through a resistance equal to the effective resistance from E to K. Further details concerning the method and results of these experiments may be found in Wiedemann, Bd. ii. § 744, &c.

Maxwell's method.

A very convenient method for exhibiting and measuring the extra current is obtained by using a Wheatstone's bridge instead of a differential galvanometer. Let the bridge be balanced as usual, so that when the battery circuit is made, and the galvanometer circuit made afterwards, there is no deflection. If one of the resistances be wound so as to have a large coefficient of self-induction, and the galvanometer circuit be completed before the battery is thrown on, then, owing to the self-induction, the galvanometer needle will be suddenly deflected.

Let AC, CD, DB, BA be four conductors of resistance S, Q, P, R, arranged as a Wheatstone's bridge (see fig. 22), with a battery between A and D, and a galvanometer G between B and C. Let L be the coefficient of self-induction of the coil S. Then, A, C, &c., denoting the potentials at A, C, &c., x and y the currents in AC and AB, and z the current in G, we have

$$A-C=Sy+L\frac{dy}{dt},\quad A-B=Rx,\quad C-D=Q(y+z),$$

&c. Eliminating, as in Maxwell, vol. i. p. 399, or above, p. 43, we get

$$z=\frac{PS-QR+PL\frac{d}{dt}}{D'}E \quad . \quad . \quad . \quad . \quad . \quad (28),$$

where $\frac{d}{dt}$ is a separated symbol, and D' is the determinant of the system of resistances with $S+\frac{d}{dt}$ written for S. We may therefore write

$$D'=D+HL\frac{d}{dt},$$

D being the ordinary determinant, and H a function of PQR, &c., which we need not determine. Equation (28) may therefore be written

$$Dz+HL\frac{dz}{dt}=PL\frac{dE}{dt} \quad . \quad . \quad . \quad . \quad . \quad (29),$$

provided the bridge be balanced, *i.e.* if PS − QR be zero. Suppose now the galvanometer circuit is closed, and then the battery circuit closed; then, integrating equation (29), from the instant before the battery is thrown in up to a time τ when all the currents have become steady and no further current flows through the galvanometer, we get

$$D\int_0^\tau z\,dt=PLE,$$

$$\text{or}\qquad z_1=\frac{PLE}{D} \quad . \quad . \quad . \quad . \quad (30),$$

where z_1 denotes $\int_0^\tau z\,dt$, *i.e.* the whole amount of electricity that flows through the galvanometer owing to the induced current. If now we derange the balance in the bridge by increasing S by a small quantity x, and decreasing Q by as much, we get a steady current through the galvanometer given by

$$z=\frac{(P+R)xE}{D}$$

Hence

$$\frac{z_1}{z}=\frac{PL}{(P+R)x} \quad . \quad (31).$$

Now, if β be the first swing of the galvanometer needle, owing to the induction current, α the deflection under the steady current, and T the time of oscillation of the needle under the earth's force alone (T is supposed to be so large that the duration of the induced current is very small compared with it); then it may be shown that

$$\frac{z_1}{z}=\frac{T\sin\frac{1}{2}\beta}{\pi\tan\alpha} \quad . \quad . \quad . \quad . \quad (32).[1]$$

Hence

$$L=\frac{(P+R)xT\sin\frac{1}{2}\beta}{P\pi\tan\alpha} \quad . \quad . \quad (33).$$

We thus get L in terms of quantities which can be easily measured. This method of finding L is due to Maxwell.

[1] Certain corrections would in general be necessary in practice, but we need not discuss them here.

Calculations of Helmholtz.

The application of the equations (26) to determine the march of the current in certain simple cases leads to results of great interest.

Suppose that an electromotive force E begins to act in a circuit of resistance R and coefficient of self-induction L. The equation for the current strength i at any time t after it has begun to act, is

$$L\frac{di}{dt}+Ri=E \quad . \quad . \quad . \quad . \quad . \quad . \quad (34).$$

The integral of this is $i=\frac{E}{R}(1-e^{-\frac{R}{L}t})$ . . . . (35),

the constant of integration being determined by the condition $i=\frac{E}{R}$ (= steady current) when $t=\infty$.

Hence the current starts with the value zero, and increases continuously till it reaches the steady value $\frac{E}{R}$. The part $-\frac{E}{R}e^{-\frac{R}{L}t}$ is due to self-induction, and is called the extra current. The whole amount of electricity passing in this part of the current is

$$-\frac{EL}{R^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (36).$$

Time-constant of a coil.

The quantity $\frac{L}{R}$ is of the same dimension as t, and is called the time constant of the coil. According as the time constant is greater or less, the longer or shorter time will the current take to rise to a given fraction of its steady value. If we desire therefore to prolong the induction and to increase it as well, we must make L large and R small, two conditions which in the extremes are inconsistent. Calculations of the form of coil for maximum inductive effects might be made, but this is not the place to enter on them.

