the Eucharist is not only a commemoration of a past event, but also the pledge and seal of something then actually present. As bread and wine sustain our earthly body, so the body and blood of Christ nourish and refresh our spiritual nature (Hagenbach, *u.s.*, p. 302). With regard to the participation of unbelievers, the Helvetic Confession lays down definitely that they who approach the Lord's Table without faith partake of the sacrament alone, but have no share in the "res sacramenti" which is the source of life and salvation (*Corpus Confession.*, p. 73).

The doctrine of the Church of England, as set forth in her 28th Article, is that "the supper of the Lord is not only a sign of the love that Christians ought to have among themselves one to another, but rather is a sacrament of our redemption by Christ's death, insomuch that, to such as rightly, worthily, and with faith receive the same, the bread which we break is a partaking of the body of Christ, and likewise the cup of blessing is a partaking of the blood of Christ. . . . . The body of Christ is given, taken, and eaten in the supper only after an heavenly and spiritual manner, and the means whereby the body of Christ is received and eaten in the supper is faith." The teaching of the Catechism is to the same effect, viz., that the sacrament of the Lord's supper was ordained "for the continual remembrance of the sacrifice of the death of Christ, and of the benefits which we receive thereby." It teaches also that "the body and blood of Christ are verily and indeed taken and received by the faithful," to "the strengthening and refreshing of our souls by the body and blood of Christ as our bodies are by the bread and wine."

The doctrine of the Presbyterian Churches of Scotland, as declared in the *Confession of Faith*, agreed upon by the Assembly of Divines at Westminster, and approved by the General Assembly in 1647, and established by Acts of Parliament in 1649 and 1690, as "the publick and avowed confession of the Church of Scotland," is that the Lord's supper was instituted by Christ, to be observed to the end of the world "for the perpetual remembrance of the sacrifice of Himself in His death; the sealing all benefits thereof to true believers; their spiritual nourishment and growth in Him; their further engagement in and to all duties which they owe to Him; and to be a bond and pledge of their communion with Him, and with each other as members of His mystical body. In the sacrament Christ is not offered up to His Father, nor any real sacrifice made at all for remission of sins of the quick or dead, but only a commemoration of that one offering up of Himself, by Himself upon the cross, once for all, and a spiritual oblation of all possible praise unto God for the same. . . . . The outward elements in this sacrament, duly set apart to the uses ordained by Christ, have such relation to Him crucified as that truly, yet sacramentally only, they are sometimes called by the name of the things they represent,—to wit, the body and blood of Christ,—although in substance and nature they still remain truly and only bread and wine. Worthy receivers outwardly partaking of the visible elements in this sacrament do then also inwardly by faith, really and indeed, yet not carnally and corporally, but spiritually, receive and feed upon Christ crucified and all benefits of His death, the body and blood of Christ being then not corporally or carnally in, with, or under the bread and wine, yet as really, though spiritually, present to the faith of believers in that ordinance as the elements themselves are to their outward senses" (chapters xxix. §§ 1, 2, 5, 7).

Authorities.—Hagenbach, *History of Doctrines*, vol. ii.; Scudamore, *Notitia Eucharistica*; Hooker, *Eccles. Polity*, bk. v.; Barrow, *Doctrine of the Sacraments*; Jeremy Taylor, *Real Presence of Christ*; Waterland *On the Eucharist*; Wilberforce, *Doctrine of the Eucharist*; Calvin, *Institutio*, lib. iv.; *Confessionum Fidei diversarum Ecclesiarum Corpus*; *Concilii Tridentini Decreta*; *Catechismus ad Parochos*; Kimmel, *Monum. Fidei Eccl. Orient.* (E. V.)

EUCHRE, a game at cards, much played in America. Euchre is said to be a corruption of the word écarté; the game is believed to have been first played by the French settlers in Louisiana, but at what date is uncertain. Euchre is played with thirty-two cards, the twos, threes, fours, fives, and sixes being rejected from a complete pack. The players cut for deal, and the lowest deals. The non-dealer then cuts to his opponent, who deals five cards to each, by two at a time and three at a time, or *vice versa*. The dealer turns up the top of the undealt cards for trumps. In suits not trumps the cards rank as at whist; in the trump suit the knave (called the *right bower*) is the highest trump, and the other knave of the same colour, black or red, as the case may be (called the *left bower*), is the next highest, this card being, of course, omitted from the suit to which it would otherwise belong. The other trumps rank as already stated, the queen being next above the ten.

