toises—an unexpected result, which has led to the modern remeasurement of the arc by Sir Thomas Maclear.

Passing over the measurements made between Rome and Rimini and on the plains of Piedmont by the Jesuits Boscovich and Beccaria, and also the arc measured with deal rods in North America by Messrs Mason and Dixon, we come to the commencement of the English triangulation. In 1783, in consequence of a representation from Cassini de Thury on the advantages that would accrue to science from the geodetic connection of Paris and Greenwich, General Roy was with the king's approval appointed by the Royal Society to conduct the operations on the part of this country,—Count Cassini, Mechain, and Delambre being appointed on the French side. And now a precision previously unknown was brought into geodesy by the use of Ramsden's splendid theodolite, which was the first to make the spherical excess of triangles measurable. The wooden rods with which the first base was measured were speedily replaced by glass rods, which again were rejected for the steel chain of Ramsden. The details of this operation are fully given in the *Account of the Trigonometrical Survey of England and Wales.* Shortly after this, the National Convention of France, having agreed to remodel their system of weights and measures, chose, as applicable to all countries, for their unit of length the ten-millionth part of the meridian quadrant. In order to obtain this length precisely, the remeasurement of the French meridian was resolved on, and deputed to Delambre and Mechain. The details of this great operation will be found in the *Base du Système Métrique Décimale.* The arc was subsequently extended by MM. Biot and Arago to the island of Iviza.

The appearance in 1838 of Bessel's classical work entitled *Gradmessung in Ostpreussen* marks an era in the science of geodesy. Here we find the method of least squares, a branch of the theory of probabilities, applied to the calculation of a network of triangles and the reduction of the observations generally. This work has been looked on as a model ever since, and probably it will not soon be superseded as such. The systematic manner in which all the observations were taken with the view of securing final results of extreme accuracy is admirable. The triangulation, which is a small one, extends about a degree and a half along the shores of the Baltic in a N.N.E. direction. The compound bars with which he measured his base line may be understood by the following brief description. On the surface of an iron bar two toises in length is laid a zinc bar, both being very perfectly planed and in free contact—the zinc bar being slightly shorter than the iron bar. They are united at one end only, and as the temperature varies the difference of length of the bars as seen at the other end varies; this difference of length is a thermometrical indication whereby a correction for temperature can be applied to the bars so as to reduce their length to that at the standard temperature. The bars in measuring were not allowed to come into contact, but the intervals left were measured by the interposition of a glass wedge. The results of all the comparisons of the four measuring rods with one another, and with the standards, are elaborately worked out by least squares. The angles were observed with theodolites of 12 and 15 inches diameter, and the latitudes determined by means of the transit instrument in the prime vertical—a method much used in Germany. The formulæ employed in the reduction of the astronomical observations are very elegant. The reduction of the triangulation was carried out in the most thorough manner,—the sum of the squares of all the actual theodolite observations being made a minimum. As it is usual now to follow this method (sometimes only approximately) in all triangulations where great precision is required, we here give a brief description of the method. The equations of condition of a triangulation are those which exist between the supernumerary observed quantities and their calculated values, for, after there are just sufficient observations to fix all the points, then any angle that may be subsequently observed can be compared with its calculated value. If a triangulation consist of $n+2$ points, two of which are the ends of a base line, then to fix the n points $2n$ angles suffice; so that if m be the actual number of angles really observed, the triangulation must afford $m-2n$ equations of condition. To show how these arise, suppose that from a number m of fixed points A, B, C . . . a new point P is observed, which m points are again observed from P, then there will be formed $m-1$ triangles, in each of which the sum of the observed angles is $=180°$ + the spherical excess; this gives at once $m-1$ equations of condition. The $m-2$ distances will each afford an equation of the form