Next, let the electromotive force E suddenly cease to act, the resistance of the circuit being unchanged. This may be realized experimentally within certain limits by throwing the battery out of the circuit, and at the same time substituting for it a wire of equal resistance. It is easy to show as above that the extra current at a time t after E ceases to act is

$$\frac{E}{R}e^{-\frac{R}{L}t},$$

and the whole amount of electricity which passes is $+\frac{EL}{R^2}$.

Experiments Helmholtz.

Helmholtz,[2] who was the first to treat this subject both experimentally and mathematically, operated as follows:—

(1) The battery was thrown into the circuit, and after a time t the circuit was broken.

(2) The battery was thrown in, and after a time t replaced by a circuit of equal resistance.

These changes were effected by means of a system of levers, which it is not necessary to describe here. An account of the apparatus will be found in the paper quoted.

The amount of electricity which passes through the circuit is measured by a galvanometer whose time of oscillation is long compared with t. In the first case the amount is

$$A=\frac{Et}{R}-\frac{EL}{R}\left(1-e^{-\frac{R}{L}t}\right);$$

in the second

$$B=\frac{E}{R}t,$$

because here the two extra currents just counterbalance each other.

The observed value of B in each case enables us to calculate t. E and R being found by separate observations, one observed value of A enables us to calculate L. Using these values of E, R and L, a series of values of t, and hence A, can be calculated from the observed values of B, and the result compared with the observed value of A. The agreement between theory and experiment was sufficiently close to justify the application of the principles from which the above formulæ were deduced. Among these principles may be mentioned the validity of Ohm's law for transient currents.

The reader will find in the original paper details concerning the above and other similar results arrived at by Helmholtz.

[2] *Pogg. Ann.*, 1851.

The case of two circuits of invariable form and position is of great interest, from the light it throws on the action of the induction coil. We shall suppose that we have no soft iron core, and that the break in the primary is instantaneous. The latter condition is very far from being realized in practice, even with the best arrangements, so that our case is an ideal one.

Let i and j be the current strengths in primary and secondary, R and S the resistances, L, M, N the coefficients of induction, E the electromotive force in the primary. The equations are

$$L\frac{di}{dt}+M\frac{dj}{dt}+Ri=E \quad . \quad . \quad . \quad . \quad . \quad (37),$$

$$M\frac{di}{dt}+N\frac{dj}{dt}+Sj=0 \quad . \quad . \quad . \quad . \quad . \quad (38).$$

It is easy, in the first place, to show that the whole amounts of electricity which traverse the secondary at make and break of the primary are equal but of opposite signs. In fact, if we integrate (38) from the instant before make to a time when the induction currents both in primary and secondary have subsided, we get

$$\int j\,dt=-\frac{M}{S}I=-\frac{ME}{SR} \quad . \quad . \quad . \quad . \quad . \quad (39).$$

where I denotes the steady current in the primary. Similarly integrating over the break, we get

$$\int j\,dt=+\frac{M}{S}I=\frac{ME}{SR}$$

In the second place, if we assume the break instantaneous, we can find the initial value of the direct current in S. Thus integrate[1] (38) from the instant before break to the time τ after it, τ being infinitely short compared with the duration of the induction currents, then

$$-MI+Nj_0+S\int_0^\tau j\,dt=0.$$

Now the last term may be neglected, because τ is infinitely small and j is not infinite, hence we have, for the initial value of j,

$$j_0=\frac{M}{N}I=\frac{ME}{NR} \quad . \quad . \quad . \quad . \quad . \quad (40).$$

It is very easy now to determine the farther course of the current in S. The equation for j reduces to

$$N\frac{dj}{dt}+Sj=0;$$

and we get, using (40),

$$j=\frac{ME}{NR}e^{-\frac{S}{N}t} \quad . \quad . \quad . \quad . \quad . \quad (41).$$

The direct induced current (current at break), therefore, starts in our ideal case with an intensity which is to the intensity of the steady current in the primary as the coefficient of mutual induction of the coils is to the coefficient of self-induction of the secondary, and then dies away in a continuous manner like any other current left to itself in a circuit of given resistance and self-induction.

Since we have already given enough of these calculations to serve as a specimen, we content ourselves with stating the result for the current at make. Owing to the self-induction of R, &c., the current in R rises continuously from zero to the value I; the induced current in S therefore begins also from zero, rises to a maximum, and then dies away. The mathematical expression for it contains, as might be expected, *two* exponential terms.

It is instructive, in connection with what has already been said concerning the electrokinetic energy of two moving circuits, to examine what becomes of the energy in the case of two fixed circuits of invariable form.