Two-handed Euchre.—The non-dealer looks at his hand and decides whether he will play it. If content, *i.e.*, if he thinks he can win three tricks, he says "order it up." The dealer then puts out from his hand any card he pleases, face downwards, and is entitled to take the trump card into his hand; but the card is generally left on the pack until wanted in the course of play. If the non-dealer is not content, he says "pass." The dealer then has the option of taking up the trump as before, or of passing also. If the trump is *ordered up* or *taken up* the play of the hand commences; if both pass, the dealer places the trump card face upwards under the pack, called *turning it down*. The non-dealer has then the option of *making it*, *i.e.*, of naming any suit, except the one turned up, saying, "make it spades," or any suit he prefers, and that suit becomes trumps, or of passing again, saying, "pass again." If he makes it, the play begins; if he passes again, the dealer has similarly the option of making it. If both pass a second time the hand is thrown up, and the other player deals. When the turn up is red and the trump is made red it is called *making it next*; the same if black is made black. If the trump is made of a different colour from the turn up, it is called *crossing the suit*. If the hand is played, the non-dealer leads; the dealer plays to the card led. He must follow suit if able, otherwise he may play any card he pleases. If the left bower is led a trump must be played to it. The highest card of the suit led wins the trick; trumps win other suits. The winner of the trick leads to the next. If the player who orders up, takes up, or makes the trump, wins five tricks, he scores two, called a *march*; if he makes three or four tricks he scores one, called the *point*. If he fails to make three tricks he is *euchred*, and his opponent scores two. The game is five up. By agreement, a player who makes more than five may carry the surplus (called a *lap*) to the next game. Also it is sometimes agreed that a love game (or *lurch*) shall count double. The game may be reckoned without reference to the adverse score; or it may be played with points, that is, the winner receives from the loser as many points as he wants of game.

Three-handed or Cut-throat Euchre.—The option of playing or passing goes to each in rotation, beginning with the player to the dealer's left. Three cards, one from each hand, constitute a trick. The player who orders up, takes up, or makes the trump plays against the other two except at *independent euchre*, when each plays for himself. If the attacking player is euchred, he is *set back* two points. Thus if he is love, and is euchred, he has seven points to make instead of five.

Four-handed Euchre is generally played with partners, who are cut for and sit opposite each other as at whist. If the first hand passes, the second may say "I assist," which means that the dealer (his partner) is to take up the trump.

The hand is then played as at whist, four cards constituting a trick. The eldest hand has the next deal. If a player has a very strong hand he may *play alone* single-handed against the two adversaries. His partner cannot object. A player can declare to play alone when he or his partner orders up, or when his partner assists, or when he makes the trump, or (if dealer) when he takes up the trump, but not when the adversary orders up, assists, or makes the trump. If the lone player wins a march he scores four, if he wins three or four tricks he scores one; if he fails to win three tricks the opponents score two.

Hints.—1. The chances are that the dealer has one trump in hand; if you order up, you must expect to meet two trumps. Therefore, you should not order up unless your hand gives you a two to one chance of winning three tricks against two trumps, and your cards are such that you would have a worse chance if you made the trump. If strong in trumps and equally strong in another suit, it is always right to pass. Also, if you have the point certain, whether you make the trump or not, you should pass, in hopes the dealer may take up the trump.

2. If you pass and the dealer turns it down, you should not make the trump unless you have a two to one chance of winning three tricks against one trump.

3. If you hold good cards in two suits of different colours, and you make the trump, you should make it next. For, the dealer having turned it down in one colour, is less likely to hold a bower of that colour than of the other. At the four-handed game the non dealer and his partner should also avoid crossing the suit. But if the dealer's partner makes the trump, he should not hesitate to cross the suit, as the dealer, having turned it down, has probably no bower in that suit.

4. At four-handed euchre, the eldest hand should be very strong to order it up; but the second player should assist if he has something more than one trick, *e.g.*, an ace and a trump, or two aces. If, however, he is strong in the non-trump suits, he should not assist unless he can be pretty sure of making two tricks. The third hand should be cautious of ordering up, as his partner, having passed, must be weak. This applies with still more force to taking up by the dealer, as his partner, not having assisted, must be very weak. To take up the dealer should be pretty sure of two tricks, and have a chance of a third.

5. If the dealer takes up the trump he should keep two cards of a suit, unless his single card is an ace. Thus, with queen, seven of one suit and king single of another, the king should be discarded.

6. Lead from a guarded suit unless in fear of losing a march, when lead your highest single card. Lead from a sequence of three trumps. At four-handed euchre always lead a trump with three. Also lead a trump if you have made it next; if your left hand adversary has assisted (unless a bower is turned up); and if your partner orders up, assists, takes up, or makes the trump. Further, lead a trump if you have lost two tricks and won the third, unless your partner has dealt and still has the turn up in hand.

7. As a rule make tricks when able. Passing or finessing is seldom good play.

8. If your partner orders up, assists, takes up, or makes the trump, trump the trick whenever you can.

9. In discarding during the play, as a rule, keep a guarded card in preference to a single one, except a single ace.

10. If the adversary is at three do not order up unless you have very good cards. If the adversary is at four take up the trump on a light hand.

11. At four-handed euchre, if the dealer is one or two, and the eldest hand four, he should order up, unless he has one certain trick, in order to prevent the opponent from playing alone. This position is called *the bridge*.

12. At four-all, if the eldest hand or third hand has a trick and the chance of a second, and such cards that he would be no better off if he made the trump, he should order it up.

13. The eldest hand, and next to him the dealer, may play alone on weaker hands than the other players. The leader, with a lone hand, should lead his winning trumps; if two tricks are thus made, and the leader has a losing trump, he should then lead his best card out of trumps. When playing against a lone hand, lead an ace. If you have not one, lead your highest card out of trumps, except with a guarded king and another suit, when lead the latter. Also, keep cards of the suits your partner discards, but do not throw an ace, even if your partner keeps your ace suit.

