liquid therefore rotates in the direction of the hands of a watch.

Action of magnet on electric discharge. *Magnetic Action on the Electric Discharge in Gases.*—A large number of very interesting results have been obtained concerning the behaviour of the electric discharge in a field of magnetic force. We can only make a brief allusion to the matter here. The key to the phenomena lies in the remark that the electric discharge in vacuum tubes may be regarded as an electric current in a very flexible elastic conductor. It is clear that such a conductor would be an equilibrium if it lay in a line of magnetic force passing through both its fixed ends. Again, if the flexible conductor be constrained to remain on a given surface, it will not be in equilibrium until it has so arranged itself that the resultant electromagnetic force at each point is *perpendicular* to the surface. At each point, therefore, the magnetic force must be tangential to the surface.[1]

A perfectly flexible but inextensible conductor, two points of which are fixed, will take such a form that the electromagnetic force at each point is balanced by the tension. Le Roux fastened a thin platinum wire to two stout copper terminals, and caused it to glow by passing a current through it. When the terminals were placed equatorially between the flat poles of an electromagnet, the wire bent into the form of a circular arc joining the terminals. When the terminals were placed axially, it assumed a helical form. (See also Spottiswoode and Stokes, *Proc. R. S.*, 1875.)

Rotation of electric discharge. The behaviour of the light emanating from the positive pole may be explained in general as lying between the two cases which we have just discussed. One of the most remarkable of these phenomena is the rotation of the discharge discovered by Walker, and much experimented on by De la Rive. This may be exhibited by means of the apparatus shown in fig. 44, consisting essentially of an exhausted vessel, one of the electrodes in which is ring-shaped; a bar of soft iron, covered with some insulating material, is passed through the ring and fixed to the stand. When this apparatus is placed on the pole of a powerful magnet, the discharge rotates as a wire hinged to the upper electrode would do.

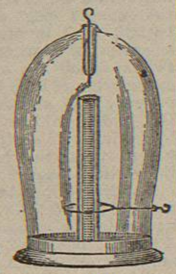

Fig. 44.

Plücker's experiments. Owing to the distinct character of the negative light, the action of the magnet on it is different from that on the positive light. Plücker found that the general character of the phenomena may be thus described:—The negative light is bounded by magnetic curves that issue from the electrode and cut the walls of the tube.

The two diagrams in fig. 45 will convey an idea of the appearance of the phenomenon. Although much tempted to follow the subject further, we must be content to refer the reader to the interesting papers of Plücker[2] and Hittorf.[3] An excellent summary will be found in Wiedemann.

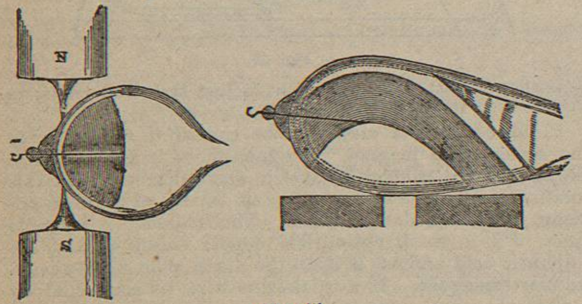

Fig. 45.

Ampère's theory. *Ampère's Method.*—Before quitting the subject of electromagnetism, it will be useful, for the sake of comparison, to give a brief sketch of the method of Ampère, or rather of that modification of the original method now commonly found in Continental books, which was suggested by Ampère himself, in a note to the *Théorie des Phénomènes Électrodynamiques.* Ampère starts with the idea that the electrodynamic action of two circuits is the sum of the actions at a distance between every pair of their elements. He supposes, as the simplest and most natural assumption, that the force between two elements is in the line joining them. Besides this assumption, his theory rests on four experiments.[4] The first of these shows that, when a wire is doubled on itself, the electrodynamic action of any current in it is *nil.* The second experiment shows that this is also true, even if one of the halves of the wire be bent or twisted in any way, so as never to be far removed from the other. The third experiment proves that the action of any closed circuit on an element of another circuit is perpendicular to the element. In the fourth experiment it is shown that the force between two conductors remains the same when all the lines in the system are increased in the same ratio, the currents remaining the same. From the assumption, together with the first experiment, it follows that the force between two elements is proportional to the product of the lengths of the elements, multiplied by the product of the strengths of the currents and by some function of the mutual distance and of the angles which determine their relative position. Hence it may be shown, from the fourth experiment, that the force between the elements must vary inversely as the square of the distance between them. The second experiment shows that we may replace any element of a circuit by the projections of the element on three rectangular axes.

From these results it is found that the force between ds and $d\sigma$ must be

$$\frac{Aii'dsd\sigma}{D^2}(\cos\epsilon - k\cos\theta\cos\theta')$$

The constant k is then determined from the result of the third experiment; and it is found that k must be equal to $\frac{3}{2}$. The formula is thus completely determined, with the exception of A, which depends on the unit of current which is chosen. The action of a closed circuit on an element is then calculated, and a vector found, which Ampère calls the "directrix," from which this action can be found in exactly the same way as we derived this same action from the magnetic induction. The theory is then applied to small plane circuits, solenoids, and so on.