Equations (37) and (38) may be used if, for generality, F be written instead of 0 in (38), so that there is electromotive force (say of constant batteries) in both circuits. Multiplying (37) by i and (38) by j, adding, and integrating from the time before E and F begin to act to a time τ when the currents have all become steady, we get

$$\int_0^\tau(Ei-Ri^2)dt+\int^\tau(Fj-Sj^2)dt=\tfrac{1}{2}(Li^2+2Mij+Nj^2) \quad . \quad (42).$$

In words, the *excess* of the chemical energy exhausted in the batteries over the amount of energy which appears as heat in the circuits is $\frac{1}{2}(Li^2+2Mij+Nj^2)$, which we denote by K. Similar remarks to those made at p. 76 apply here. K is the amount of electrokinetic energy stored up in the medium surrounding the circuits during the time that E and F are raising the currents against self and mutual induction.

If we integrate similarly over the break of both currents, we find the *defect* of the chemical energy exhausted under the heat evolved in the circuit to be again K. Much of the energy thus discharged from the system at break usually appears in the spark.

Thomson's theory.

Electrical Oscillations.—Helmholtz[2] seems to have been the first to conceive that the discharge of a condenser might consist of a backward and forward motion of the electricity between the coatings, or of a series of electric currents alternately in opposite directions. Sir William Thomson[3] took up the subject independently, and investigated mathematically the conditions of the phenomenon.

Let q be the charge of the condenser at time t, C its capacity, E the difference of potentials between the armatures, i the current in the wire connecting the armatures, R its resistance,[4] L the coefficient of self-induction. Then we have

$$q=CE,\quad i=-\frac{dq}{dt},$$

$$\text{and}\qquad L\frac{di}{dt}+Ri=E,$$

$$\textit{i.e.},\qquad \frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{1}{LC}q=0 \quad . \quad . \quad . \quad . \quad . \quad (43)$$

The solution of this equation is

$$q=e^{-mt}(Ae^{nt}+Be^{-nt}), \quad . \quad . \quad . \quad . \quad . \quad (44)$$

$$\text{where}\qquad m=\frac{R}{2L},\quad n=\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}.$$

A and B are constants to be determined by the conditions $q=Q$ and $\frac{dq}{dt}=0$ when $t=0$.

Two distinct cases arise.

(1.) Let R be greater than $\sqrt{\frac{4L}{C}}$; then the exponentials in (44) are real, the discharge is continuous, all in one direction, and involves no essentially new features.

(2.) Let R be less than $\sqrt{\frac{4L}{C}}$; then the appropriate form of the solution is

$$q=e^{-mt}(A\cos nt+B\sin nt),$$

where m has the same meaning as before, but n stands now for $\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$. If we determine A and B by the initial conditions, we get

$$q=e^{-mt}\left(\cos nt+\frac{m}{n}\sin nt\right)Q \quad . \quad . \quad . \quad . \quad (45).$$

The current is given by

$$i=\frac{Q}{NLC}e^{-mt}\sin nt, \quad . \quad . \quad . \quad . \quad (46).$$

It follows from these equations that, when $R<\sqrt{\frac{4L}{C}}$, the charge of one armature of the condenser passes through a series of oscillations. The different maxima are

$$Q,\quad -Qe^{-\frac{m\pi}{n}},\quad Qe^{-\frac{2m\pi}{n}},\quad Qe^{-\frac{3m\pi}{n}},\ \text{\&c.},$$

occurring at times

$$0,\quad \frac{\pi}{n},\quad \frac{2\pi}{n},\quad \frac{3\pi}{n},\ \text{\&c.}$$

[1] The reader might suppose that this process of integration might be equally applied to (37). This is not so, however, owing to the variability of R at the break.

[2] *Die Erhaltung der Kraft*, 1847.

[3] *Phil. Mag.*, 1855. This paper is a very remarkable one in many respects. The methods used in the beginning to arrive at the equation (43) are well worth the reader's study.

[4] R here must be understood to represent the mean resistance of the circuit during the discharge.

When the charge has any of these maximum values, the current is zero. The current maxima form a similar descending geometric series, the times of occurrence being

$$\frac{\theta}{n},\ \frac{\theta+\pi}{n},\ \frac{\theta+2\pi}{n},\ \frac{\theta+3\pi}{n}.\ \&c.,$$

where θ is the acute angle $\tan^{-1}\frac{n}{m}$.

The interval between any positive and the next negative maximum, whether of charge or current, is therefore $\frac{\pi}{n}$.

We need not insist on the evident importance of this result. Thomson, in his original paper, points out the various applications of which it is capable. He predicts the phenomena afterwards observed by Feddersen; in fact, he suggests the use of Wheatstone's mirror to detect it. Its bearing on the anomalous magnetization of steel needles by jar discharges, and on the anomalous evolution of gas by statical discharges, when electrodes of small surface are used (in Wollaston's manner), are also dwelt upon.