Laws of Euchre.—*Dealing.*—1. If the dealer gives too many or too few cards to any player, or if he turns up two cards, it is a misdeal, and the next player deals. 2. If the dealer exposes a card, or if there is a faced card in the pack, there must be a fresh deal. *Playing.*—3. Any one playing with the wrong number of cards can score nothing that hand. The same if, when the trump is ordered up, the dealer omits to discard before he or his partner plays. 4. When more than two play, exposed cards can be called. Also a card led out of turn may be called, or a suit from the side offending at their next lead. 5. A player not following suit when able may correct his mistake before the trick is turned and quitted or he or his partner plays to the next trick, the card played in error being an exposed card. If the error is not corrected a revoke is established. A player revoking is euchred, and cannot score anything that hand. 6. A player making the trump must abide by the suit first named. 7. If, after the trump is turned, a player reminds his partner that they are at the point of the bridge, the latter loses the right to order up. 8. Each player has a right to see the last trick. (H. J.)

EUCLID. Of the lives of the Greek mathematicians generally very little is known, and among the number Euclid is no exception; we are ignorant not only of the dates of his birth and death, but also of his parentage, his teachers, and the residence of his early years. In some of the editions of his works, as will be seen, he is called *Megarensis*, as if he had been born at Megara in Greece, a mistake which arose from confounding him with another Euclid, a disciple of Socrates. Proclus, the Neo-platonist (412–485 A.D.), is the authority for most of our information regarding Euclid, which is contained in his commentary on the first book of the *Elements*. He there states that Euclid lived in the time of Ptolemy I., king of Egypt, who reigned from 323 to 285 B.C., that he was younger than the associates of Plato, but older than Eratosthenes (276–196 B.C.) and Archimedes (287–212 B.C.) Euclid is said to have founded the mathematical school of Alexandria, which was at that time becoming a centre, not only of commerce, but of learning and research, and for this service to the cause of exact science he would have deserved commemoration, even if his writings had not secured him a worthier title to fame. Proclus preserves a reply made by Euclid to King Ptolemy, who asked whether he could not learn geometry more easily than by studying the *Elements*—"There is no royal road to geometry." Pappus of Alexandria, whose date is rather uncertain, but is probably a century earlier than that of Proclus, says that Euclid was a man of mild and inoffensive temperament, unpretending, and kind to all genuine students of mathematics. This being all that is known of the life and character of Euclid, it only remains therefore to speak of his works.