As was remarked in the historical sketch, a variety of other elementary laws may be substituted for that of Ampère, all of which lead to the same result for closed circuits.

Generalization of theory. Maxwell has presented Ampère's theory in a more general form, in which the assumption about the direction of the elementary action is not made. Neglecting couples, he finds for the most general form of the components of the force exerted by $d\sigma$ on ds,

$$\left.\begin{array}{l} R = \frac{1}{D^2}\left(\frac{dD}{ds}\frac{dD}{d\sigma} - 2D\frac{d^2D}{dsd\sigma}\right)ii'dsd\sigma + D\frac{d^2Q}{dsd\sigma}ii'dsd\sigma \\ \text{in the direction of D,} \\ \text{and} \qquad S = -\frac{dQ}{d\sigma}ii'dsd\sigma, \quad S' = \frac{dQ}{ds}ii'dsd\sigma \\ \text{in the direction of } ds \text{ and } d\sigma \text{ respectively.} \end{array}\right\} \quad 22).$$

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[1] Loci having this property were called by Plücker epipolic curves.

[2] *Pogg. Ann.*, ciii., civ., cv., cvii., cxiii., 1858, &c.

[3] *Pogg. Ann.*, cxxxvi., 1869.

[4] Details respecting these experiments, and other matter connected with Ampère's theory, may be found in Maxwell, vol. ii. § 502, &c., and in almost any Continental work on experimental physics.

In these expressions Q is a function to be determined only by further assumption. Q = constant gives Ampère's formula; $Q = -\frac{1}{2r}$ gives the formula of Grassmann, and so on. We may in fact construct an infinite variety of different elementary formulæ. The reader interested in this subject may consult Wiedemann, Bd. ii. §§ 26, 27, 45–54, &c., and Tait, *Proc. R.S.E.*, 1873.

In our account of the magnetic action of electric currents no mention has been made of the effect of the proximity of soft iron. Under the magnetic action of the electric circuit soft iron is magnetized inductively. The distribution of the lines of force is in general greatly affected thereby. The general feature of the phenomenon is a concentration of the lines upon the iron. By the proper use of this effect electromagnetic forces of great power may be developed. It is not easy to give a mathematically accurate account of the action, owing to our ignorance of the exact law of magnetic induction in powerfully paramagnetic bodies. The discussion of this subject, however, belongs to MAGNETISM (which see).

The Induction of Electric Currents.

Faraday's laws of induction. A brief account has already been given (see Historical Sketch, p. 11) of Faraday's discovery[1] of the induction of electric currents. The results he arrived at may be summed up as follows.

Let there be two linear circuits, ABKE (the primary) and CDG (the secondary), two portions of which, AB and CD, are parallel, and near each other.

I. When a current is started in AB, a transient current flows through CD in the opposite direction to the current in AB; when the current in AB is steady, no current in CD can be detected; when the current in AB is stopped, a transient current flows through CD in the same direction as the current in AB. These currents in CD are said to be induced, and may be called inverse and direct currents respectively, the reference being to the direction of the primary. Both inverse and direct currents last for a very short time, and the quantity of electricity which passes in each of them is the same.

II. If the circuit AB, in which a steady current is flowing, be caused to approach CD, an inverse current is thereby induced in CD; when the circuit AB, under similar circumstances, recedes from CD, a direct current is induced in CD. We have already mentioned that when AB is at rest, and the current in it does not vary, there is no current in CD. AB has been supposed to approach and recede from CD, but the same statement applies when CD approaches and recedes from AB.

III. When a magnet is magnetized or demagnetized in the neighbourhood of a circuit, or approaches or recedes from the circuit, the effect is the same as if an equivalent[2] current approached or receded from the circuit. For example, imagine a small circular circuit placed horizontally, and a vertical bar magnet lowered in the axis of the circuit with its north pole pointing down upon the circuit, the magnet may be replaced by a series of coaxial circular currents (see above, p. 71), and the motion will induce a current passing round the circuit against the hands of a watch.

Faraday showed how the direction of the induced current can be predicted when the variation of the magnetic field or the motion of the conductor in it is known, and he gave, in his own manner, indications how the magnitude of the current could be inferred.

Maxwell's statement. Maxwell has thrown the law of Faraday into the following form:—"The total electromotive force acting round a circuit at any instant is measured by the rate of decrease of the number of lines of magnetic force which pass through it."

Or, integrating with respect to the time:—"The time integral of the total electromotive force acting round any circuit, together with the number of lines of magnetic force which pass through the circuit, is a constant quantity."

For "number of lines of force" may of course be substituted the equivalent expressions, "induction through the circuit," or "surface integral of magnetic induction," taken over any surface bounded by the circuit.