Experiments of Feddersen, &c.

Several physicists have taken up the experimental investigation of this matter. Feddersen's experiments realize the case above discussed, if we abstract the disturbance owing to the air interval, of the effect of which it is not easy to give an accurate account. Feddersen's results are in good general agreement with theory. He finds, for instance, that the critical resistance at which the discharge begins to assume the oscillatory character varies inversely as the square root of the capacity of the battery from which the discharge is taken. A good account of the researches of Paalzow,[1] Bernstein,[2] and Blaserna, and of the older researches of Helmholtz,[3] remarkable for the use of his pendulum interruptor, will be found in Wiedemann, §§ 801, &c. Schiller, in a very interesting paper,[4] describes a variety of measurements of the period of oscillation, and the damping of the alternating currents in a secondary coil, when the current of the primary is broken. By means of the pendulum interruptor of Helmholtz (for description of which see his paper) the primary is broken, and at a measured interval thereafter the secondary circuit, which contains a condenser and a Thomson's electrometer, is also broken. The deflection of the electrometer indicates the charge of the condenser at the instant when the secondary is broken. The interval between two null points separated by a whole number of oscillations can thus be found, and hence the time of oscillation of the coil calculated. The agreement of Schiller's results with calculation is very remarkable, and must be regarded as a highly satisfactory proof of the validity of the theoretical principles involved.

Induction in masses of metal.

Induction in Masses of Metal and Magnetism of Rotation.—Hitherto we have dealt only with linear circuits; but it is obvious that currents will also be induced in a mass of metal present in a varying magnetic field. If the variation of the field be due to relative motion between the mass of metal and the system to which the field is due, the electromagnetic action of the induced currents will oppose the motion. Many instances might be given of this principle. If a magnet be suspended over a copper disc, or, better still, in a small cavity inside a mass of copper, its vibrations are opposed by a force due to the induced currents which for small motions varies as the angular velocity of the needle. Accordingly, it comes much sooner to rest than it would do if suspended in the air at a distance from conducting masses; it moves beside the copper as if it were immersed in a viscous fluid. Plücker suspended a cube of copper between the poles of a powerful electromagnet, and set it spinning about a vertical axis; directly the magnet was excited it stopped dead. Foucault arranged a flat copper disc between the flat poles of an electromagnet placed at such a distance apart as just to admit it between them. The disc was set in rotation by means of a driving gear. So long as the magnet was not excited, the driver had comparatively little work to do; but as soon as the magnet was excited, the work required to keep up any considerable velocity greatly increased. The additional work thus expended appears in the heat developed in the disc by the induced currents. Tyndall demonstrates this very neatly by causing a small cylindrical vessel of thin copper filled with fusible metal to rotate between the poles of an electromagnet, when enough heat is quickly developed to melt the metal.

On the other hand, when a mass of metal moves in the neighbourhood of a magnet, the electromagnetic action of the induced currents will cause the magnet to move, if it be free to do so. Thus, if we suspend a magnet with its axis horizontal over a disc which can be set in rotation about a vertical axis, owing to the electromagnetic action of the induced current, the needle will tend to rotate in the same direction as the disc. If the velocity be great enough, or the needle be rendered astatic, it may be carried round and round continuously. This action was discovered by Arago, and excited the attention of many philosophers, till it was at last explained by Faraday (see Historical Sketch). Many of the observations made by Faraday's predecessors, and some made by himself, are at once seen to support the conclusion that the phenomenon is simply a case of Lenz's law. Thus Snow Harris found that the deflecting couple on a suspended needle varied approximately as the velocity of the disc directly, and as the square of the distance of the needle from the disc inversely. It was also found that the action of the disc was directly proportional to the conductivity of the metal of which it was made, an exception occurring in the case of iron, whose action was disproportionately great. Cutting radial slits in the disc diminished the action very much.

Faraday's explanation.

Besides the component tangential to the disc, it is found that there is a repulsive normal action on the pole of the magnet, and also a radial action, which may be towards or from the centre of the disc, according as the pole is nearer or farther from the centre of the disc. These actions look at first sight somewhat more difficult to explain; but a little consideration will show that the laws of induction account for these also.

Let us first suppose the induced currents to appear and die away instantly after the small motion of the disc which produces them (we may suppose the motion of the disc to take place by an infinite number of small jumps). Thus the currents of induction are obviously symmetrical with respect to the diameter through the foot of the perpendicular from the magnetic pole on the disc, and we may roughly represent the electromagnetic action by a magnet placed perpendicular to the diameter at a certain distance from the centre of the disc, its south pole pointing in the direction of the disc's motion if the inducing pole be a north pole. Let OX (fig. 52) be the direction of the diameter in the same vertical plane as the pole, NS the representative magnet, OY being the direction of motion. By our present supposition the inducing pole M lies in the plane of ZOX, in which case it is obvious that the resultant action reduces to a tangential component T parallel to OY.