Among those which have come down to us the most remarkable is the *Elements* (Στοιχεῖα). They consist of thirteen books; two more are frequently added, but there is reason to believe that they are the work of a later mathematician, Hypsicles of Alexandria. At the outset of the first book occur the definitions or explanations of the meanings of the terms employed; the postulates, which limit the instruments to be used in the constructions to the ruler and the compasses; and the axioms or common notions, the fundamental principles from which mathematical truths are deduced. The propositions, which consist of both theorems and problems, deal with rectilineal figures, principally the triangle and the parallelogram, and the book concludes with the celebrated Pythagorean theorem and its converse. The second book is occupied with the consideration of the rectangular parallelograms contained by the segments of straight lines, and their relation to certain squares. It contains only two problems, the one to divide a straight line in medial section ("the divine section," as it was afterwards called), and the other which shows how to effect the quadrature of any rectilineal area. The third book, prefaced with a few definitions, discusses the properties of circles. The fourth book contains no theorems. The problems are on the inscription in, and circumscription about, circles of triangles, squares, and certain regular polygons, and on the inscription of circles in, and the circumscription of circles about, some of these figures. Though, in the definitions preliminary to this book, Euclid explains when a rectilineal figure is inscribed in and circumscribed about another rectilineal figure, he has given no proposition showing how in any case such inscription or circumscription may be effected. The equilateral triangle, the square, the regular pentagon, and such regular polygons as can be derived from these, were the only regular figures known to be inscriptible in a circle by means of elementary geometry, till Gauss discovered, in 1796, that the circumference of a circle could be divided into 17 equal parts. In his *Disquisitiones Arithmeticæ*, published in 1801, it is proved that there can be inscribed in a circle any regular polygon, the number of whose sides is prime, and is denoted by 2^n+1. Euclid's second book presupposes, that is, depends to some extent upon, the first; the third presupposes both the first and second; the fourth presupposes the first three; and all four are largely concerned with the discussion of the absolute equality or inequality of certain magnitudes. The fifth book stands alone, depending upon none of the preceding books, and contains the theory of proportion, with respect not merely to geometrical magnitudes, such as lines, angles, areas, solids, but to any magnitudes of which multiples can be formed. The diagrams consist of straight lines, probably for convenience of construction, but the enunciations of the propositions and the reasoning are perfectly general. With the exception of his treatment of parallels, Euclid's doctrine of proportion has been the subject of more discussion than any other part of the *Elements*. The foundation of the doctrine is the criterion of proportionality laid down in the famous fifth definition. The necessity or the appropriateness of such a criterion can be seen only when the distinction between number and magnitude has been clearly apprehended, or, what amounts to the same thing, when an adequate conception has been formed of incommensurables. The ordinary arithmetical test of proportionality will then be found to suit only certain cases which occur—those, namely, where the magnitudes considered are commensurable, and if the theory of proportion is to be rigorous and complete (as Euclid's is), it must be extended to incommensurables by the notions of continuity and limits. The difficulty therefore which is felt by readers of the fifth book in grasping Euclid's doctrine is due mainly to the nature of the subject, and no very material simplification of the full treatment of proportion is possible. The sixth book contains the application of the theory of proportion, mostly to rectilineal figures. In the last proposition, the second part of which is due to Theon, it is noteworthy that the restricted definition of an angle, given in the first book, and adhered to throughout, is tacitly abandoned. The seventh, eighth, and ninth books are arithmetical, that is, treat of the properties of numbers. The definitions relating to them occur at the beginning of the seventh book, and some of these show perhaps the tendency of the Greeks, natural enough to a scientific people with a defective numerical notation, to consider quantity from a geometrical point of view. A number composed of two factors was called a plane number, one composed of three a solid number, and the factors themselves were called sides. The test by which numbers are recognized to be proportionals is different from that given in the fifth book, for here it requires to be applied only to quantities which are commensurable, namely, integers. The last proposition of the ninth book shows how to construct a perfect number, that is, a number which is equal to the sum of all its divisors; for example, $6=1+2+3$, $28=1+2+4+7+14$, &c. The tenth book is the longest of the *Elements*. It is occupied with the consideration of commensurable and incommensurable magnitudes, and ends with the proposition that the diagonal and the side of a square are incommensurable. With regard to straight lines, Euclid distinguishes between those which are commensurable or incommensurable in length, and those which are so in power, the latter being defined to be straight lines the squares on which have or have not a common measure. There are three sets of definitions to this book, the second set inserted before the forty-ninth proposition, and the third before the eighty-sixth. The eleventh, twelfth, and thirteenth books treat mainly of solid geometry. In the eleventh are given the definitions which serve for the three books, the principal properties of straight lines and planes, of solid angles, and of parallelepipeds. The twelfth book begins with two theorems of plane geometry, and then discusses chiefly the properties of pyramids, cones, and cylinders. The last two propositions relate to spheres, the last being to prove that spheres have to one another the triplicate ratio of their diameters. In this book is exemplified the method of Exhaustions, which is founded on the principle that by taking away from a magnitude more than its half, from the remainder more than its half, and so on, a remainder is at length reached which is less than any assignable magnitude (book x. prop. 1). Other applications of this method, the nearest approach made by the ancients to the differential calculus, are to be found in the works of Archimedes (see his *Measurement of the Circle, Conoids and Spheroids, Sphere and Cylinder*). The thirteenth book treats of lines divided in extreme and mean ratio, of some regular figures inscribed in circles, and of the five regular solids, the last proposition being to exhibit the edges of these five solids, and to compare them with one another.

The question has often been mooted, to what extent Euclid, in his *Elements*, is a discoverer or a compiler. To this question no entirely satisfactory answer can be given, for scarcely any of the writings of earlier geometers have come down to our times. We are dependent on Pappus and Proclus for the scanty notices we have of Euclid's predecessors, and of the problems which engaged their attention; for the solution of problems, and not the discovery of theorems, would seem to have been their principal object. From these authors we learn that the property of the right-angled triangle had been found out, the principles of geometrical analysis laid down, the restriction of constructions in plane geometry to the straight line and the circle agreed upon, the doctrine of proportion, as well as loci, plane and solid, and some of the properties of the conic sections investigated, the five regular solids (often called the Platonic bodies) and the relation between the volume of a cone or pyramid and that of its circumscribed cylinder or prism discovered. Elementary works had been written, and the famous problem of the duplication of the cube reduced to the determination of two mean proportionals between two given straight lines. Notwithstanding this amount of discovery, and all that it implied, Euclid must have made a great advance beyond his predecessors (we are told that "he arranged the discoveries of Eudoxus, perfected those of Theætetus, and reduced to invincible demonstration many things that had previously been more loosely proved"), for his *Elements* supplanted all similar treatises, and, as Apollonius received the title of "the great geometer," so Euclid has come down to later ages as "the elementator."

The first six and, less frequently, the eleventh and twelfth books are the only parts of the *Elements* which are now read in the schools or universities of the United Kingdom; and, within recent years, strenuous endeavours have been made by the Association for the Improvement of Geometrical Teaching to supersede even these. On the Continent, Euclid has for many years been abandoned, and his place supplied by numerous treatises, certainly not models of geometrical rigour and arrangement. The fact that for twenty centuries the *Elements*, or parts of them, have held their ground as an introduction to geometry is a proof that they are, at any rate, not unsuitable for such a purpose. They are, speaking generally, not too difficult for novices in the science; the demonstrations are rigorous, ingenious, and often elegant; the mixture of problems and theorems gives perhaps some variety, and makes their study less monotonous; and, if regard be had merely to the metrical properties of space as distinguished from the graphical, hardly any cardinal geometrical truths are omitted. With these excellences are combined a good many defects, some of them inevitable to a system based on a very few axioms and postulates. Thus the arrangement of his propositions seems arbitrary; associated theorems and problems are not grouped together; the classification, in short, is imperfect. That is the main objection to the retention of Euclid as a school-book. Other objections, not to mention minor blemishes, are the prolixity of his style, arising partly from a defective nomenclature, his treatment of parallels depending on an axiom which is not axiomatic, and his sparing use of superposition as a method of proof. A text-book of geometry which shall be free from Euclid's faults, and not contain others of a graver character, and which shall at the same time be better adapted to purposes of elementary instruction, is much to be desired, and remains yet to be written.