Some care must be taken in determining the positive direction round the circuit. The following is a correct process:—Assume one direction (D, fig. 46) *through* the circuit as positive, then the positive direction *round* (R) is determined by the right-handed screw relation; if the number of lines of force reckoned positive in direction D is decreasing, then the electromotive force is in direction R; if that number is increasing, the electromotive force is in the opposite direction.

Fig. 46.

This will be clearer if we consider the following simple example. Let ABCD (fig. 47) be a horizontal rectangular circuit (AB next the reader). In a northern latitude, the vertical component Z of the earth's magnetic force is downwards; if, therefore, the positive direction through the circuit be taken downwards, the positive direction round is ADCB, and the number of lines of force through it is Z.AB.BC. If BC slide on DC and AB parallel to itself through a small distance BB′ in time τ, Z.AB.BC *increases* by Z.CB.BB′; hence the electromotive force is Z.BC.BB′ ÷ τ, and acts in the direction ABCD. If v be the velocity of BC, we may write for the electromotive force Z.BC.v. That is, the electromotive force at any instant is proportional to the velocity.

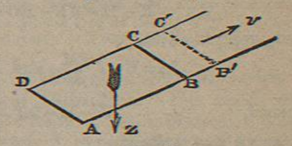

Fig. 47.

The law of Faraday leads to a complete determination of the induced current in all cases. We may regard it as resting on the experiments of Faraday, and of those who followed out his results.

Theory of Helmholtz and Thomson. Another view of the matter of great importance was enunciated independently and about the same time by Helmholtz[3] and Sir William Thomson.[4]

Let a circuit carrying a current[5] i move in an invariable magnetic field, so that the number of lines of magnetic force passing through it is increased by dN, then the work[6] done by the electromagnetic forces on the circuit is by Ampère's theory idN; also, if R be the resistance of the circuit, Ri^2dt is the heat generated in time dt. Now if E be the electromotive force of the battery which maintains the current i, the whole energy supplied is $Eidt$; hence we must have

$$idN + Ri^2dt = Eidt$$

$$\text{and } i = \left(E - \frac{dN}{dt}\right)\frac{1}{R} \quad . \quad . \quad . \quad . \quad (23).$$

Hence there is an electromotive force $-\frac{dN}{dt}$ in the moving circuit. Now $\frac{dN}{dt}$ is the rate of increase of the number of lines of force passing through the circuit.

We have therefore deduced the law stated above from Ampère's theory and the principle of the conservation of energy; at least we have done so for the case of induction by permanent magnets, and the same reasoning will also apply to the case where the alteration of the magnetic field, owing to the induced current in the primary circuit, is so small that it may be neglected.

Electromagnetic unit of electromotive force. We have now the means of stating in a convenient form the electromagnetic unit of electromotive force. It is the electromotive force of induction in a circuit the number of lines of magnetic force through which is increasing at the rate of one per second.

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[1] *Exp. Res.*, ser. i., ii., (ix.), xxviii., xxix., 1831–32, 1851. The general statement in the text is given for the reader's convenience, and is not meant to be historical.

[2] Equivalent in the sense of producing the same magnetic field.

[3] *Ueber die Erhaltung der Kraft*, 1847.

[4] *Rep. Brit. Ass.*, 1848, and *Phil. Mag.*, 1851.

[5] All the quantities are supposed to be measured in electromagnetic absolute units.

[6] We may suppose this work spent in raising a weight, &c.

In the case where the field is due to a current i', we have by formulæ (4) and (14) of last division

$$N = i'M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24),$$

where M now stands for $\iint \frac{\cos \epsilon}{D} ds d\sigma$ extended all over the two circuits. M, which depends merely on the configuration and relative position of the two circuits, is called the coefficient of mutual induction.

An application of the principle of the conservation of energy of great importance was made by Sir William Thomson to the case of two electric circuits of any form, in which the currents are kept constant.

Let two such circuits, the currents in which are ii', be displaced so that the coefficient of mutual induction M increases by dM. Let us suppose that the currents i and i' are maintained by two constant batteries of electromotive forces E and E′, and that the motion takes place *so slowly* that the currents may be regarded as constant throughout. If R and R′ be the resistances of the circuits, Hdt the mechanical equivalent of the whole heat generated, and Kdt the whole expenditure of chemical energy in the batteries in time dt,

$$H = Ri^2 + R'i'^2, \text{ and } K = Ei + E'i',$$
$$K - H = i(E - Ri) + i'(E' - R'i').$$

Now, applying (23),

$$Ri = E - i'\frac{dM}{dt}, \text{ and } R'i' = E' - i\frac{dM}{dt};$$

whence

$$K - H = 2ii'\frac{dM}{dt},$$

or, as we may write it,

$$(K - H)dt = 2ii'dM \quad . \quad . \quad . \quad . \quad (25).$$

Now $ii'dM$ is the work done by the electromagnetic forces during the displacement which we may suppose spent in lifting a weight.

Hence, when two electric currents are allowed slowly to approach each other, being kept constant and doing work the while, over and above the work which is spent in generating heat in the conductors, an amount of energy is drawn from the batteries equivalent to twice the work done by the electromagnetic forces.