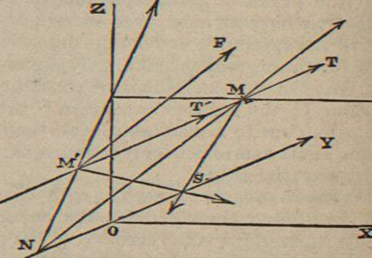

Fig. 52.

But, owing to the inductive action on each other of the currents in the disc, the induced currents do not rise and fall instantaneously, but endure for a sensible time. We may roughly represent the effect of this by supposing the representative magnet NS carried onwards a little with the disc, or, which amounts to the same thing, we may suppose the pole M to lag a little behind at M′ (lying, say, on MM′ perpendicular to ZOY.) The action of N will now preponderate, and the resultant force on M′ will be in the direction M′F. This force, when resolved parallel to OY, OZ, OX, gives a tangential component as before, a repulsive normal component, and a radial component, which will be directed to or from the centre of the disc, according as the representative magnet lies farther from or nearer to the centre of the disc than the foot of the perpendicular from M′.

[1] *Pogg. Ann.*, 1861.
[2] *Pogg. Ann.*, 1871.
[3] *Monatsber. der Berl. Akad.*, 1874.
[4] *Pogg. Ann.*, 1874.

The original explanation of rotation magnetism (Faraday, *Exp. Res.*, 81, &c.) should be consulted by the reader who wishes to pursue the subject. An account of the researches of Nobili, Matteucci, and others will be found in Wiedemann, Bd. ii. § 860, &c. The mathematical theory has been treated by Jochmann, who neglected the inductive action of the currents on each other (*Crelle's Journ.*, 1864; *Pogg. Ann.*, 1864; also Wiedemann, *l.c.*). A complete theory of the induction of currents in a plane conducting sheet has been arrived at by Maxwell by means of an extremely elegant application of the method of images (*Proc. R.S.*, 1872; also *Electricity and Magnetism*, vol. ii. §§ 668, 669).

On the Origin of Electromotive Force.

It remains for us now to view the transformations of energy which take place in the voltaic circuit from the other side, and to inquire whence comes the energy that is evolved in so many different forms by the electric current. Two distinct questions are here involved. First—What form of energy is being absorbed, and at what part of the circuit does the absorption take place? Secondly—Where is the electromotive force which drives the current situated?

Conservation of energy.

To the first of these questions experiment has given, on the whole, a very satisfactory answer. The electric circuit is, indeed, one of the best instances of the great law of conservation, which states that the appearance of energy anywhere is always accompanied by the disappearance somewhere of energy to an equal amount. No general discussion of this first question is necessary; it will be sufficient to indicate the application of the general principle when we deal with particular instances.

Unfortunately the answers, both experimental and theoretical, that have been at different times given to our second question, are not so concordant as could be desired. The reader is, therefore, cautioned against accepting without due examination[1] anything that may be here advanced.

Contact force.

Perhaps the most general principle concerning the origin of electromotive force recognized by physicists of the present day is the following:—

When two different substances are in contact, there exists in general an electromotive force at the surface of separation, tending to displace electricity across that surface.

This electromotive force is commonly called the "electromotive force of contact," or simply the "contact force." In the particular case of two conductors in contact, the effect of this force would simply be to maintain a certain difference of potential between them.

Although the earliest known case of electrification—viz. amber rubbed with woollen cloth—is an instance in point, and although many experiments on electrification by the friction of different substances were made, yet this principle was not recognized fully till the experiments of Galvani and Volta directed the attention of men of science to the matter.

Contact of metals. Volta's experiments.

Volta was the first to demonstrate clearly the existence of the contact force in the case of metals. A simplified form of his fundamental experiment is the following. The upper and lower plates of a condensing electroscope (see above, p. 34) are made of different metals, say copper and zinc. Let the upper plate be laid upon the lower, and the metallic contact ensured by connecting them for an instant by means of a wire. If the upper plate be now lifted vertically upwards, the gold leaves of the electroscope diverge, indicating that the zinc plate is now positively electrified to a considerable potential. This is explained as being due to the contact force at the junction of the copper wire with the zinc plate, by virtue of which the zinc is at a higher potential than the copper. Suppose the upper plate to be connected with the earth, then if E be the contact force, the potential of the zinc plate is E. Now E is very small, but as soon as the upper plate is raised the potential of the lower plate is increased in the same ratio as its capacity is diminished; hence the divergence of the leaves. Volta found that he could arrange the metals in series, thus—

Zn...........	0	Fe...........	1
Pb...........	5	Cu...........	2
Sn...........	1	Ag...........	3

such that, when any metal is placed in contact with one below it in the series, it takes a higher potential; and he found that the electromotive force between any two metals in the series is the sum of the electromotive forces between every adjacent intervening pair. Thus, if Zn|Pb denote the electromotive force from lead to zinc, we get from the above table,

$$\begin{aligned}&Zn|Pb=5,\ Pb|Sn=1,\\&Zn|Sn=Zn|Pb+Pb|Sn=6,\\&Pb|Cu=Pb|Sn+Sn|Fe+Fe|Cu=6,\end{aligned}$$

and so on.