Of the thirty-three ancient books subservient to geometrical analysis, Pappus enumerates first the *Data* (Δεδομένα) of Euclid. He says it contained 90 propositions, the scope of which he describes; it now consists of 95. It is not easy to explain this discrepancy, unless we suppose that some of the propositions, as they existed in the time of Pappus, have since been split into two, or that what were once scholia have since been erected into propositions. The object of the *Data* is to show that when certain things—lines, angles, spaces, ratios, &c.—are given by hypothesis, certain other things are given, that is, are determinable. The book, as we are expressly told, and as we may gather from its contents, was intended for the investigation of problems; and it has been conjectured that Euclid must have extended the method of the *Data* to the investigation of theorems. What prompts this conjecture is the similarity between the analysis of a theorem and the method, common enough in the *Elements*, of *reductio ad absurdum*,—the one setting out from the supposition that the theorem is true, the other from the supposition that it is false, thence in both cases deducing a chain of consequences which ends in a conclusion previously known to be true or false.

The *Introduction to Harmony* (Εἰσαγωγη Αρμονική), and the *Section of the Scale* (Κατατομὴ Κανόνος), treat of music. There is good reason for believing that one at any rate, and probably both, of these books are not by Euclid. No mention is made of them by any writer previous to Ptolemy (140 A.D.), or by Ptolemy himself, and in no ancient codex are they ascribed to Euclid.

The *Phænomena* (Φαινόμενα) contains an exposition of the appearances produced by the motion attributed to the celestial sphere. Pappus, in the few remarks prefatory to his sixth book, complains of the faults, both of omission and commission, of writers on astronomy, and cites as an example of the former the second theorem of Euclid's *Phænomena*, whence, and from the interpolation of other proofs, Gregory infers that this treatise is corrupt.

The *Optics* and *Catoptrics* (Ὀπτικά, Κατοπτρικά) are ascribed to Euclid by Proclus, and by Marinus in his preface to the *Data*, but no mention is made of them by Pappus. This latter circumstance, taken in connexion with the fact that two of the propositions in the sixth book of the *Mathematical Collection* prove the same things as three in the *Optics*, is one of the reasons given by Gregory for deeming that work spurious. Several other reasons will be found in Gregory's preface to his edition of Euclid's works.

In some editions of Euclid's works there is given a book on the *Divisions of Superficies*, which consists of a few propositions, showing how a straight line may be drawn to divide in a given ratio triangles, quadrilaterals, and pentagons. This was supposed by John Dee of London, who transcribed or translated it, and entrusted it for publication to his friend Federic Commandine of Urbino, to be the treatise of Euclid referred to by Proclus as τὸ περὶ διαιρέσεων βιβλίον. Dee mentions that, in the copy from which he wrote, the book was ascribed to Machomet of Bagdad, and adduces two or three reasons for thinking it to be Euclid's. This opinion, however, he does not seem to have held very strongly, nor does it appear that it was adopted by Commandine. The book does not exist in Greek.

The fragment, in Latin, *De Levi et Ponderoso*, which is of no value, and was printed at the end of Gregory's edition only in order that nothing might be left out, is mentioned neither by Pappus nor Proclus, and occurs first in Zamberti's edition of 1537. There is no reason for supposing it to be genuine.

The following works attributed to Euclid are not now extant:—

1. Three books on *Porisms* (Περὶ τῶν Πορισμάτων) are mentioned both by Pappus and Proclus, and the former gives an abstract of them, with the lemmas assumed. A porism, according to Pappus, was neither a theorem nor a problem, but something of an intermediate form, which yet could be enunciated as a theorem or as a problem. Later geometers, he says, defined it to be a local theorem wanting part of the hypothesis, but this definition he censures as imperfect. After the publication of Commandine's translation of Pappus (1588), many attempts were made to extract from this unsatisfactory description a clear idea of what a porism was, and, with the help of the lemmas, to restore Euclid's books. The mystery, which baffled the penetration even of Edmund Halley, was not resolved till the time of Simson, who, in 1722, gained some insight into the subject, and whose posthumous treatise *De Porismatibus* appeared in 1776. Simson's views have been objected to by recent French writers, such as M. Paul Breton, and M. Michel Chasles; but for a discussion of the subject recourse must be had to the article PORISM. Here it will be sufficient to state Simson's definition, which is,—"A porism is a proposition in which it is proposed to demonstrate that one or more things are given, between which and every one of innumerable other things, not given but assumed according to a given law, a certain relation, described in the proposition, is to be shown to take place;" and to refer to Simson's *Opera Reliqua*; Playfair's paper *On the Origin and Investigation of Porisms*; Trail's *Life of Dr Simson*; Breton's *Recherches Nouvelles sur les Porismes d'Euclide*, and his *Question des Porismes*; Vincent's *Considérations sur les Porismes*; and Chasles's *Les Trois Livres de Porismes d'Euclide*.