There remains therefore an amount of work as yet unaccounted for. What becomes of it? The answer is, that the energy, or, as Sir W. Thomson calls it, the "mechanical value," of the current is increased. But how increased? When a material system (and we may consider the two circuits, the batteries, the lifted weight, &c., as such) is left to itself, it moves so that its *potential* energy *decreases*. In this case, therefore, there must have been an increase of *kinetic* energy somewhere. This energy may be called the electrokinetic energy of the system; according to Maxwell's theory, this kinetic energy has its seat in the medium surrounding the wire. The energy thus stored up is accounted for in the increased development of heat, &c., when the two currents are broken in succession.

Returning now to our general law of induction, let us write down in the most general form the equations which determine the course of the currents in two circuits (A, B), in which the form and relative positions of the circuits, as well as the current strengths, are variable. The number of lines of force which pass through a circuit depends partly on neighbouring circuits, partly on the circuit itself. Retaining the notation used above, we may, in the case of two circuits, write the first part Mi', and the second part Li; where L is a double integral of the same form as M, only both elements ds and $d\sigma$ now belong to the same circuit. We have, therefore, for the whole number of lines of force passing through the circuit A, $Mi' + Li$. Similarly we have for B, $Mi + Ni'$. We have therefore by our general law,

$$\left.\begin{aligned} E - \frac{d}{dt}(Mi' + Li) &= Ri \\ E' - \frac{d}{dt}(Mi + Ni') &= R'i' \end{aligned}\right\} \quad . \quad . \quad . \quad (26).$$

These are the general equations for the induction of two circuits. The electromotive force of induction in A can be divided into two parts: one of these, viz., $\frac{d}{dt}(Mi')$ is due to the circuit B, the other $\frac{d}{dt}(Li)$ is due to the circuit A itself, and is called the electromotive force of self-induction. L is called the coefficient of self-induction for A. Similarly $\frac{d}{dt}(Ni')$ is the electromotive force, and N the coefficient force of self-induction for B.

If we have only one circuit then $M = 0$, and the equation for the course of the current is

$$E - \frac{d}{dt}(Li) = Ri;$$

here there is *only* self-induction.

Theory of Neumann. Law of Lenz.

F. E. Neumann, to whom belongs the honour of first stating with mathematical accuracy the laws of induction, adopted a foundation for his theory very different from the one chosen above. His method was based on the law of Lenz[1], enunciated very soon after the great discovery of Faraday, which lays down that, in all cases of induction by the motion of magnets or currents, the induced current has a direction such that its electromagnetic action on the inducing system tends to oppose the motion producing it.

Besides its historical importance, this law affords a very convenient guide in many practical applications of the theory of induction. The reader will find no difficulty in verifying it on the elementary cases given at the beginning of this division. It can be deduced at once from our general law. Consider any circuit in which a current i is flowing, and let the direction of the current be the positive direction round the circuit. Suppose the circuit to move so that the number of lines of force passing through it increases, this is the way the circuit would tend to move under the electromagnetic forces when traversed by a current i; but the electromotive force of induction is in the negative direction round the circuit by the general law, and would therefore produce a current opposite in direction to i. The electromagnetic action on this current would be opposite to that on i, that is, would tend to hinder the displacement. It is a curious fact that a law exactly like this had been announced shortly before Lenz by Ritchie, only with the direction of the action *reversed* in every case.

The results of Neumann are identical with those given above. The double integral M, which is here called the coefficient of mutual induction of two circuits, Neumann calls the mutual *potential* of the two circuits, and what has been called above the coefficient of self-induction of a circuit he calls the *potential of the circuit on itself*. Accounts of his theory will be found in Wiedemann's *Galvanismus*, and in most Continental works on electricity.

Experimental Verification of the Laws of Mutual Induction.—It will be observed that, in the law of induction for linear circuits, no statement is made respecting the material or thickness of the circuit in which the electromotive force of induction acts, or of the non-conducting medium across which induction takes place.

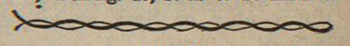

Fig. 48.

Faraday showed that the material of the circuit has no effect. He found, for instance, that when two wires of different metals were joined and twisted up together, as in fig. 48, so as to be insulated from each other, no induced current could be obtained by passing the arrangement between the poles of a powerful magnet. The same result was obtained when one of the branches of the circuit was an electrolyte. Lenz[3] connected two spirals of wire in circuit with each other, and placed first one then the other, on the soft iron keeper of a horse-shoe magnet; so long as the number of turns on each spiral was the same, the induced current was the same, no matter what the material or thickness of the wire in each spiral. Since in this case the whole resistance of the circuit was always the same, the electromotive force of induction must have been the same.

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[1] *Pogg. Ann.*, 1834.

[2] *Exp. Res.*, 193, &c., 1832; also 3143, &c., 1851.

[3] *Pogg. Ann.*, 1835.