It follows from Volta's law that, if a number of metals be connected up in series, the difference of potentials between the extreme metals is independent of the intermediate metals, and, in particular, is zero if the extreme metals be the same. We cannot, therefore, have a resultant electromotive force in a closed circuit consisting of metals merely. This is entirely in accordance with experiment, provided the temperature be the same everywhere.

Experiments by Kohlrausch.

While one party of physicists neglected or attempted to explain away Volta's contact force, another took up the investigation, and endeavoured to obtain precise measurements of it in different cases. Careful experiments of this kind were made by Kohlrausch[2] and Gerland,[3] by a method due to the former.

A condenser is used whose plates are made of the metals to be tested, say zinc and platinum. The plates are first placed parallel to each other at a very small distance apart, and touched simultaneously with a wire (say of platinum). A difference of potentials is thereby established, so that if the potential of the Pt be zero that of the Zn is Zn|Pt. (Here we neglect the contact force between air and zinc and between air and platinum. No experimental proof that we know of has been given in support of this, see below, p. 85). In consequence of this difference of potentials the Zn plate becomes positively charged. The wire is now removed, the plates of the condenser separated to a considerable distance, and the Zn plate connected with one electrode of a Dellmann's electrometer, the other electrode of which is connected to earth. The reading is proportional to the potential difference Zn|Pt increased in the ratio in which the capacity of the Zn plate has been decreased by the separation. Hence, if A be the reading,

$$Zn|Pt=\lambda A \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

The condenser plates are now brought into their former position, and connected through a Daniell's cell, consisting

[1] This applies particularly to any indications of the views of living physicists.

[2] *Pogg. Ann.*, 1853.
[3] *Pogg. Ann.*, 1868.

of a strip of zinc immersed in a porous vessel filled with zinc sulphate, which is itself immersed in a vessel containing copper sulphate, into which dips a strip of copper. In the first instance, the copper strip is connected with the zinc plate, and the zinc strip with the copper plate of the condenser. The difference between the potentials of the condenser plates is easily found by an application of Volta's law[1] to be $D + Zn|Pt$, where D denotes the difference between the potentials of the two pieces of *copper* forming the terminals of a Daniell's cell; hence if B be the electrometer reading, after removing the Daniell and separating the plates as before, we have

$$D + Zn|Pt = \lambda B \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

If we connect up the Daniell the opposite way with the condenser, then we get a reading C, such that

$$D - Zn|Pt = \lambda C \quad . \quad . \quad . \quad . \quad . \quad (3).$$

From (2) and (3) we get

$$Zn|Pt = \frac{B-C}{B+C}D \quad . \quad . \quad . \quad . \quad . \quad (4),$$

which gives the contact force Zn|Pt in terms of the electromotive force of a Daniell. From (1), (2), (3) we get

$$B - C = 2A,$$

an identical relation which the observations ought to satisfy, and which, therefore, affords the means of testing their accuracy.

In this way Kohlrausch found for Zn|Cu the value ·48D, or in other words, that the contact force from copper to zinc is about equal to half the electromotive force of a Daniell's cell. As an instance of the general nature of the results, we give two series of numbers from the observations of Kohlrausch. The contact force is between zinc and the metal mentioned in first column in each case, and Zn Cu is taken = 100.

Cu	100	100
Au	112	115
Ag	105	109
Pt	106	123
Fe	75	—

In the second set of experiments the metals were carefully cleaned, whereas in the first set they may have been a little oxidized. This may very well account for the differences, for Kohlrausch found oxidized zinc strongly negative[2] to freshly cleaned zinc. In fact, he found Zn|ZnO = about ·4Zn|Cu.

In order to test Volta's law, a further series of observations was made, giving the contact force between iron and several metals. The following table gives the results observed directly and calculated from the table last given:—

	Observed.	Calculated.
Cu	31·9	25·3
Pt	32·3	32·3
Au	39·7	38·0
Ag	29·8	30·9

It will be seen that, with the exception of the values for Fe Cu, the agreement is very fair.

It is not necessary to give here the results of Gerland and Hankel.[3] The latter made a great number of very careful experiments. He showed that the results depend on the nature of the surface of the bodies, being different when the surface is filed and when it is polished with rouge or other powder. His tables also show the gradual change effected in the contact force when the plates are exposed to the action of the air.