2. Two books are mentioned, named Τόπων προς ἐπιφανείᾳ, which is rendered *Locorum ad Superficiem* by Commandine and subsequent geometers. These books were subservient to the analysis of loci, but the four lemmas which refer to them, and which occur at the end of the seventh book of the *Mathematical Collection*, throw very little light on their contents. Simson's opinion was that they treated of curves of double curvature, and he intended at one time to write a treatise on the subject. (See Trail's *Life*, pp. 60–62, 100–105).

3. Pappus says that Euclid wrote four books on the *Conic Sections* (βιβλία τέσσαρα Κωνικῶν), which Apollonius amplified, and to which he added other four. It is known that, in the time of Euclid, the parabola was considered as the section of a right-angled cone, the ellipse that of an acute-angled cone, the hyperbola that of an obtuse-angled

cone, and that Apollonius was the first who showed that the three sections could be obtained from any cone. There is good ground therefore for supposing that the first four books of Apollonius's *Conics*, which are still extant, resemble Euclid's *Conics* even less than Euclid's *Elements* do those of Eudoxus and Theætetus.

4. A book on *Fallacies* (Περὶ ψευδαρίων) is mentioned by Proclus, who says that Euclid wrote it for the purpose of exercising beginners in the detection of errors in reasoning.

This notice of Euclid would be incomplete without some account of the earliest and the most important editions of his works. Passing over the commentators of the Alexandrian school, the first European translator of any part of Euclid is Boetius (500 A.D.), author of the *De Consolatione Philosophiæ*. His *Euclidis Megarensis Geometriæ libri duo* contain nearly all the definitions of the first three books of the *Elements*, the postulates, and most of the axioms. The enunciations, with diagrams but no proofs, are given of most of the propositions in the first, second, and fourth books, and a few from the third.

Some centuries afterwards, Euclid was translated into Arabic, but the only printed version in that language is the one made of the thirteen books of the *Elements* by Nasir Al-Dín Al-Túsí (13th century), which appeared at Rome in 1594. Judging from the unusual number of diagrams in this edition, the translation of Euclid's text is probably rather free.

The first printed edition of Euclid was a translation of the fifteen books of the *Elements* from the Arabic, made, it is supposed, by Adelard of Bath (12th century), with the comments of Campanus of Novara. It appeared at Venice in 1482, printed by Erhardus Ratdolt, and dedicated to the doge Giovanni Mocenigo. This edition represents Euclid very inadequately; the comments are often foolish, propositions are sometimes omitted, sometimes joined together, useless cases are interpolated, and now and then Euclid's order changed.

The first printed translation from the Greek is that of Bartholomew Zamberti, which appeared at Venice in 1505. Its contents will be seen from the title: *Euclidis megarēsis philosophi platonici Mathematicaruȝ disciplinarū Janitoris: Habent in hoc volumine quicūqȝ ad mathematicā substantiā aspirāt: elemētorum libros xiii cū expositione Theonis insignis mathematici . . . . . Quibus . . . . adjuncta. Deputatum scilicet Euclidi volumē xiiii cū expositiōe Hypsi. Alex. Itidēqȝ Phaeno. Specu. Perspe. cum expositione Theonis. ac mirandus ille liber Datorum cum expositiōe Pappi Mechanici una cū Marini dialectici protheoria. Bar. Zāber. Vene. Interp̄te.*

The first printed Greek text was published at Basel, in 1533, with the title Εὐκλείδου Στοιχείων *βιβλ. ιέ ἐκ τῶν Θέωνος συνουσιῶν*. It was edited by Simon Grynæus from two MSS. sent to him, the one from Venice by Lazarus Bayfius, and the other from Paris by John Ruellius. The four books of Proclus's commentary are given at the end from an Oxford MS. supplied by John Claymundus.

The English edition, the only one which contains all the extant works attributed to Euclid, is that of Dr David Gregory, published at Oxford in 1703, with the title, Εὐκλείδου τὰ σωζόμενα. *Euclidis quæ supersunt omnia.* The text is that of the Basel edition, corrected from the MSS. bequeathed by Sir Henry Savile, and from Savile's annotations on his own copy. The Latin translation, which accompanies the Greek on the same page, is for the most part that of Commandine.

The French edition has the title, *Les Oeuvres d'Euclide, traduites en Latin et en Français, d'après un manuscrit très-ancien qui était resté inconnu jusqu' à nos jours. Par F. Peyrard, Traducteur des oeuvres d'Archimède.* It was published at Paris in three volumes, the first of which appeared in 1814, the second in 1816, and the third in 1818. It contains the *Elements* and the *Data*, which are, says the editor, certainly the only works which remain to us of this ever-celebrated geometer. The texts of the Basel and Oxford editions were collated with 23 MSS., one of which belonged to the library of the Vatican, but had been sent to Paris by the Comte de Peluse (Monge). The Vatican MS. was supposed to date from the 9th century; and to its readings Peyrard gave the greatest weight.