We conclude, therefore, that the electromotive force[1] of induction is independent of the material, and also of the thickness of the wire, so long as the latter is so small that we may consider the wire as a linear circuit.

Lenz made quantitative determinations of the induced current by means of the above arrangement.

The soft iron keeper, with a coil of n windings, was rapidly detached from the magnet, and the first swing a of a galvanometer in circuit with the coil was measured. The quantity of electricity which passes in the induced current is measured by $\sin \frac{1}{2}a$, provided the whole duration of the current is small compared with the time of oscillation of the galvanometer needle (see art. GALVANOMETER). Again, when the keeper is attached to the magnet, very nearly all the lines of magnetic induction[2] pass through the keeper; hence the number of lines of induction which pass through the coil is very nearly proportional to the number of windings, and therefore, if the resistance of the circuit be kept the same, the whole amount of electricity which passes will be proportional to n. In the actual experiment the wire was wound and unwound from the keeper, so that the whole resistance did remain the same. The following is a set of Lenz's results:—[3]

No. of Windings.	2	4	8	12	16	20
$\sin \frac{1}{2}a$	0·0491	0·1045	0·2156	0·3319	0·4470	0·5594
$\sin \frac{1}{2}a \div n$	0·0245	0·0261	0·0270	0·0276	0·0279	0·0280

The value of $\sin \frac{1}{2}a \div n$ is very nearly constant. It increases a little as the number of windings increases, as ought to be the case, for, although most of the lines of induction pass through the keeper, yet all do not, and a few more are included when the number of turns is increased.

Effect of medium.

Faraday made special investigations in search of the effect of the medium across which induction is exerted. He found[4] that no effect on the integral current was produced by inserting shellac, sulphur, copper, &c. between the primary and secondary coils. The insertion of iron or any strongly magnetic body, of course, produces an effect, because the distribution of the lines of magnetic force is thereby altered, and therefore, by our general law, the electromotive force of induction will be correspondingly affected. We conclude, therefore, that the electromotive force of induction is independent of the medium across which it is exerted.[5]

It must be remarked, however, that in the case of conducting media, the statement is subject to a certain limitation, the nature of which follows from the law of induction itself. For there will be induced currents in the intervening medium if it be a conductor, and these currents will disturb the lines of force while they continue to flow. These currents are *transient*, however, so that their integral effect on the number of lines of force passing through the secondary is zero. It is obvious, therefore, that, if we replace "electromotive force" by "time integral of electromotive force extended over the whole time that the induction currents last," the statement will still be true. The only effect, therefore, of interposed conducting media is on the *time* which the induced currents take to rise and fall.

Weber[6] applied his electrodynamometer to test the laws of induction.

The suspended coil was caused to oscillate when there was no current either in it or in the fixed coil, and the logarithmic decrement[7] of its oscillations carefully determined. This decrement, due to the friction of the air, &c., was found to be constant for different lengths of the arc of oscillation. The terminals of the suspended coil were next connected so that it formed a closed circuit, and a constant current was sent through the fixed coil. Induction currents were now generated in the suspended coil, whose electrodynamic action constantly opposed its motion. It was found that the logarithmic decrement was still constant, but greater than before. Weber therefore concluded that the induced current at each instant was proportional to the velocity of the coil. Since the resistance does not vary, this is in accordance with the general law.

Weber further showed that the induced current is the same whether it is produced by a current in the fixed coil or by a magnet, which exercises the same electromagnetic action as that current on the suspended coil, when the latter is traversed by a current of unit strength.

The electrodynamometer may also be used to demonstrate the equality of the whole amounts of electricity which pass in the direct and inverse currents. If the induced currents from a secondary coil whose primary is being "made and broken" be passed through both coils of the instrument, there will be a deflection, since the action depends on the square of the current; but if the induced current be sent through the suspended coil alone, and a constant current be sent through the fixed coil, there will be no deflection, which shows that the quantities of electricity passing in the alternate currents of the secondary coil are equal and of opposite sign.

Felici.

Felici (1852 and 1859) made an extended series of experiments on the laws of induction. He used null methods, and his experiments bear a resemblance in some respects to the electrodynamical experiments of Ampère. Maxwell[8] has given a summary of Felici's results.

It is found, for instance, that the electromotive force of induction of a circuit A on another B is independent of the material or section of the conductors, that it is proportional to the current in A and to the number of windings in B. The induction of A on B is the same as that of B on A, when the inducing current i is the same in both cases. Any portion of A or B may be replaced by a zig-zag portion, which nowhere deviates far from it. In pairs of circuits geometrically similar, the electromotive force of induction is proportional to the linear dimensions, and so on.

If B be so situated with respect to A that starting or stopping a current in A produces no induced current in B, B is said to be conjugate to A. There are an infinite number of such conjugate positions of B; and Felici shows that, if B be moved from one of these P_1 into another P_2 very quickly, no effect is produced on the galvanometer. If B be moved from P_1 to any position P (not a conjugate position), the effect on the galvanometer is the same as if the current i were suddenly started in A, B being in the position P.