$$\frac{PC}{PB} \cdot \frac{PB}{PA} \cdot \frac{PA}{PC} = 1,$$

not, however, limited to three factors. Should P observe the m points and not be observed back, there will be $m-3$ equations of the above form (they are called side equations). In a similar manner other cases can be treated. In practice the ratios of sides are replaced by the ratios of the sines of the corresponding opposite angles. To each observed angle a symbolical correction is applied, so that if α be an observed angle and $\alpha+x$ the true or most probable angle, $\sin(\alpha+x)=\sin\alpha(1+x\cot\alpha)$, x being a small angle whose square is neglected. Thus the side equation takes the form $\beta+\beta_1x_1+\beta_2x_2+\ldots\beta_rx_r=0$. In the case of equations formed by adding together the three observed angles of a triangle the co-efficients are of course unity. The problem then is this: Given n equations

$$\begin{aligned}
\beta + \beta_1x_1 + \beta_2x_2 + \ldots \beta_mx_m &= 0\\
\beta' + \beta_1'x_1 + \beta_2'x_2 + \ldots \beta_m'x_m &= 0\\
\beta'' + \beta_1''x_1 + \beta_2''x_2 + \ldots \beta_m''x_m &= 0\\
\cdots\cdots\cdots\cdots\cdots\cdots &
\end{aligned}$$

between $m(m>n)$ unknown quantities $x_1 \ldots x_m$, which are the corrections (expressed in seconds of arc) to the observed angles, it is required to determine these quantities so as to render the function $w_1x_1^2+w_2x_2^2+w_3x_3^2+\ldots w_mx_m^2$ a minimum, where $w_1 \ldots w_m$ are the *weights* of the determinations of the angles to which the corresponding corrections belong. The corrections $x_1 \ldots x_m$ fulfilling this condition of minimum have, according to the theory of least squares, a higher probability than any other system of corrections that merely satisfy the equations of condition. Multiply the n equations by multipliers $\lambda_1, \lambda_2, \ldots \lambda_n$, and we obtain by the theory of maxima and minima m equations

$$\begin{aligned}
w_1x_1 &= \beta_1\lambda_1 + \beta_1'\lambda_2 + \beta_1''\lambda_3 + \ldots\\
w_2x_2 &= \beta_2\lambda_1 + \beta_2'\lambda_2 + \beta_2''\lambda_3 + \ldots,\\
\vdots\quad & \quad\vdots\\
w_mx_m &= \beta_m\lambda_1 + \beta_m'\lambda_2 + \beta_m''\lambda_3 + \ldots
\end{aligned}$$

The values of $x_1 \ldots x_m$ obtained from these equations are to be substituted in the original equations of condition, and then there will be n equations between the n multipliers $\lambda_1 \ldots \lambda_n$. These being solved, the numerical values of $\lambda_1 \ldots \lambda_n$ will be obtained, and on substituting these in the last equations written down, the values of $x_1 \ldots x_m$ will follow. The process is a long and tedious one; but it is inevitable if we wish very good results.

The great meridian arc in India was commenced by Colonel Lambton at Punnœ in latitude 8° 9′. Following generally the methods of the English survey, he carried his triangulation as far north as 20° 30′. The work then passed into the able hands of Sir George (then Captain) Everest, who continued it to the latitude of 29° 30′. Two admirably written volumes by Sir George Everest, published in 1830 and in 1847, give all the details of the vast undertaking. The great trigonometrical survey of India is now being prosecuted with great scientific skill by Colonel Walker, R.E., and it may be expected that we shall soon have some valuable contributions to the great problem of geodesy. The working out of the Indian chains of triangle by the method of least squares presents peculiar difficulties, but enormous in extent as the work is, it is being thoroughly carried out. The ten base lines on which the survey depends were measured with Colby's compensation bars.

These compensation bars were also used by Sir Thomas Maclear in the measurement of the base line in his extension of Lacaille's arc at the Cape. The account of this operation will be found in a volume entitled *Verification and Extension of Lacaille's Arc of Meridian at the Cape of Good Hope*, by Sir Thomas Maclear, published in 1866. Lacaille's amplitude is verified, but not his terrestrial measurement.