What may be called the German edition has the title Εὐκλείδου Στοιχεῖα. *Euclidis Elementa ex optimis libris in usum Tironum Græce edita ab Ernesto Ferdinando August.* It was published at Berlin in two parts, the first of which appeared in 1826, and the second in 1829. All the above-mentioned texts were collated with three other MSS.

Of translations of the *Elements* into modern languages the number is very large. The first English translation, published at London in 1570, has the title, *The Elements of Geometrie of the most auncient Philosopher Euclide of Megara. Faithfully (now first) translated into the Englishe toung, by H. Billingsley, Citizen of London. Whereunto are annexed certaine Scholies, Annotations, and Inventions, of the best Mathematiciens, both of time past, and in this our age.* The first French translation of the whole of the *Elements* has the title, *Les Quinze Livres des Elements d'Euclide. Traduicts de Latin en François. Par D. Henrion, Mathematicien.* The first edition of it was printed in 1614, and a second, corrected and augmented, was published at Paris in 1623. An Italian translation, with the title, *Euclide Megarense acutissimo philosopho solo introduttore delle Scientie Mathematice. Diligentemente rassettato, et alla integrità ridotto, per il degno professore di tal Scientie Nicolò Tartalea Brisciano*, was published at Venice in 1569; a Spanish version, *Los Seis Libros primeros de la geometria de Euclides. Traduzidos en lēgua Española por Rodrigo Çamorano, Astrologo y Mathematico*, at Seville in 1576; and a Turkish one at Bulak in 1825. Dr Robert Simson's editions of the first six and the eleventh and twelfth books of the *Elements*, and of the *Data*, which form the basis of all the modern school texts of Euclid, are so common that it is not considered necessary to describe them.

Authorities.—The authors and editions above referred to; Fabricii *Bibliotheca Græca*, vol. iv.; Murhard's *Litteratur der Mathematischen Wissenschaften*; Heilbronner's *Historia Matheseos Universæ*; De Morgan's article "Eucleides" in Smith's *Dictionary of Biography and Mythology*. (J. S. M.)

EUCLID, of Megara, a Greek philosopher, the founder of the Megarian school, was born in the latter half of the 5th century B.C., probably at Megara, though Gela in Sicily has also been named as his birth-place. He was one of the most devoted of the disciples of Socrates. If we may believe Aulus Gellius, such was his enthusiasm that, when a decree was passed forbidding the Megarians to enter Athens, he regularly visited his master by night in the disguise of a woman; and he was one of the little band of intimate friends who had the privilege of listening to the hero's last discourse. After his master's death, he withdrew, with a number of his fellow-disciples, to Megara; and it has been conjectured, though there is no direct evidence, that this was the period of Plato's residence in Megara, of which indications appear in the *Theætetus*. The fundamental principle of Euclid's philosophy was a combination of the Eleatic conception of Being—the One and All, and the Socratic conception of the Good. Being is immaterial and unchangeable, and is identical with the Good, which is the same as God, as Reason, and (following the Socratic doctrine) as Wisdom, and which alone truly exists. Thus the existence of evil was denied; and the main object of the Megarian, as it was of the Eleatic dialectic, was to prove the conceptions of division, number, becoming, motion, and possibility to be self-contradictory and false. With Plato, Euclid taught that sense has cognizance of the changeable and unreal only, while thought penetrates to unchangeable Being, to the Good. The Megarian school prided itself first of all upon its dialectic. Euclid's dialectic differed greatly from that of his master Socrates, in marked contrast to whom he repudiated the principle of analogical reasoning as unsound. His favourite method of attacking an opponent was by the *reductio ad absurdum*, which was also a favourite method with his followers, whose arguments degenerated into trivial sophisms, which laid them frequently open to an attack with their own weapon, and which earned for them the contemptuous name of the Ἐριστικοί or "wranglers." Of Euclid's followers the chief were Eubulides, who taught Demosthenes, wrote against Aristotle, and invented several trifling but ingenious paradoxes, of which the most famous is the *Sorites*; Diodorus Chronus, the author of certain arguments to prove the impossibility of motion; Philo; and, most famous of all, Stilpo, who was distinguished by the attractiveness of his lectures.

Our knowledge of Euclid's philosophy is borrowed from scattered passages in Plato, and from Diogenes Laertius. See Zeller, *Socrates and the Socratic Schools*; Dyeck, *De Megaricorum Doctrina* (Bonn, 1827); Mallet, *Histoire de l'École de Mégare* (Paris, 1845); Ritter, *Ueber die Philosophie der Meg. Schule*; Prantl, *Geschichte der Logik*, i., 33; Henne, *L'École de Mégare* (Paris, 1843).