All these results are direct consequences of our general law, and indeed might be used as a foundation for it.[9]

Faraday's exploring conductor.

In his later researches on electromagnetic induction (series xxvii. and xxix.), Faraday develops in considerable detail his ideas on the connection between the lines of magnetic force and the induced current, and gives increased precision to the experimental methods that flow therefrom. He points out the great value of methods, such as the use of iron filings, for exhibiting in a visible form the course of the lines of magnetic force. He also insists on the great use of a small moving circuit, which can be used to explore the magnetic field under circumstances which render the application of other methods impossible.

The direction of a line of force may be determined in various ways by means of the moving conductor. Maxwell[10] gives four such ways:—(1) if a conductor be moved along a line of force parallel to itself, it will experience no electromotive force; (2) if a conductor carrying a current be free to move along a line of force, it will show no tendency to do so; (3) if a linear conductor coincide with a line of force and be moved parallel to itself in any direction, it will experience no electromotive force in the direction of its length; (4) if a linear conductor carrying an electric current coincide in direction with a line of magnetic induction, it will not experience any mechanical force.

In these researches Faraday treats at considerable length a case of the induction of electric currents, to which Continental writers have given the somewhat mysterious name of "unipolar induction." It belongs to a class of cases on

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[1] Of course, the same is not true of the current of induction, which depends on the resistance of the circuit.

[2] In Maxwell's sense; we might say "lines of magnetic force" in Faraday's sense; see art. MAGNETISM.

[3] Wiedemann, Bd. ii. § 706.

[4] *Exp. Res.*, 1709, &c., 1838.

[5] Other investigators have sought for such effects, and some have affirmed their existence; but there is no body of concurrent testimony on the point.

[6] *Maassbestimm.*, §§ 10 and 11, 1846.

[7] See art. GALVANOMETER.

[8] Vol. ii. § 536; see also Wiedemann, Bd. ii. § 709.

[9] See Maxwell, *l.c.*

[10] Vol. ii. § 597.

which they have rightly dwelt as being in a sense the reverse of the electromagnetic rotations. The following theory of the phenomenon will make this clearer :—

Referring back to figure 40, let AB be part of a conducting circuit arranged as there described, and let it be caused to move in the direction Pp. Then if E be the electromotive force in the circuit in the direction AB, N the number of lines of force passing through the circuit, ϕ the angle through which AB moves (from X to Y) about OZ, we have, by our general law,

$$E=-\frac{dN}{dt}=-\frac{dN}{d\phi}\frac{d\phi}{dt}.$$

Now, by Ampère's theory, $K=\frac{idN}{d\phi}$, hence (p. 73)

$$\begin{aligned}E&=-\frac{K}{i}\frac{d\phi}{dt}\\&=-m\,(\cos\beta_1-\cos\alpha_1-\cos\beta_2+\cos\alpha_2)\frac{d\phi}{dt}\quad.\quad(27).\end{aligned}$$

Hence, if the conductor AB be caused to move with given angular velocity about the magnet SN, in that direction which it would take under the action of the magnet if it carried a current i, then there will be an electromotive force of induction along the circuit of which AB forms part, whose direction is opposite to that of i, and whose magnitude is found by dividing the couple acting on AB (when traversed by i) by i, and multiplying it by the given angular velocity. This result is a beautiful instance of the law of Lenz.

A great variety of experimental arrangements may be imagined to realize the case thus described. Every apparatus devised to produce an electromagnetic rotation may be used to illustrate it.

The following case may be taken as typical. SN (fig. 49) is a bar magnet whose action may be represented by two poles, N and S. At the middle point of its axis is fixed a disc BA, against which presses the terminal of a wire CA in metallic connection with the axis through the pivot at S. If CA be caused to rotate in the direction of the arrow p, the disc standing still, there will be an induced current in CABC in the direction of the arrow q. If CA and the disc revolve together, there will be no current. If CA stand still, and the disc rotate in the direction of the arrow, there will be a current in the opposite direction; for this is clearly the same as if the disc stood still, and CA rotated in the opposite direction.[1] The electromotive force in each case is independent of the form of CA, and is given by $2m(1-\cos\alpha)\omega$, where m is the strength of the pole N, α the angle ANB, and ω the angular velocity.

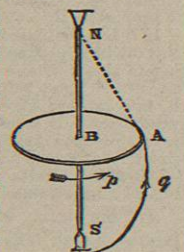

Fig. 49.

It is well to remind the reader that the lines of force are closed curves, every one of which passes up the axis of the magnet from S to N, and back through the outside medium to S. If this be forgotten, and an attempt be made to determine the electromotive force of induction by considering the motion of the *disc*, an error will easily be made. If we take the simpler course above, and consider the motion of the *conductor*, there is then no danger of mistake.

Coils with iron core.