The number of stations in the principal triangulation of Great Britain and Ireland is about 250. At 32 of these the latitudes were determined with Ramsden's and Airy's zenith sectors. The theodolites used for this work were, in addition to the two great theodolites of Ramsden which were used by General Roy and Captain Kater (and which are now in as good condition as when they came from the hands of the maker), a smaller theodolite of 18 inches diameter by the same mechanician, and another of 24 inches diameter by Messrs Troughton and Simms. Observations for determination of absolute azimuth were made with these instruments at a large number of stations; the stars α, δ, and λ Ursæ Minoris and 51 Cephei being those observed, always at the greatest azimuths. At six of these stations the probable error of the result is under 0″·4, at twelve under 0″·5, at thirty-four under 0″·7: so that the absolute azimuth of the whole network is determined with extreme accuracy. Of the seven base lines which have been measured, five were by means of steel chains and two with Colby's compensation bars. This is a system of six compound bars self-correcting for temperature. The compound bar may be thus described. Two bars, one of brass and the other of iron, are laid side by side, parallel, and firmly united at their centres, from which they are free to expand or contract; at the standard temperature they are of the same length. Let AB be one bar, A′B′ the other; draw a line through the corresponding extremities A, A′ to P, and a line through the other extremities B, B′ to Q, make A′P = B′Q, AA′ being = BB′. Now if A′P is to AP as the rate of expansion of the bar A′B′ to the rate of expansion of the bar AB, then clearly the distance PQ will be invariable, or very nearly so. In the actual instrument P and Q are finely engraved dots at the distance of ·10 feet apart. In the measurement the bars when aligned do not come into contact; an interval of six inches is left between each bar and its neighbour. This small space is measured by an ingenious micrometrical arrangement constructed on exactly the same principle as the bars themselves. The triangulation was computed by least squares. The total number of equations of condition for the triangulation is 920; if therefore the whole had been reduced in one mass, as it should have been, the solution of an equation of 920 unknown quantities would have occurred as a part of the work. To avoid this an approximation was resorted to; the triangulation was divided into twenty-one parts or figures; four of these, not adjacent, were first adjusted by the method explained, and the corrections thus determined in these figures carried into the equations of condition of the adjacent figures. The average number of equations in a figure is 44; the largest equation is one of 77 unknown quantities.[1]

Airy's Zenith Sector is too well known to need description. The vertical limb is read by four microscopes; altogether, in the complete observation of a star there are 10 micrometer readings and 12 level readings. In some recent observations in Scotland for latitude the Zenith Telescope has been used with very great success; it is very portable; and a complete determination of latitude, affected with the mean of the declination errors of two stars, is effected by two micrometer readings and four level readings. The observation consists in measuring with the telescope micrometer the *difference* of zenith distances of two stars which cross the meridian, one to the north and the other to the south of the observer at zenith distances which differ by not much more than 10′ or 15′, the interval of the times of transit being not less than one nor more than twenty minutes. The advantages are that, with simplicity in the construction of the instrument and facility in the manipulation, refraction is eliminated (or nearly so, as the stars are generally selected within 25° of the zenith), and there is no large divided circle. The telescope, which is counterpoised on one side of the vertical axis, has a small circle for finding, and there is also a small horizontal circle. This instrument is universally used in American geodesy.

The United States Coast Survey has a principal triangulation extending for about 9° 30′ along the coast, but the final results are not yet published.

In 1860 was published F. G. Struve's *Arc du Méridien de* 25° 20′ *entre le Danube et la Mer Glaciale mesuré depuis* 1816 *jusqu'en* 1855. This work is the record of a vast amount of scientific labour and is the greatest contribution yet made to the question of the figure of the earth. The latitudes of the thirteen astronomical stations of this arc were determined partly with vertical circles and partly by means of the transit instrument in the prime vertical. The triangulation, a great part of which, however, is a simple chain of triangles, is reduced by the method of least squares, and the probable errors of the resulting distances of parallels is given; the probable error of the whole arc in length is ± 6·2 toises. Ten base lines were measured. The sum of the lengths of the ten measured bases is 29,863 toises, so that the average length of a base line is 19,100 feet. The azimuths were observed at fourteen stations. In high latitudes the determination of the meridian is a matter of great difficulty; nevertheless the azimuths at all the northern stations were successfully determined,—the probable error of the result at Fuglenœs being ± 0″·53.