EUDOCIA, the wife of Theodosius II., was the daughter of the Athenian sophist Leontius, or Leon. It is impossible to fix the date of her birth more precisely than in the last decade of the 4th century, though by an inference from a statement of Nicephorus Callistus (xiv. 50) the year 393–4 has been fixed upon. She was called Athenais prior to her conversion to Christianity. By her father she was carefully instructed in literature and the sciences; and so high an estimate did the philosopher form of her beauty and merit that, thinking any other endowment unnecessary, he divided his whole patrimony between his two sons. Athenais, however, resented this as an injustice, and carried her plea to Constantinople before the emperor. Here she gained access to Pulcheria, the sister of Theodosius, and by her she was secretly destined to be the wife of the emperor. The probable date of her marriage is 421. Before her elevation to the throne, she renounced paganism and was baptized. It was not, however, till the birth of a daughter that she received the title of Augusta (423). Her brothers she not only forgave, but raised to the dignity of consuls and præfects. About 438 Eudocia made an ostentatious pilgrimage to the Holy Land, distributing alms and donations for pious purposes with a munificence which exceeded that of the great Helena, and she returned to Constantinople in the following year with precious relics of St Stephen, St Peter, and the Virgin. Her peace, however, was soon after disturbed by the jealousy of her husband, on account, it is said, of his observing a beautiful apple which he had presented to her in the hands of Paulinus, his master of the offices. The execution of the supposed favourite, and the retirement of Eudocia in 449 to Jerusalem, did not appease the anger of the emperor, who despatched a messenger for the purpose of putting to death two ecclesiastics who had gained her confidence. The assassination of his envoy provoked the emperor still further, and Eudocia was stripped of her royal honours, and degraded in the eyes of the nation. In Jerusalem Eudocia became infected with the Eutychian heresy, and through her influence it made considerable progress in Syria, but the misfortunes of her daughter Licinia Eudocia led her to obtain a reconciliation with Pulcheria, and through her mediation and that of her brothers she afterwards returned to the communion of the church. She died at Jerusalem about 460, and was buried in the church of St Stephen. With her latest breath she protested that she had never transgressed the bounds of innocence and friendship. Eudocia continued through life to cultivate her early literary tastes. She composed a paraphrase on the Octateuch in heroic verse, a paraphrase of Daniel and Zechariah, and a poem on the martyrdom of St Cyprian. To these are added a poem on her husband's victory over the Persians, and, according to Zonaras, a cento of the verses of Homer applied to the life and miracles of Christ, but her authorship of the latter is generally disputed by critics.

EUDOCIA AUGUSTA, of Macrembolis, lived in the second half of the 11th century. She was the wife of the emperor Constantine XI., and after his death of Romanus IV. She had sworn to her first husband on his deathbed not to marry again, and had even imprisoned and exiled Romanus, who was suspected of aspiring to the throne. Perceiving, however, that she was not able unaided to avert the invasions which threatened the eastern frontier of the empire, she revoked her oath, married Romanus, and with his assistance dispelled the impending danger. She did not live very happily with her new husband, who was warlike and self-willed, and when he was taken prisoner by the Turks she was compelled to vacate the throne in favour of her son Michael and retire to a convent, where she died at an advanced age. She compiled a dictionary of mythology entitled Ἰωνιά (*Collection of Violets*), which has been published by Villoison in his *Anecdota Græca*, Venice, 1781.

EUDOXUS, a physical philosopher, was a native of Cnidus, and flourished about the middle of the 4th century B.C. It is chiefly in his quality of astronomer that his name has descended to our times. What particular service he rendered to that science beyond introducing the Egyptian sphere into Greece, and correcting the length of the year, cannot now be ascertained. Of his personal history it is known, from a life by Diogenes Laertius, that he studied at Athens under Plato, but being dismissed by that philosopher, passed over into Egypt, where he remained for sixteen months, and that he then went to Cyzicus and the Propontis, where he taught physics, and ultimately migrated with a band of pupils to Athens, where he died in the fifty-third year of his age. Eudoxus is frequently referred to by ancient writers. Strabo attributes to him the introduction of the odd quarter day into the year. According to Vitruvius he invented a solar dial. The *Phænomena* of Aratus is a poetical account of the astronomical observations of Eudoxus. Several works have been attributed to him, but they are all lost.

EUDOXUS, of Cyzicus, a Greek navigator who flourished about 130 B.C. He was employed by Ptolemy Euergetes to make a voyage to India. After two of these he circumnavigated Africa from the Red Sea to Gades. An attempt to make the return voyage was unsuccessful.

EUGENE, FRANÇOIS (1663–1736), commonly called PRINCE EUGENE OF SAVOY, one of the greatest generals of his time, born at Paris on the 18th October 1663, was the fifth son of Eugene Maurice, count of Soissons, who was grandson of the duke of Savoy, Charles Emmanuel I., and of Olympia Mancini, niece of Cardinal Mazarin. Originally destined for the church, Eugene was known at court as the *petit abbé*; but his own predilection was strongly for the army. His mother, however, had fallen into disgrace at court, and his application for a commission, repeated more than once, was refused by the king, Louis XIV., prompted probably by the minister Louvois. This engendered in him what proved to be a life-long resentment against the king and his native country. Having quitted France in disgust, he proceeded to Vienna, where the emperor Leopold, who was allied to his family, received him kindly, and granted him permission, along with several other Frenchmen of distinction, to serve against the Turks