According to Volta, the contact forces between metals and liquids are either very small, and do not follow the same law as the contact forces between metals, or else are absolutely non-existent. Subsequent observers, however, demonstrated the existence of contact forces in this case also, but showed that they do not obey Volta's law like the contact forces in the case of metals.

Becquerel[4] placed the fluid to be examined in a capsule of the metal, say copper. The capsule was placed on the upper plate of a condenser consisting of two copper plates in connection with a gold-leaf electroscope. The fluid and the lower plate of the condenser were touched each with a finger for a short time, and then the upper plate was removed. The divergence of the gold leaves was taken to indicate the contact force. In this way Becquerel found that zinc, copper, and platinum were mostly negative to alkaline solutions; but the metals were in general positive to concentrated sulphuric acid. It is obvious, however, that the result of the experiment is complicated by the contact of the hand with the liquid and with the metal of the condenser.

Similar objections apply to the results of Pfaff[5] and Peclet.[6]

Buff[7] made the lower plate of his condenser of the metal to be examined, of zinc for example; upon this was laid a thin glass plate on which was spread a thin layer of the liquid to be examined, or a piece of filter paper soaked with it. A zinc wire was used to bring the liquid and the lower plate of the condenser into communication; this wire was then removed and the glass plate with the liquid lifted. The divergence of the leaves was taken to indicate the contact force between zinc and the liquid. Although this method is an improvement on the methods of Becquerel and Peclet, it is still unsatisfactory, owing to the presence of the glass.

The most extensive and at the same time most careful experiments at present on record are those of Hankel.[8] Hankel's method.

The fluid (L) to be examined was placed in a wide-mouthed funnel. The condenser was formed by the surface of the liquid and a copper plate, which could be placed parallel to it at a very short distance apart, and raised as usual. The stem of the funnel was bent at a right angle twice, and ended in a wider portion, into which dipped a strip of the metal (M) to be examined. M was connected for a moment by a platinum wire with the copper plate and also with the earth. The wire was then removed, the plate was raised, and its potential determined by means of Hankel's dry pile electroscope. The reading (A) is proportional to $Cu|Pt + Pt|M + M|L$, or, by Volta's law, to $Cu|M + M|L$. Hence

$$Cu|M + M|L = \lambda A.$$

In the next place, the funnel is emptied and a plate of the metal M placed on its mouth. The copper plate is lowered so that it is at the same distance as before, contact established by means of the platinum wire, and so on. The reading being B, we have

$$Cu|M = \lambda B.$$

The plate of M is replaced by a plate of zinc, and the experiment repeated, and we have, C being the third reading,

$$Cu|Zn = \lambda C.$$

[1] The truth of which, therefore, is assumed. The assumption of course is justified *a posteriori*.

[2] A metal is said to be negative to another when it assumes the lower potential in contact, and positive when it assumes the higher potential.

[3] *Abh. der Königl. Sächs. Gesellschaft*, 1861. 1865.

[4] *Ann. de Chim. et de Phys.*, 1824.

[5] *Pogg. Ann.*, 1840.

[6] *Ann. de Chim. et de Phys.*, 1841.

[7] *Ann. der Chem. u. Pharm.*, 1842.

[8] *Abh. der. Königl. Sächs. Gesellschaft*, 1865

From these three results we get

$$M|L = \frac{A-B}{C} Cu|Zn.$$

It is not necessary to quote Hankel's results here. The reader may refer to Wiedemann's *Galvanismus*, or to Hankel's paper.

Sir William Thomson has given a new proof of the existence of Volta's contact force as follows.[1] A ring is formed, one-half of which is copper the other half zinc. This ring is placed horizontally, and a needle made of thin sheet metal is so balanced as to form a radius of the ring. If when the needle is unelectrified it be adjusted so as to be over the junction of the two metals, then, when it is positively electrified, it will deviate towards the copper, and when negatively electrified, towards the zinc. Again, if a whole, instead of a half needle as above, be suspended over a disc made of alternate quadrants of zinc and copper, or, better still, inside a flat cylindrical box constructed in a similar way, so that when the needle is unelectrified its axis coincides with one of the diameters in which the disc is divided, then when the needle is positively electrified it will take up a position such that its axis bisects the copper quadrants; if it be negatively electrified, its axis will bisect the zinc quadrants.

Thomson has also given an elegant demonstration of the contact force between copper and zinc by means of an apparatus which is a modification of his water-dropping apparatus.[2] A copper funnel is placed in a cylinder of zinc, and drops copper filings at a point near the centre of the cylinder. The filings are charged negatively by induction as they fall, owing to the excess of the potential of the zinc cylinder over that of the copper. If therefore the filings be caught in an insulated metal can, they will communicate to it a continually increasing negative charge, while the zinc cylinder and the copper funnel will become charged more and more positively.