In most of the experiments we have hitherto been describing, the object has been to obtain indications of the direction of the currents of induction, or to measure the electromotive force of induction under definite circumstances; if, however, we desire to exhibit the effects of induction in a striking manner, in order to convey belief to the spectator, or to serve some practical purpose, recourse is had to a different kind of apparatus. We may wind our primary and secondary coils on bobbins, and insert the former within the latter, so as to get the greatest possible number of turns of wire into proximity. The number of turns on the primary is usually made small, in order that the current in it may not be weakened by a large resistance, and that its coefficient of self-induction (see below) may be small. Mention has already been made of the effect of soft iron in increasing the number of lines of force that pass through a circuit. It is easy to see that it will produce a corresponding effect in strengthening induction. The precise amount of it is very hard to calculate, owing to the irregularities in the magnetization and demagnetization that arise from residual magnetism. The question belongs, however, to magnetism. The effect can be demonstrated practically by observing the alteration in the inductive action produced by inserting a bundle of iron wires[2] into our primary coil.

Physiological effects.

The physiological effects of induced currents are very striking; indeed, the nerve and muscle preparation of the physiologist affords a very delicate method for detecting them. If the human body form part of the circuit of the secondary coil of such an induction apparatus as we have just indicated, and the primary current be stopped and started in rapid succession, say by stripping one terminal of the circuit on a toothed wheel attached to the other, a sensation is experienced which, with a moderately powerful apparatus furnished with a core, is so painful and peculiar that the patient is not likely to forget either it or its cause. The tetanic muscular contractions produced in this way have formed the subject of much physiological investigation, of which an account will be found in the proper place (see article PHYSIOLOGY).

The flat spirals of Henry, formed of flat bands of copper insulated from each other with silk ribbon, are also very convenient for demonstrating the existence of induced currents.

The most powerful inductive apparatus for furnishing large quantities of electricity are the various magneto-electric machines which have now been brought to great perfection (see Historical Sketch).

By means of these and similar appliances, all the effects of the electric current and the electric discharge may be shown in the greatest perfection.

Induction by statical discharge.

Induction by Discharge of Statical Electricity.—The phenomena of induction can be exhibited with the transient current of electricity in the discharge of a Leyden jar or other accumulator of statical electricity. There is a difficulty in exhibiting the effect, owing to the great differences of potential between different parts of the circuit, which render the application of a coil of silk-covered wire useless. A common way of getting over the difficulty consists in cutting two spiral grooves in two flat ebonite discs. Wires are embedded in these, and they are then put together with a thin plate of glass between, so that the spirals are opposite each other. When a jar is discharged through one spiral, an induction current passes in the other, and may be indicated by a galvanometer, or, better still, by a frog preparation. The induced current is, however, in general a complicated phenomenon, owing to the oscillatory nature of the discharge (see above, p. 65).

It would lead us too far to go into these and kindred subjects: the reader who desires to pursue the matter will find excellent accounts in Mascart, t. ii. §§ 611–825, and Riess, Bd. ii. §§ 780–906. Particularly interesting are the researches of Verdet, an account of which will be found in his works, along with many indications of what others have done in the same field.

Induced Currents of Higher Orders.—Induced currents may in their turn induce other currents, and these again others, and so on.[1] This can be brought about by forming part of the secondary circuit of one inductive apparatus into the primary of the next, and so on. As may be supposed, the successive induced currents diminish very rapidly in strength, and require special means for their detection. But the phenomenon also goes on increasing in complicacy. Suppose we start the current in the first primary, there is a single inverse current of the "first order" which rises and then falls; there will, therefore, be two currents of the "second order"—first a direct, then an inverse; each of these rising and falling causes two currents of the third order, and so on in geometric progression. These currents have been detected in certain cases by means of their physiological action and their magnetizing powers. The latter effects present some points of interest in connection with magnetism, but we cannot spare space for the matter here.

Self-Induction.—The existence of self-induction has been deduced as a theoretical consequence of the general law of induction. It was not so discovered, however. It was first arrived at by Faraday[2] from experimental considerations. The observation from which he started was the following fact communicated to him by Mr Jenkin, who had shortly before discovered it:—Although it is impossible with a short circuit of wire and a single battery cell to obtain a shock by making and breaking contact, yet a very powerful shock is obtained if the coil of an electromagnet be included in the circuit. This may be shown thus:—Let ZC (fig. 50) be a battery of a single cell, CABDEF a circuit with a cross branch BF, in which at G the human body, &c., may be inserted. Contacts can be made and broken at A, very rapidly if need be, by means of a toothed wheel. When BDEF consists of a short single wire, nothing particular is felt at G, but when the coil of an electromagnet is inserted in DE, the patient at G experiences a series of powerful shocks comparable to that obtained from the secondary coil of an inductive apparatus in the manner already described.

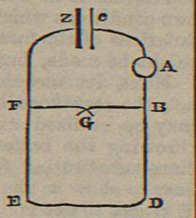

Fig. 50.