[1] See the volume of the Ordnance Survey, entitled *Account of the Principal Triangulation of Great Britain and Ireland*, by Captain A. R. Clarke, R.E., F.R.S., 1858.

Mechanical Theory.

Newton appears to have been the first to apply his own newly-discovered doctrine of gravitation, combined with the so-called centrifugal force, to the question of the figure of the earth. Assuming that an oblate ellipsoid of rotation is a form of equilibrium for a homogeneous fluid rotating with uniform angular velocity, he obtained the ratio of the axes 229 : 230, and the law of variation of gravity on the surface. A few years later Huyghens published an investigation of the figure of the earth, supposing the attraction of every particle to be towards the centre of the earth, obtaining as a result that the proportion of the axes should be 578 : 579. In 1740 Maclaurin wrote his celebrated essay on the tides, one of the most elegant geometrical investigations ever made. He demonstrated that the oblate ellipsoid of revolution is a figure which satisfies the conditions of equilibrium in the case of a revolving homogeneous fluid mass whose particles attract one another according to the law of the inverse square of the distance; he gave the equation connecting the ellipticity with the proportion of the centrifugal force at the equator to gravity, and he determined the attraction on a particle situated anywhere on the surface of such a body. Some few years afterwards Clairaut published (1743) his *Théorie de la Figure de la Terre*, which contains, among other results, demonstrated with singular elegance, a very remarkable theorem which establishes a relation between the ellipticity of the earth and the variations of gravity at different points of its surface. Assuming that the earth is composed of concentric ellipsoidal strata having a common axis of rotation each stratum homogeneous in itself, but the ellipticities and densities of the successive strata varying according to any law, and that the superficial stratum has the same form as if it were fluid, he proves the very important theorem contained in the equation

$$\frac{g'-g}{g}+e=\frac{5}{2}m,$$

Where g, g' are the amounts of gravity at the equator and at the pole respectively, e the ellipticity of the meridian, and m the ratio of the centrifugal force at the equator to g. Clairaut also proved that the increase of gravity in proceeding from the equator to the poles is as the square of the sine of the latitude. This, taken with the former theorem, gives the means of determining the earth's ellipticity from observation of the comparative force of gravity at any two places. Clairaut would seem almost to have exhausted the subject, for although much has been written since by mathematicians of the greatest eminence, yet, practically, very little of importance has been added. Laplace, himself a prince of mathematicians, who had devoted much of his own time to the same subject, remarks on Clairaut's work that "the importance of all his results and the elegance with which they are presented place this work amongst the most beautiful of mathematical productions" (Todhunter's *History of the Mathematical Theories of Attraction and the Figure of the Earth*, vol. i. p. 229).

The problem of the figure of the earth treated as a question of mechanics or hydrostatics is one of great difficulty, and it would be quite impracticable but for the circumstance that the surface differs but little from a sphere. In order to express the forces at any point of the body arising from the attraction of its particles, the form of the surface is required, but this form is the very one which it is the object of the investigation to discover; hence the complexity of the subject, and even with all the present resources of mathematicians only a partial and imperfect solution can be obtained, and that not without some labour. We may, however, here briefly indicate the line of reasoning by which some of the most important of the results we

have alluded to above may be obtained. The principles of hydrostatics show us that if X, Y, Z be the components parallel to three rectangular axes of the forces acting on a particle of a fluid mass at the point x, y, z, then, p being the pressure there, and ρ the density,