Thomson finds, in agreement with Kohlrausch, that, when the copper and zinc are bright, the electromotive force of contact is about half that of a Daniell's cell. When the copper is oxidized by heating in air, the contact force is equal to a Daniell or more. He has gone a step farther, and shown that when two bright pieces of copper and zinc are connected by a drop of distilled water, their potentials are as nearly as can be observed the same.[3]

The subject of contact electricity has been taken up quite recently by Clifton.[4] He experiments on the contact-force between a metal and a liquid by a method which is a simplification of Hankel's.

Two horizontal plates are used of the metal M; the liquid L is placed in a glass vessel on the lower plate and connected with the lower plate by a strip of the same metal which dips into it. The upper plate is lowered to a distance of 0·1 or 0·2 mm. from the surface of the liquid, which acts as the lower surface of the condenser, and the upper plate and lower plate are connected by a copper wire. The difference of potential between the two surfaces of the condenser is therefore M|L. The copper wire is then removed, the upper plate raised, and its potential measured with a Thomson's electrometer. In this way a contact force equal to the thousandth part of Zn|Cu can be detected.

Clifton finds zinc and copper to be both positive to water to about the same degree, and both very slightly negative to dilute sulphuric acid. He concludes therefore that zinc and copper dipping in water will be at the same potential. This he verifies directly, finding that any difference of potential, if it exist, must be less than ·00079 of the electromotive force of a Daniell. The result of Sir William Thomson is therefore confirmed.

There are many other points of interest in Clifton's paper, but, as the results are given in most instances as preliminary, we need not discuss the matter farther.

Before leaving this subject, it may be well to notice that there is one point which is not touched by all these experiments, viz., the question whether there is or is not a contact force between metals or even liquids and air. It has not yet been shown that the results of the experiments which are supposed to demonstrate that Zn|Cu is about half the electromotive force of a Daniell could not be equally well explained by supposing the difference of potential to be[5] $Cu|A + A|Zn + Cu|Zn$, whence Cu|Zn is very small compared with Cu|A and A|Zn. This supposition would not invalidate Volta's law; nor would it contradict Clifton's results, for we have, in accordance with his experiments, on the new hypothesis,

$$Aq|A + Cu|Aq + Aq|Cu = Aq|A + Zn|Aq + A|Zn,$$

therefore, transposing,

$$Zn|Aq + Aq|Cu + Cu|A + A|Zn = 0,$$

which, according to the new hypothesis, means that copper and zinc immersed in water are at the same potential. In this view, the important part of the contact force usually observed between zinc and copper would be $Cu|A + A|Zn$,[6] which must therefore, by Sir Wm. Thomson's result, be the same as $Cu|Aq + Aq|Zn$.

It is not very easy to see how this point is to be settled by direct measurements of electromotive force. Supposing, however, that it were settled, and that the contact force between two given metals A and B, and between each of them and a given liquid L, were known, then the difference of potentials between the two metals when immersed in any liquid could be predicted in all cases, and also the initial electromotive force tending to send a current through a circuit made by connecting the metals together and dipping them into the liquid.

A number of cases of this kind have been investigated by Gerland;[7] but satisfactory agreement between theory and observation has been attained in but a few cases. Researches of this kind are beset with a double array of almost insuperable difficulties. Gerland's results.

The direction of the initial resultant electromotive force in any circuit of two metals and one fluid may be found by observing the first swing of a galvanometer through which the circuit is suddenly connected. Considerable precautions must be taken to obtain consistent results, and when all care has been taken, it is not found that the results of one observer always agree with those of another. This is scarcely to be wondered at, when we consider the difficulty of making sure that in two different experiments we are operating with absolutely the same metals and fluid in absolutely the same conditions as to surface.

When the current tends to pass from one metal to another through the liquid in which they are immersed, the former metal is said to be positive to the latter. If the whole electromotive force in the circuit be the sum of all the contact forces at the various junctions, then it follows easily that we ought to be able to arrange the metals in a series, such that any one in the series is positive to any following one and negative to any preceding when both are dipped in the same liquid. It does not follow that the order of the series is the same for different liquids; this would be so if the metallic contacts alone were operative.

Many electromotive series of this kind have been given by different experimenters; but they have lost much of their

[1] *Proc. Lit. and Phil. Soc. of Manchester*, 1862, or *Reprint*, p. 319.

[2] *Reprint*, p. 324.

[3] Jenkin, *Electr. and Mag.*, § 22.

[4] *Proc. R. S.*, June 1877.

[5] A stands for air.

[6] See Maxwell's *Electricity*, vol. i. § 249.

[7] See Wiedemann, Bd. i. § 36.