If the cross circuit be done away with, a powerful spark is obtained at A on breaking contact, but none on making. This spark is particularly bright if a mercury contact be used, owing to the combustion of the mercury. If, however, the electromagnet be removed from DE, and a short wire substituted, the spark becomes quite insignificant, although the whole circuit may be now very hot, owing to the increased current. Faraday found that the same effect, only smaller, was produced when a simple helix without a core was substituted for the electromagnet; and a similar effect, only still smaller, was obtained when a very long straight wire was used. Faraday soon recognized that these effects are consequences of the laws at which he had arrived in his first series of researches on induction. When the current is rising in a circuit, the number of lines of magnetic force passing through it is on the increase, hence an electromotive force is generated which opposes that of the battery, and causes the current to rise slowly; again, when the current begins to decrease the number of lines of force begins to decrease, and an electromotive force of induction is called forth which tends to prolong the current. We have, therefore, a weakening of the electromotive force at starting and an exaltation at stopping, which accounts for the absence of the spark or shock at make, and the presence of one or other at break. Such inductive effects are obviously heightened when the current is wound into a spiral form; if, however, the spiral were wound double, and the current sent through the two wires in opposite directions, the inductive effects would annul each other, and, in fact, with this arrangement the spark and shock are extremely small.

Faraday demonstrated the existence of these electromotive forces by means of the currents which they produce in derived circuits,[3] when the battery contact is broken or made.

He used the arrangement given in figure 50. A galvanometer was inserted at G, and the needle stopped by pins properly placed from deviating as urged by the branch of the battery current from B to F, but left free to move in the opposite direction. It was found that the needle deviated sharply when contact was broken at A, in a direction indicating a current from F to B. Again the contact was made, and the needle stopped at the deviation due to the current from B to F, so that it could not return to zero. The contact was then broken and made again, and it was found that at the make the needle tended to go beyond the position due to the steady current in BF. Faraday also arranged a platinum wire at G, so that it did not glow under the steady current in BF, but immediately ignited when the contact at A was broken. Chemical action was produced in a similar manner. In fact we may, by taking advantage of the self-induction, cause a single cell to produce decomposition of water and evolution of gas, which it could not do alone consistently with the conservation of energy. This may be managed[4] by inserting at A (fig. 50), instead of the contact breaker, the coil of an electromagnet, and placing the decomposing cell in DE. Let contact be made and broken at G (say by an automatic break); when the contact is made the current flows through the coil and through BF, when it is broken the electromotive force of induction added to that of the battery enables the current to pass through the cell and liberate the ions. At the make there is no such effect; there results therefore continued chemical decomposition.

Edlund's measurements of extra current.

Edlund[5] investigated the integral electromotive forces of self-induction at the opening and closing of a circuit, and showed that they are equal. His experimental arrangement is very ingenious:—

G (fig. 51) is a differential galvanometer, A a coil whose self-induction is to be examined, C a wire wound in a zig-zag[6] so as to have no self-induction. The battery E is connected at B and D with the circuit composed of G, A, and C, so that the currents in BcdCD and BbaAD pass round the coils of G in opposite directions. The resistance C is so arranged that there is no deflection of the needle in G. If now the current be stopped by breaking the circuit EB at K, the electromotive force due to the self-induction of A causes an extra current to flow round the circuit AabBcdCD, traversing the coils of G in the same direction. We therefore get a deflection D_1. In a similar manner if we make contact at K we get another deflection D_2, due to the starting of the current in A. There is no difficulty in showing that, if E_1, E_2 be the time integrals of the electromotive force in the two cases, then

$$\frac{E_1}{E}=\frac{D_1}{D_2}.$$

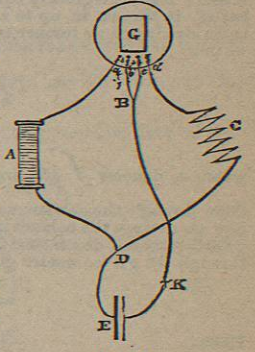

Fig. 51.

One of the difficulties encountered in such experiments is the increase of the electromotive force of the battery E when it is left open for a time; this causes the extra current at make to be greater than that at break. Rijke, who made experiments similar to those of Edlund, avoids this difficulty by circuiting the battery, when BK is broken,

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[1] If the reader wish for a proximate rule for the direction of the electromotive force of induction, the following will serve. Stand with the body in the line of magnetic force with the head pointing in the positive direction, look in the direction in which the part of the circuit on which the feet are is moving; the E. M. F. along the circuit is towards the right hand.

[2] The iron is broken up into wires to prevent the formation of induced currents in the body of the metal. These currents retard the rise of the induced currents.

[1] Some physicists have called these currents induced currents of the second and third orders, &c.

[2] *Exp. Res.*, 1048. &c., 1834.

[3] These currents are sometimes called extra currents, and the name is applied even when there is no alternative circuit. The extra currents are then the defect or excess of the currents at the make and break, considered with reference to the steady current.

[4] De la Rive, Wiedemann, Bd. ii. § 740.

[5] *Pogg. Ann.*, 1849.

[6] The best arrangement would be to use insulated wire and double it on itself.