$$dp=\rho(\mathrm{X}dx+\mathrm{Y}dy+\mathrm{Z}dz);$$

and for equilibrium the necessary conditions are, that $\rho(\mathrm{X}dx + \mathrm{Y}dy + \mathrm{Z}dx)$ be a complete differential, and at the free surface $\mathrm{X}dx + \mathrm{Y}dy + \mathrm{Z}dz = 0$. This equation implies that the resultant of the forces is normal to the surface at every point, and in a homogeneous fluid it is obviously the differential equation of all surfaces of equal pressure. If the fluid be heterogeneous then it is to be remarked that for forces of attraction according to the ordinary law of gravitation, if X, Y, Z be the components of the attraction of a mass whose potential is V, then

$$\mathrm{X}dx+\mathrm{Y}dy+\mathrm{Z}dz=\frac{d\mathrm{V}}{dx}dx+\frac{d\mathrm{V}}{dy}dy+\frac{d\mathrm{V}}{dz}dz,$$

which is a complete differential. And in the case of a fluid rotating with uniform velocity, in which the so-called centrifugal force enters as a force acting on each particle proportional to its distance from the axis of rotation, the corresponding part of $\mathrm{X}dx+\mathrm{Y}dy+\mathrm{Z}dz$ is obviously a complete differential. Therefore for the forces with which we are now concerned $\mathrm{X}dx+\mathrm{Y}dy+\mathrm{Z}dz=d\mathrm{U}$, where U is some function of x, y, z, and it is necessary for equilibrium that $dp=\rho d\mathrm{U}$ be a complete differential; that is, ρ must be a function of U or a function of p, and so also p a function of U. So that $d\mathrm{U}=0$ is the differential equation of surfaces of equal pressure and density.

We may now show that a homogeneous fluid mass in the form of an oblate ellipsoid of revolution having a uniform velocity of rotation can be in equilibrium. It may be proved that the attraction of the ellipsoid $x^2+y^2+z^2(1+\epsilon^2)=c^2(1+\epsilon^2)$ upon a particle P of its mass at x, y, z has for components

$$\mathrm{X}=\mathrm{A}x,\quad \mathrm{Y}=\mathrm{A}y,\quad \mathrm{Z}=\mathrm{C}z,$$

where

$$\mathrm{A}=2\pi\rho\left(\frac{1+\epsilon^2}{\epsilon^3}\tan^{-1}\epsilon-\frac{1}{\epsilon^2}\right)$$

$$\mathrm{C}=4\pi\rho\left(\frac{1+\epsilon^2}{\epsilon^2}-\frac{1+\epsilon^2}{\epsilon^3}\tan^{-1}\epsilon\right)$$

Besides the attraction of the mass of the ellipsoid, the centrifugal force at P has for components $-x\omega^2, -y\omega^2, 0$; then the condition of fluid equilibrium is

$$(\mathrm{A}-\omega^2)x dx+(\mathrm{A}-\omega^2)y dy+\mathrm{C}z dz=0,$$

which by integrating gives

$$(\mathrm{A}-\omega^2)(x^2+y^2)+\mathrm{C}z^2=\text{constant}.$$

This is the equation of an ellipsoid of rotation, and therefore the equilibrium is possible. The equation coincides with that of the surface of the fluid mass if we make

$$\mathrm{A}-\omega^2=\frac{\mathrm{C}}{1+\epsilon^2},$$

which gives

$$\frac{\omega^2}{2\pi\rho}=\frac{3+\epsilon^2}{\epsilon^3}\tan^{-1}\epsilon-\frac{3}{\epsilon^2}.$$

If we would determine the maximum value of ω from this equation, we find that it corresponds to the value of ϵ determined by the condition

$$\tan^{-1}\epsilon=\frac{9\epsilon+7\epsilon^3}{(1+\epsilon^2)(9+\epsilon^2)};$$

hence it may be shown that if the angular velocity exceed that calculated from $\frac{\omega^2}{2\pi\rho}=0{\cdot}2247$, equilibrium is impossible for the form of an ellipsoid of revolution. If ω fall short of this limit, there are two ellipsoids which satisfy the condition of equilibrium; in one of these the eccentricity is greater and in the other less than 0·93. In the case of the earth, which is nearly spherical, we get by expanding the expression for ω^2 in powers of ϵ^2, rejecting the higher powers, and remarking that the ellipticity $e=\frac{1}{2}\epsilon^2$,

$$\frac{\omega^2}{2\pi\rho}=\frac{4}{15}\epsilon^2=\frac{8}{15}e.$$

Now, if m be the ratio of the centrifugal force at the equator to gravity there,

$$m=\frac{c\omega^2}{\frac{4}{3}\pi\rho c-c\omega^2},\quad \therefore\ \frac{\omega^2}{2\pi\rho}=\frac{2}{3}\,\frac{m}{1+m}.$$

In the case of the earth it is a matter of observation that $m=\frac{1}{289}$, hence the ellipticity

$$e=\frac{5}{4}m=\frac{1}{231},$$

so that the ratio of the axes on the supposition of a homogeneous fluid earth is 230 : 231, as announced by Newton.

Now, to come to the case of a heterogeneous fluid, we shall assume that its surfaces of equal density are spheroids, concentric and having a common axis of rotation, and that the ellipticity of these surfaces varies from the centre to the outer surface, the density also varying. In other words, the body is composed of homogeneous spheroidal shells of variable density and ellipticity. On this supposition we shall express the attraction of the mass upon a particle in its interior, and then, taking into account the centrifugal force, form the equation expressing the condition of fluid equilibrium. The attraction of the homogeneous spheroid $x^2+y^2+z^2(1+2e)=c^2(1+2e)$, where e is the ellipticity, of which the square is neglected, on an internal particle, whose co-ordinates are $x=f$, $y=0$, $z=h$, has for its x and z components

$$\mathrm{X}'=\frac{4}{3}\pi\rho f\left(1-\frac{2}{5}e\right),\quad \mathrm{Z}'=\frac{4}{3}\pi\rho h\left(1+\frac{4}{5}e\right),$$

the Y component being of course zero. Hence we infer that the attraction of a shell whose inner surface has an ellipticity e, and its outer surface an ellipticity $e+de$, the density being ρ, is expressed by

$$d\mathrm{X}'=-\frac{4}{3}\cdot\frac{2}{5}\pi\rho f de,\quad d\mathrm{Z}'=\frac{4}{3}\cdot\frac{4}{5}\pi\rho h de.$$

To apply this to our heterogeneous spheroid; if we put c_1 for the semiaxis of that surface of equal density on which is situated the attracted point P, and c_0 for the semiaxis of the outer surface, the attraction of that portion of the body which is exterior to P, namely, of all the shells which inclose P, has for components

$$\mathrm{X}_0=-\frac{8}{15}\pi f\int^{c_0}\frac{de}{dc}dc,\quad \mathrm{Z}_0=\frac{16}{15}\pi h\int_{c_1}^{c_0}\rho\frac{de}{dc}dc,$$

both e and ρ being functions of c. Again the attraction of a homogeneous spheroid of density ρ on an *external* point f, h has the components

$$\mathrm{X}''=\frac{4}{3}\pi\rho\frac{f}{r^3}\left\{c^3(1+2e)-\lambda ec^5\right\}$$

$$\mathrm{Z}''=\frac{4}{3}\pi\rho\frac{h}{r^3}\left\{c^3(1+2e)-\lambda' ec^5\right\}$$

where $\lambda=\frac{3}{5}\cdot\frac{4h^2-f^2}{r^4}$, $\lambda'=\frac{3}{5}\cdot\frac{2h^2-3f^2}{r^4}$, and $r^2=f^2+h^2$.

Now e being considered a function of c, we can at once express the attraction of a shell (density ρ) contained between the surface defined by $c+dc$, $e+de$ and that defined by c, e upon an external point; the differentials with respect to c, viz. $d\mathrm{X}''$ $d\mathrm{Z}''$, must then be integrated with ρ under the integral sign as being a function of c. The integration will extend from $c=0$ to $c=c_1$. Thus the components of the attraction of the heterogeneous spheroid